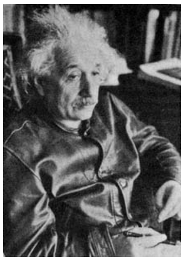
Relativity



Albert Einstein

- 1879-1953
- Was a patent office clerk in 1905.
  - This was *his* annus mirabilus, a comparable period of intense activity, comparable to Newton's in 1666, when as a young man he grasped the essential insights in a number of different subjects.

Albert Einstein, 2

- In his miracle year, Einstein published 5 papers:

1. He finished his doctoral thesis and published it.
2. He wrote a paper on Brownian motion, showing that it is visible evidence for the atomic theory of matter.
3. He explained the photoelectric effect, whereby shining a light on certain metals causes electricity to flow.
   - Characterized it as light energy knocking electrons out of matter.
   - For this he eventually got the Nobel Prize.

- and two more…

Albert Einstein, 2

- And the remaining two papers:

4. He wrote an obscure paper entitled "On the Electrodynamics of Moving Bodies."
   - This became the basis of the special theory of relativity.
5. A few months later he published a continuation of the electrodynamics paper, in which he expressed the relationship between matter and energy by the famous formula, $E=mc^2$

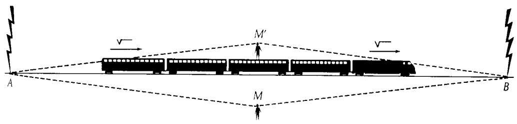
Questions about motion

- Einstein had a long-standing interest in questions about the laws of physics as they applied to objects in motion.
- Newton's unverifiable concepts of absolute time and space troubled him.
- Likewise the Maxwell theory that light was a wave motion passing through an immobile æther.

Thought experiments

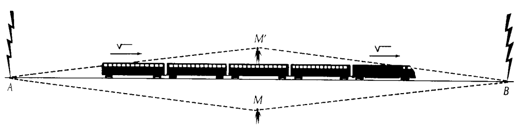
- Einstein's thought experiments.
  – Just as Galileo had explored Aristotle's physics with theoretical situations that revealed inconsistencies, Einstein used his imagination to show that the Newtonian world view led to paradoxes in quite ordinary phenomena.

The Train Station Experiment



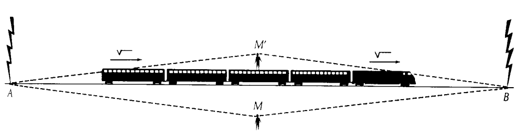
- A straight railway line runs through the station shown above. Points A and B are at opposite ends of the station platform. There are light fixtures at both ends.

The Train Station Experiment, 2



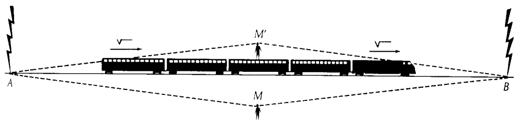
- A man is standing on the platform at point M, holding a set of mirrors joined at right angles so that he can see the lights at A and B at once.

The Train Station Experiment, 3



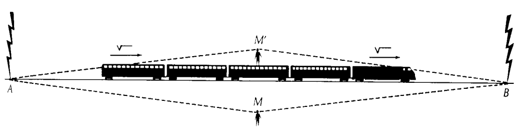
- Now suppose that the man M is looking into the mirrors and sees the lights at A and B flash on at the same time.
- M can say that the lights came on *simultaneously.*

The Train Station Experiment, 4



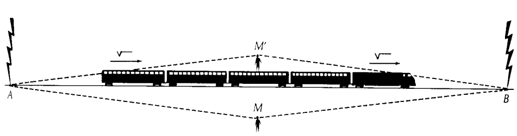
- Now imagine an express train coming through the station and not stopping. Suppose that a woman, M′, is on the train, leaning out a window, equipped with the same angled mirror device that M had.

The Train Station Experiment, 5



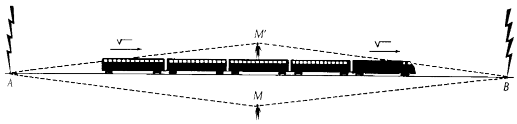
- Suppose that M′ also sees the same flashes of light that M saw.
- Will she see them at the same time?

The Train Station Experiment, 6



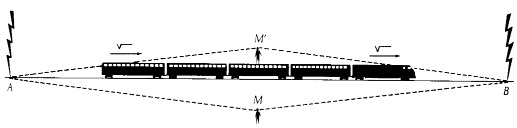
- According to Einstein, she won't.
- If light is an undulation of the æther that travels at a constant speed, it will take a certain amount of time for the light to travel from A and B toward M′.
- Meanwhile, the train, carrying observer M′, is moving toward B and away from A.

The Train Station Experiment, 6



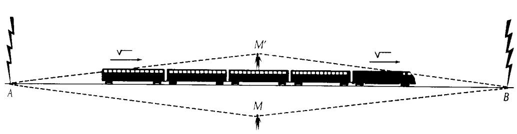
- If the flashes happened at the "same time" at A and B, then while the light was travelling toward M′, she was moving toward B.
- Therefore, she will see the light at B first, and will say that the flashes were *not* simultaneous.

The Train Station Experiment, 7



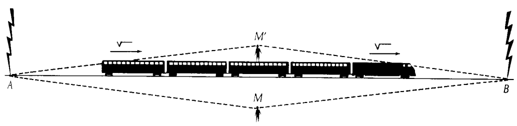
- One *could* say that the lights are on the platform and the man at M is midway between them, and it is the train that is moving, so he is right and she is wrong.
- But that requires further information about the placement of the man at M, and requires knowing that A and B are equidistant.

The Train Station Experiment, 8



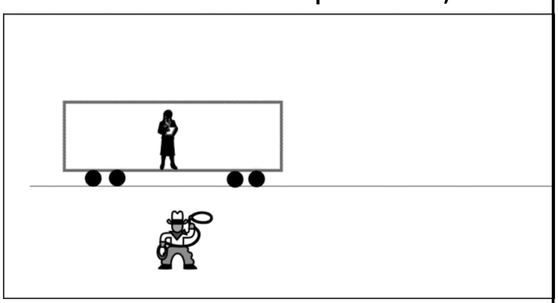
- To make the case more general, let the light flashes be lightning bolts that are in the directions A and B, but how far away is unknown.
- Now it is not so easy to say that the man at M was right.
  - The flash from A could have been much closer than the one from B, and would take less time to reach M.

The Train Station Experiment, 9



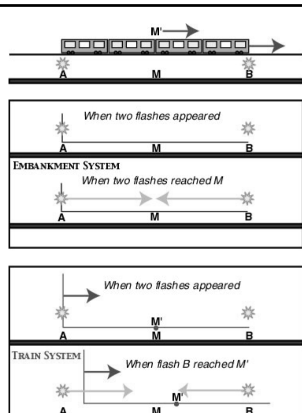
- Or, the flashes could have come from the front and back of the train and were therefore moving *with* observer M'.
- If all we know is that M saw them at the same time while M' saw them at different times, then the flashes were simultaneous *for* M and not simultaneous *for* M'.

The Train Station Experiment, 10



- An animation of the thought experiment, using lightning flashes.

The Train Station Experiment, 11



- Yet another illustration of the same.

The Train Station Experiment, 12

- What is the point here?
- Both the train station (sometimes simply called the embankment) and the train itself are frames of reference.
  - One can identify one's place in either without reference to the other.
- Each frame of reference interprets the time of events differently because they perceive them differently.
- No frame of reference can claim to have priority over another. Each is entitled to measure distance, time, and any other quantity with reference to it own reference points.

Special Relativity

- Einstein was much influenced by Ernst Mach's positivism and was inclined to discard notions from science that could not be independently detected and measured.
- Such a notion was absolute time and absolute space.
- Instead, Einstein suggested that physical theory should start with the observations that are verified.

Special Relativity, 2

- Einstein proposed a new systematic way of studying frames of reference that move with respect to each other.
- He began with the curious result of the Michelson-Morley experiment – that the speed of light appears to be the same in all frames of reference.

Special Relativity, 3

- His system is set out axiomatically, beginning with:

1. The *speed of light* is a constant in all frames of reference, moving inertially with respect to each other.
2. There is no such thing as absolute motion, or place, or time.
   - There is no privileged frame of reference.

Special Relativity, 4

- Note that Einstein begins with a definition of what will remain the same at all times – the speed of light.
  - Light is therefore an *invariant.*
  - It is essential in scientific theories that *invariants* are specified – things that remain the same while other things change.

Special Relativity, 5

- Concepts that become relative:
  - Simultaneity
    - Happening "at the same time" is not an absolute concept, but one that is relative to a frame of reference.
  - Time itself (i.e., duration)
    - Time moves more slowly for an object that is moving with respect to another object.

Special Relativity, 6

- Length (distance)
  - Distances are only determinable within a frame of reference.
  - Einstein accepted the FitzGerald-Lorentz explanation of the Michelson-Morley experiment, that matter shrinks in the direction of its motion by the factor of

$$\sqrt{1 - v^2/c^2}$$

Special Relativity, 7

- The upper limit of the speed of light:
  - Note that if the speed of the frame of reference, *v*, is the same as the speed of light, *c*, then the shrinkage factor becomes zero.

$$\sqrt{1 - v^2/c^2}$$

  - That is, at the speed of light everything shrinks to zero length. Hence the speed of light is an upper limit.

Special Relativity, 8

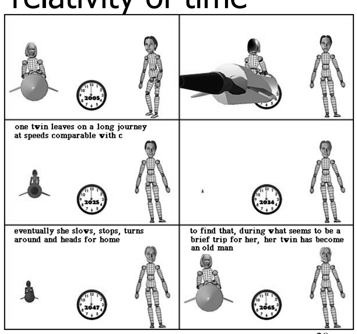
- Mass
  - Another thing which becomes relative is the mass of a body.
  - The greater the speed of a body (i.e., the greater the speed of its frame of reference is compared to another frame of reference), the larger will its mass be.
  - The mass of a body is a measure of its energy content.

Special Relativity, 9

- Energy
  - Finally, mass and energy are not independent concepts.
    - This was the subject of Einstein's continuation paper in 1905.
  - When a body radiates energy (for example, a radioactive body) of amount E, it loses mass by an amount E/c^2
  - Therefore, in principle $m = E/c^2$
  - Or, more familiarly, $E = mc^2$

The Twin Paradox – The relativity of time

- In this version, there are two twins, Jane and Joe, 25 years old.
- Jane, an astronaut travels on a long space journey to a distant location, returning, by her calculations, 5 years later. She is then 30 years old. However, on her return, she finds that Joe is 65 years old.



General Relativity

- Special Relativity concerns frames of reference that move inertially with respect to each other.
  - In a straight line and at constant speed.
- This is a special case.
- *All motion that is not inertial is accelerated.*

General Relativity, 2

- In 1905, Einstein confined his thinking to inertial frames of reference, but inertial motion is the exception, not the rule.
- For the next several years he pondered the laws of physics as they applied to bodies that were speeding up, slowing down, and changing direction.
- In 1916, he published a far more revolutionary revision of Newton's physics which we call *General Relativity.*

Acceleration and Gravity

- In Newton's physics, *inertial motion* is not perceived as different from rest.
- *Acceleration* is perceived as *an effect on inertial mass* due to a force impressed.
  - Viz., Newton's second law, $F = ma$

Acceleration and Gravity, 2

- Mass
  - Curiously, the concept of mass has two alternate measures in Newtonian physics.
  - *Inertial mass* is measured as resistance to change of motion (acceleration).
  - *Gravitational mass* is measured as attraction between bodies, causing acceleration.
  - But inertial mass = gravitational mass.
- Inertial and gravitational mass are equal in value and ultimately measured by the same effect: acceleration.

Acceleration and Gravity, 3

- The Positivist viewpoint:
  - Since the inertial and gravitational masses of a body have the same value in all cases, they must be equivalent.
  - Since the measure of gravitation is acceleration, these concepts must be equivalent.
- Einstein's thought experiment
  - In typical Einstein fashion, he explored this idea with a thought experiment.
  - He looked for a case where acceleration and gravity should produce different effects according to classical (i.e., Newtonian) physics.

Einstein's Elevator

- Einstein's choice for this thought experiment is an elevator.
  - I.e., a closed room that moves due to a force that cannot be seen from within the elevator.
  - A person riding in an elevator can see the effects of the forces causing motion, but cannot determine what they are.

Einstein's Elevator, 2

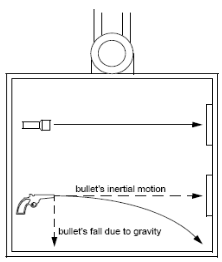
- Consider a man standing in an elevator (with the doors closed) and feeling his weight pushing down on his feet.
- This is the normal sensation if the elevator is sitting on the surface of the Earth and not moving.
- However, it would be the exact same sensation if the elevator were out in space, away from the gravitational pull of the Earth, and was accelerating upward at 9.8 m/s^2

Einstein's Elevator, 3

- The man in the elevator really cannot tell whether he is on the ground, his (gravitational) mass pulled by gravity, or accelerating through space and his (inertial) mass pushed against the floor of the elevator.
- But, if Newton is correct, he can test for this…

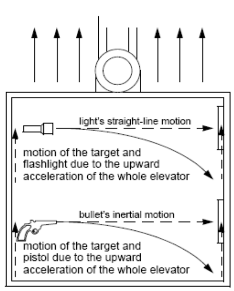
Einstein's Elevator, 4

- If the elevator is in a gravitational field, there should be a difference between the path of a ray of light, and a projectile with gravitational mass, such as a bullet.
- The man can shine a flashlight straight across the elevator at a target and it should hit it exactly, since light travels in straight lines.
- But a bullet shot straight across will (theoretically) fall in a parabolic arc since it will be attracted downward by gravity during its flight.



Einstein's Elevator, 5

- Conversely, if the elevator is accelerating out in space, both the light ray and the bullet will miss the target because, while they both travel across the elevator in a straight line, the elevator is accelerating upward, raising the target above the line that the light and bullet travel on.



Einstein's Elevator, 6

- But Einstein reasoned that this would only be true if there was a difference in kind between inertial and gravitational mass.
- Since they always equaled the same amount for any body, he argued, in Positivist fashion, that they must behave the same.
- Therefore, he argued, light must also curve in a gravitational field, though only by a very slight amount, which is why it had not been detected.

The Bending of Starlight

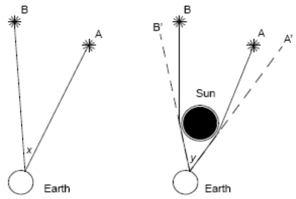
- The curving of light in the presence of a gravitational mass would be very, very slight, so Einstein needed to find an example in Nature that was on a scale that could be detected.
- He chose to predict the bending of starlight as it passes by the Sun.

Starlight during a solar eclipse

- On an ordinary night one can view any pair of distant stars and measure the apparent angle between them.
- During the day, the very same stars may be in the sky, but we cannot see them due to the sun—except during a solar eclipse.

Starlight during a solar eclipse, 2

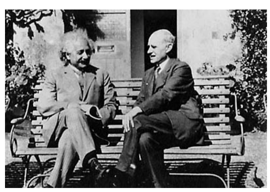
- In 1919, Einstein predicted that the Sun would bend the light from distant stars by 1.7 seconds of arc during a solar eclipse.
- This was confirmed by the astronomer Sir Arthur Eddington on the island of Principe, off the west coast of Africa.



Eddington's observations confirmed Einstein's prediction that starlight would be visibly bent by the gravitation effect of the Sun. The apparent angle between stars A and B as seen from the Earth is usually *x*. During a solar eclipse, however, the sun bends the paths of their light, increasing their apparent angular separation to *y* (1.7 seconds of arc greater than *x*), and causing the stars to appear at A' and B'.

Einstein becomes famous

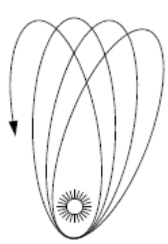
- The results of the expedition were dramatically announced at a joint meeting of the Royal Society and the Royal Astronomical Society in 1919.
- Einstein instantly became a household name and a synonym for genius.



Einstein and Eddington.

Further confirmation

- The elliptical orbits of planets do not remain in a single place, but themselves slowly revolve around the Sun. This is accounted for by Newton, but Mercury's orbit was changing more swiftly than Newton predicted.
- Einstein showed that the extra amount by which Mercury's orbit advanced was predicted by relativity.



An exagerated diagram of the orbit of Mercury showing the advance of the perihelion.

The Curvature of Space

- Light travels along the shortest path at the greatest possible speed.
- The shortest path is a straight line only in Euclidean (flat) geometry.
- In geometry of curved space, the shortest path is a *geodesic.*
- Space is curved by the presence of mass.
- Gravity, then, is not a force, but a curvature of space due to the presence of matter.

Plato lives!

- Pythagoras too!
- If matter is what causes space to curve (which we feel as gravity), maybe matter is really only highly curved space.
- Therefore matter is really geometry, i.e. mathematics.
- Energy is an abstraction known only by its measurable effect, which is also mathematical.
- Hence, *all reality is ultimately just mathematics*.