Mathematics in Mesopotamia & Babylonia

The Fertile Crescent

- The first area of the world to move to an agricultural way of life was near where Africa, Europe, and Asia meet.

Why There?

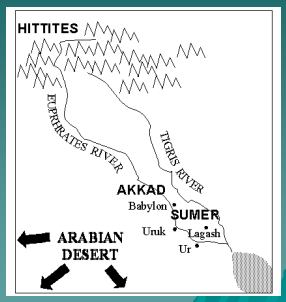
- The "Fertile Crescent" is fertile, because rivers from all three continents drain into it, bringing rich nutrients to the soil as well as water.
- It was also in an area of moderate climate, perfect for growing crops.
- For those reasons it was also more heavily populated.

Mesopotamia

- The world's first great civilization arose in the Fertile Crescent.
  - This happened in particular in an area that is now part of Iraq.
  - This area is bounded by the Tigris and the Euphrates rivers.
  - The Greeks called it Mesopotamia, meaning "between the rivers."

Sumer

- The first notable civilization in Mesopotamia was *Sumer.*
- It was located in southern Mesopotamia, near where the rivers join.



Early Settlements

- The earliest known settlement in the Euphrates floodplain dates from before 5000 BCE. It is characterized by large villages and temples.
- Relatively rapid development is attributed to the use of irrigation.

Historical Sumer

- The known history of Sumer begins in 2900 BCE, with the invention of writing.

Cuneiform

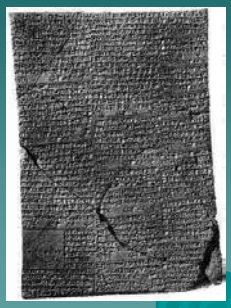
- The Mesopotamian area was rich in clay and in reeds that grew on the river banks.
- They were combined to make a medium for writing.

Cuneiform, contd.

- Clay was formed into a slab, about the size of a human hand.
- The reed stalks were cut to make a stylus.
- The stylus was pushed into the wet clay in a variety of different ways to make recognizable marks, carrying meaning.

Cuneiform, contd.

- The clay tablets were left to dry in the sun, and became very durable.
- There are thousands of cuneiform tablets still in existence.
- Hence, much is known about the history of these settlements.



Sumerian Culture

- The Sumerians were very well organized and had a complex bureaucracy, ruled by the priests in the temples.
- All the major trades of pre-industrial times developed there.

The Downside of Sumerian Culture

- The trend to rapid urbanization and blight of the environment that followed was characteristic.
- Fertile soils were quickly depleted by over use.
- Over-irrigation led to salinization.
- The accumulation of wealth attracted raiders. The area has been a battleground ever since.

Babylonia

Babylonia

- Babylonia is a civilization that developed in Mesopotamia around 1800 BCE, succeeding the Sumerian civilization, which had collapsed by then.
- The Babylonians used the cuneiform system of writing on clay tablets with reed styluses.

Babylonian Interests

- The Babylonians had a complex and prosperous culture, and pursued many interests.
- Because of the durability of cuneiform tablets, much is known about their civilization.

Babylonian Astronomy

- Some of the earliest, reasonably reliable records of the positions of the stars and planets were made by Babylonians, who developed a complex system of recording them.

Mespotamian Numbers

- Throughout the Mesopotamian civilizations, from Sumer to Babylonia, a unique number system was used based on the number 60, not on the familiar base 10 used in most other cultures.

Sexagesimal Numbers

- In the sexagesimal, i.e. 60-based, system, there are different combinations of characters for each number from 1 to 59.
- Then the symbol for 1 is used again, but this time meaning 60.
  - The symbol for 2 also means 120. The symbol for 3 also means 180, etc.

A Place-Value System

- Compared to the Egyptians, who had totally separate symbols for 2 and 20 and 200 and 2000, etc., the Mesopotamian/Babylonian system used the same symbols over for the next higher level.
- Note that we do the same, but we place zeros behind them to indicate the level.

Using the marsh reeds as a stylus

- Mesopotamian writing was done on wet clay tablets, by pushing the end of a reed stalk into the clay.

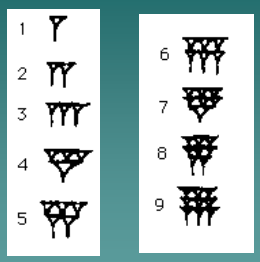
Two Characters Only

- Though there are 59 separate symbols for the numerals in a sexagesimal system, the Babylonian numbers are all written with only two different characters, but put together in different combinations.

Vertical – the Character for 1

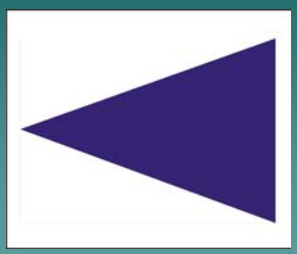
- If the reed is turned with the thick end up and the pointed end down, it is the symbol for 1.

The Numbers from 1 to 9



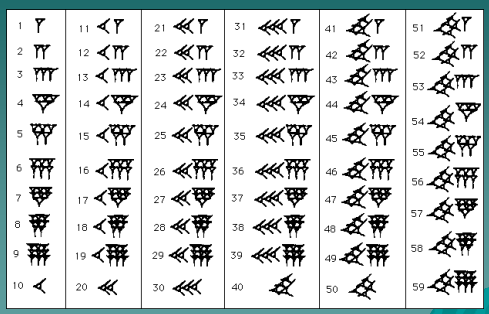
Horizontal – the Character for 10

- If the reed is turned with the thick end to the right and the pointed end to the left, it is the symbol for 10.



Counting by Tens: 10, 20, 30, 40, 50

The Numbers from 1 to 59



What comes after 59?

- 60 in the sexagesimal number system is the basic unit at the next place value.
- So it looks just like 1.
- That is, 60 = 1 x 60

Example:

- A 9 times multiplication table.

Why choose a base of 60?

- Most cultures have number systems based on 10, or perhaps 5, related to the digits on our hands.
- But 10 is a poor choice for dividing evenly into parts.
- It is only divisible by 1, 2 and 5.

Factors of 60

- The number 60 can be evenly divided by many more smaller numbers:
- 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30.
- Fractional parts are much easier to express exactly.

Fractions

- Any unit can be divided into parts of a lower place value, by dividing it by 60.
- Just as:
  - 1 minute = 60 seconds
  - ½ of a minute = 30 seconds
- Seconds is the next lower division of time after minutes.

The Sexagesimal System Today

- We still use the 60-based counting system in two places
  - Keeping time in hours, minutes, and seconds.
  - Measuring angles in degrees, minutes and seconds.

Why?

- Time-keeping and detailed astronomical observation came from the Babylonians.
- Greek science made use of Babylonian data and kept their number system for that purpose.

Place Value, with Place Holder

- In our decimal base system, we use the same numerals over and over again to mean numbers of different sizes.
  - But we can tell which size is intended by the use of zeros and decimal places.
  - E.g., 27900 is bigger than 279
  - 98.6 is smaller than 986

Place Value, but No Place Holder

- In the Mesopotamian/ Babylonian system, numbers that are 60 times larger or 60 times smaller are all written the same way.

The number 85 (as we write it) is written as 1x60 + 2x10 + 5 =

The number that we write as 1 5/12 is written as 1+ 25/60 = 1 + (2x10 + 5)/60.

This also appears as

Ambiguous in principle, but rarely in practice

- Because the orders of magnitude are separated by factors of 60, there was rarely confusion in the early centuries.
- But ultimately, this was a severe drawback in the system, as society became more complex.