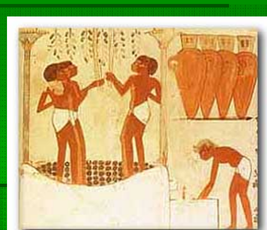
Numbers and Counting

Number

- The fundamental abstraction.
- There is archaeological evidence of counters and counting systems in some of the earliest of human cultures.
- In early civilizations, counting and measuring became necessary for administration.

Numbers and Agriculture

- Keeping track of the amount of land allocated to a farmer, the quantity of the harvest, and any taxes or duty to be paid required a well-developed system of measuring and counting.



Numbers are abstractions

- It is something to know that three sheep plus two sheep always equals five sheep.
- Or that three urns and two urns are five urns.
- It is a big step to realize that 3 of anything plus 2 more of them makes 5 of them, or, that 3+2=5.
- The pure numbers are abstractions.

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Contention

Only a civilization that has a well-developed *written* number system and has discovered rules for manipulating those numbers has the chance of moving on to science.

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A look at the number systems and rules of arithmetic of two of the great ancient civilizations:

- Egypt
- Babylonia

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Egypt

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Egypt

- Egypt is one of the world's oldest civilizations.
- The "Ancient period" was from about 3000-300 BCE, during which this civilization had agriculture, writing, and a number system.

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The Gift of the Nile

- The settled area of Egypt is a narrow strip of land along the shores of the Nile River.
- Egypt would not be possible without the waters of the Nile.

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An insular, protected country

- Because of Egypt's isolation from possible invaders, it was able to develop into a stable, prosperous country through agriculture.

The Predictable Nile

- The Nile river flooded every year in July.
- The floods provided rich nutrients and silt that made very productive soil.

Farmers and Scribes

- Egypt subsisted on organized and centralized farming in the area flooded annually by the Nile.
- Tracking and managing the allocation of land required extensive record-keeping, and written language.

Hieroglyphics

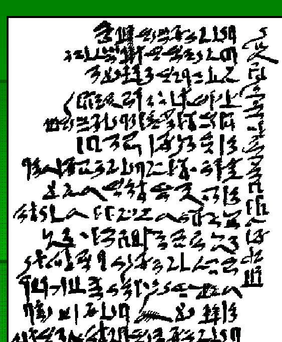
- Egypt developed a pictorial writing system called *hieroglyphics.*
- (This is from the entrance to the Great Pyramid at Giza.)

Ceremonial Writing

- Hieroglyphics were used for permanent messages.
- Some were carved inscriptions on monuments and buildings.
- Others were painted on the inside walls of buildings and tombs.

Hieratic

- For everyday use, a script form of hieroglyphics evolved called *hieratic.*
- *This is from a letter written about 1790 BCE.*



Papyrus Rolls

- Egyptians developed a sort of paper made from the pith of the papyrus reeds growing on the side of the Nile.
- These were made into long strips and then rolled and unrolled for use.

Egyptian Technology

- Egyptian “know-how” reflected their beliefs and needs.
- Many inventions, devices, and procedures supported their system of agriculture and the building of their many monuments.

The Cult of Death

- Much attention was paid to preparation for death and the life that would follow.
- Pharaohs and other important officials spent great sums on their tombs and the preparation of their bodies (mummification) for entry into the afterlife.

The Pyramids

- Most famous were the pyramids, built as tombs for great pharaohs.
- The great pyramids contain as many as 2,300,000 limestone blocks, each weighing 2.5 tonnes.

Practical Science

- Topics that would later be part of science were studied and mastered for practical ends:
  - Anatomy: for embalming, mummifying
  - Chemistry: for making cosmetics, paints, dyes, and food preservatives
  - Astronomy: for establishing a calendar for agriculture

Egyptian Astronomy

- The flooding of the Nile is so regular that it coincides with an astronomical event.
- When the star *Sirrius* appears in the sky just before dawn, the flooding of the Nile was imminent.

Egyptian Calendars

- The beginning of the year was when the Nile was predicted to flood, July on our calendars.
- Like most calendars, there was some coordination of the cycle of the sun and the moon.

The Earliest Egyptian Calendar

- This calendar had 12 months, alternating 29 days and 30 days.
  - The actual cycle of the moon is about 29 ½ days.
- The "year" was therefore 354 days.
- So, every 2 or 3 years, an additional month was added.

The Second Egyptian Calendar

- This had a 365-day year.
- All 12 months were 30 days long.
- Then an extra 5 days was added at the end.
- This calendar worked better for tracking the solar year, but the coordination with the moon cycle was lost.

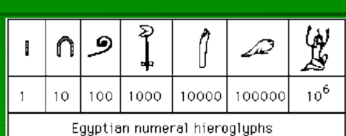
The Seasons

- The year was divided into three seasons, as suited what was important:
  - Inundation (the flooding of the Nile)
  - Emergence (of the crops)
  - Harvest

Egyptian Numbers

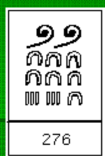
- A system of writing numbers emerged from hieroglyphics.
- A number was written as a picture of its components.
- The base of the system was 10, like ours, but the notation was completely different.

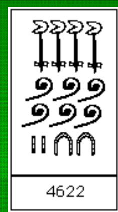
The Notation System



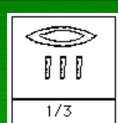
- Each power of 10 had a separate symbol.
- The order in which the symbols of a number was written was not important; i.e. no place value.

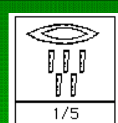
Examples of Written Numbers:





Fractions

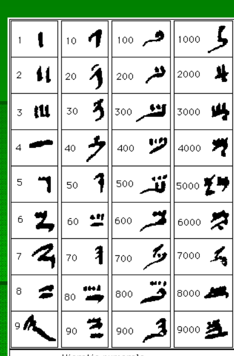




- All fractions represented a single part of a larger whole, e.g. 1/3 and 1/5, as above. (There was an exception made for 2/3.)
- The symbol for a fraction was to place an open mouth above the denominator.

Hieratic numbers

- The number system was cumbersome, so a shorthand version was developed for use in Hieratic.
- But the Hieratic version had even more symbols, and still no place value.
- 1, 2, 3, …, 10, 20, 30, …, 100, 200, 300, … all were separate symbols.



Hieratic numerals

Egyptian Arithmetic

- Despite the cumbersome notation system, the Egyptians developed an extraordinarily efficient method of doing arithmetical calculations.

Multiplication and Division by Doubling

- Calculations were done by a series of steps requiring doubling numbers, and then adding up some of the results.
- Knowledge required: how to add, and how to multiply by two.
- *Not required:* how to multiply by 3, or 4, or 5, or any other number.

Example: 13 x 24

- In two columns, write the number 1 in the left column and one of the above numbers in the right column.
- Generally choosing the larger number to write down works best.
- In this example, the 13 will be called the "other" number.

1	24

Example: 13 x 24, contd.

- Double each of the numbers in the first line, and write the result in the next line.
- Do the same to the numbers in the new line.
- Continue until the number in the bottom left position is more than one half the other number (in this case, 13).

1	24
2	48
4	96
8	192

Example: 13 x 24, contd.

- Now, place a tick mark by numbers in the left column that add up to the other number.
- The best procedure is to start from the bottom.
- Here 8, 4 and 1 are chosen, because 8+4+1=13.

✓	1	24
	2	48
✓	4	96
✓	8	192

Example: 13 x 24, contd.

- For every line with a tick mark, copy the number in the second column out to the right.
- Add up the numbers in the right-hand column.

✓	1	24	24
	2	48	
✓	4	96	96
✓	8	192	192
			312

Example: 13 x 24, contd.

- This works because (1 x 24) + (4 x 24) + (8 x 24) =

  (1 + 4 + 8) x 24 =

  13 x 24.

✓	1	24	24
	2	48	
✓	4	96	96
✓	8	192	192
			312

Now consider a more complicated example

- This works well for larger numbers too, and compares favourably with our manual system of multiplication.
- Try the numbers 246 x 7635.

Example: 246 x 7635

- Choose the larger number to double. The doubling is more difficult, but manageable.

1	7 635
2	15 270
4	30 540
8	61 080
16	122 160
32	244 320
64	488 640
128	977 280

Example: 246 x 7635, contd.

- Tick off the entries in the left column that add to 246, write the corresponding right column entries off to the side and add them up.

	1	7 635	
✓	2	15 270	15 270
✓	4	30 540	30 540
	8	61 080	
✓	16	122 160	122 160
✓	32	244 320	244 320
✓	64	488 640	488 640
✓	128	977 280	977 280
			1 878 210

Division via Doubling

- Use the same process for division, but go about it somewhat differently.
- This time you double the divisor successively, stopping just before the number reached would be greater than the dividend.
  - *Terminology:* For 100÷25=4, 100 is the dividend, 25 is the divisor, and 4 is the quotient.

Example: 300 ÷ 14

- In two columns, write the number 1 in the left column and the divisor in the right.
- Now, double the numbers in both columns until the last entry on the right is more than half of the dividend.
- Here, the last entry is 224, since doubling it gives more than 300.

1	14
2	28
4	56
8	112
16	224

Example: 300 ÷ 14

- Place tick marks beside the entries in the *right* column that add up as close as possible to the dividend, without exceeding it.
- Then copy the numbers in the left column on the same line as the ticks into a separate column and add them up.
- This gives the quotient 21.

1	1	14	✓
	2	28	
4	4	56	✓
	8	112	
16	16	224	✓
21			

Example: 300 ÷ 14

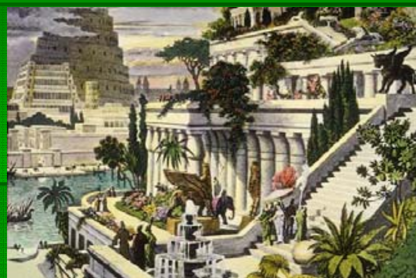
- As a check, add up the ticked numbers in the right column.
- This gives 294.
- So 14 goes into 300 a full 21 times, with a remainder of 6.
- The division process does not give exact answers but it is good enough.

1	1	14	✓	14
	2	28		
4	4	56	✓	56
	8	112		
16	16	224	✓	224
21				294

An arithmetic system for practical use

- The main problems that a scribe would have to solve were such things as determining the area of a plot of land assigned to a farmer – a multiplication problem.
- Or dividing up some commodity into equal portions – a division problem.

Babylonia



Babylonia

- Babylonia is a civilization that developed in Mesopotamia around 1800 BCE, succeeding the Sumerian civilization, which had collapsed by then.
- The Babylonians used the cuneiform system of writing on clay tablets with reed styluses.

Babylonian Interests

- The Babylonians had a complex and prosperous culture, and pursued many interests.
- Because of the durability of cuneiform tablets, much is known about their civilization.

Babylonian Astronomy

- Some of the earliest, reasonably reliable records of the positions of the stars and planets were made by Babylonians, who developed a complex system of recording them.

Mespotamian Numbers

- Throughout the Mesopotamian civilizations, from Sumer to Babylonia, a unique number system was used based on the number 60, not on the familiar base 10 used in most other cultures.

Sexagesimal Numbers

- In the sexagesimal, i.e. 60-based, system, there are different combinations of characters for each number from 1 to 59.
- Then the symbol for 1 is used again, but this time meaning 60.
  - The symbol for 2 also means 120. The symbol for 3 also means 180, etc.

A Place-Value System

- Compared to the Egyptians, who had totally separate symbols for 2 and 20 and 200 and 2000, etc., the Mesopotamian/Babylonian system used the same symbols over for the next higher level.
- Note that we do the same, but we place zeros behind them to indicate the level.

Using the marsh reeds as a stylus

- Mesopotamian writing was done on wet clay tablets, by pushing the end of a reed stalk into the clay.

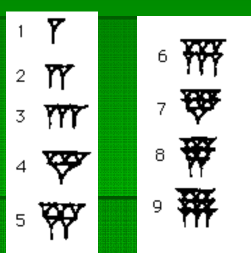
Two Characters Only

- Though there are 59 separate symbols for the numerals in a sexagesimal system, the Babylonian numbers are all written with only two different characters, but put together in different combinations.

Vertical – the Character for 1

- If the reed is turned with the thick end up and the pointed end down, it is the symbol for 1.

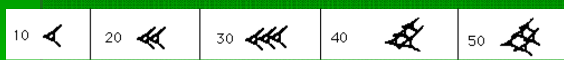
The Numbers from 1 to 9



Horizontal – the Character for 10

- If the reed is turned with the thick end to the right and the pointed end to the left, it is the symbol for 10.

Counting by Tens: 10, 20, 30, 40, 50



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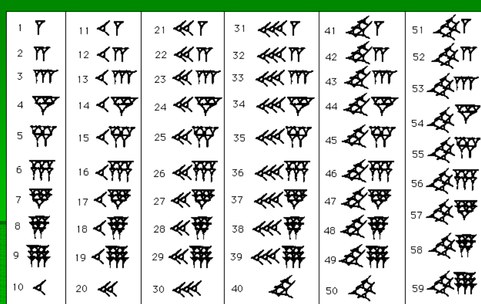
The Numbers from 1 to 59



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What comes after 59?

- 60 in the sexagesimal number system is the basic unit at the next place value.
- So it looks just like 1.
- That is, 60 = 1 x 60

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60

Example:

- A 9 times multiplication table.

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Why choose a base of 60?

- Most cultures have number systems based on 10, or perhaps 5, related to the digits on our hands.
- But 10 is a poor choice for dividing evenly into parts.
- It is only divisible by 1, 2 and 5.

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Factors of 60

- The number 60 can be evenly divided by many more smaller numbers:
- 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30.
- Fractional parts are much easier to express exactly.

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Fractions

- Any unit can be divided into parts of a lower place value, by dividing it by 60.
- Just as:
  - 1 minute = 60 seconds
  - ½ of a minute = 30 seconds
- Seconds is the next lower division of time after minutes.

The Sexagesimal System Today

- We still use the 60-based counting system in two places
  - Keeping time in hours, minutes, and seconds.
  - Measuring angles in degrees, minutes and seconds.

Why?

- Time-keeping and detailed astronomical observation came from the Babylonians.
- Greek science made use of Babylonian data and kept their number system for that purpose.

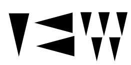
Place Value, with Place Holder

- In our decimal base system, we use the same numerals over and over again to mean numbers of different sizes.
  - But we can tell which size is intended by the use of zeros and decimal places.
  - E.g., 27900 is bigger than 279
  - 98.6 is smaller than 986

Place Value, but No Place Holder

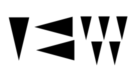
- In the Mesopotamian/ Babylonian system, numbers that are 60 times larger or 60 times smaller are all written the same way.

The number 85 (as we write it) is written as 1x60 + 2x10 + 5 =



The number that we write as 1 5/12 is written as 1+ 25/60 = 1 + (2x10 + 5)/60.

This also appears as



Ambiguous in principle, but rarely in practice

- Because the orders of magnitude are separated by factors of 60, there was rarely confusion in the early centuries.
- But ultimately, this was a severe drawback in the system, as society became more complex.