Claudius Ptolemy

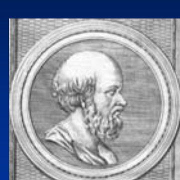
Saving the Heavens

Euclid's *Elements* at work

- Euclid's *Elements* quickly became the standard text for teaching mathematics at the Museum at Alexandria.
- Philosophical questions about the world could now be attacked with exact mathematical reasoning.

Eratosthenes of Cyrene

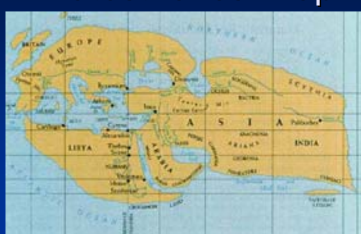
- 276 - 194 BCE
- Born in Cyrene, in North Africa (now in Lybia).
- Studied at Plato's Academy.
- Appointed Librarian at the Museum in Alexandria.



"Beta"

- Eratosthenes was prolific. He worked in many fields. He was a:
  - Poet
  - Historian
  - Mathematician
  - Astronomer
  - Geographer
- He was nicknamed "Beta."
  - Not the best at anything, but the second best at many things.

Eratosthenes' Map



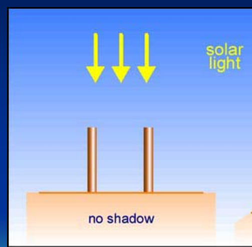
- He coined the word "geography" and drew one of the first maps of the world (above).

Using Euclid

- Eratosthenes made very clever use of a few scant observations, plus a theorem from Euclid to decide one of the great unanswered questions about the world.

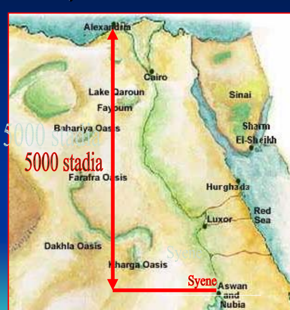
His data

- Eratosthenes had heard that in the town of Syene (now Aswan) in the south of Egypt, at noon on the summer solstice (June 21 for us) the sun was directly overhead.
  - I.e. A perfectly upright pole (a gnomon) cast no shadow.
  - Or, one could look directly down in a well and see one's reflection.



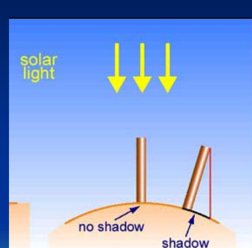
His data, 2

- Based on reports from on a heavily travelled trade route, Eratosthenes calculated that Alexandria was 5000 stadia north of Syene.

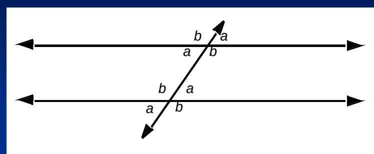


His data, 3

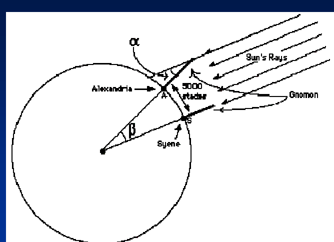
- Eratosthenes then measured the angle formed by the sun's rays and the upright pole (gnomon) at noon at the solstice in Alexandria. (Noon marked by when the shadow is shortest.)
- The angle was 7°12'.



Proposition I.29 from Euclid



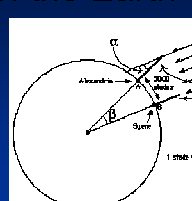
A straight line falling on parallel straight lines makes the alternate angles equal to one another, the exterior angle equal to the interior and opposite angle, and the interior angles on the same side equal to two right angles.



- Eratosthenes reasoned that by I.29, the angle produced by the sun's rays falling on the gnomon at Alexandria is equal to the angle between Syene and Alexandria at the centre of the Earth.

Calculating the size of the Earth

- The angle at the gnomon, α, was 7°12', therefore the angle at the centre of the Earth, β, was is also 7°12' which is 1/50 of a complete circle.
- Therefore the circumference of the Earth had to be stadia = 250,000 stadia.



7°12' x 50 = 360°

50 x 5000 = 250,000

Eratosthenes' working assumptions

- 1. The Sun is very far away, so any light coming from it can be regarded as traveling in parallel lines.
- 2. The Earth is a perfect sphere.
- 3. A vertical shaft or a gnomon extended downwards will pass directly through the center of the Earth.
- 4. Alexandria is directly north of Syene, or close enough for these purposes.

A slight correction

- Later Eratosthenes made a somewhat finer observation and calculation and concluded that the circumference was 252,000 stadia.
- So, how good was his estimate.
  - It depends….

What, exactly, are *stadia*?

- Stadia are long measures of length in ancient times.
- A *stade* (singular of stadia) is the length of a stadium.
  - And that was…?

Stadium lengths

- In Greece the typical stadium was 185 metres.
- In Egypt, where Eratosthenes was, the stade unit was 157.5 metres.

Comparative figures

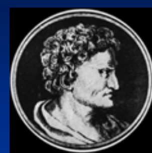
Stade Length	Circumference	
	In Stadia	In km
157.5 m	250,000	39,375
157.5 m	252,000	39,690
185 m	250,000	46,250
185 m	252,000	46,620

Compared to the modern figure for polar circumference of 39,942 km, Eratosthenes was off by at worst 17% and at best by under 1%.

An astounding achievement

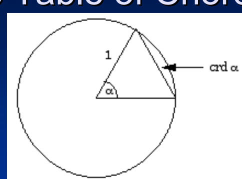
- Eratosthenes showed that relatively simple mathematics was sufficient to determine answers to many of the perplexing questions about nature.

Hipparchus of Rhodes



- Hipparchus of Rhodes
- Became a famous astronomer in Alexandria.
- Around 150 BCE developed a new tool for measuring relative distances of the stars from each other by the visual angle between them.

The Table of Chords



- Hipparchus invented the table of chords, a list of the ratio of the size of the chord of a circle to its radius associated with the angle from the centre of the circle that spans the chord.
- The equivalent of the sine function in trigonometry.

Precession of the equinoxes

- Hipparchus also calculated that there is a very slow shift in the heavens that makes the solar year not quite match the siderial ("star") year.
  - This is called precession of the equinoxes. He noted that the equinoxes come slightly earlier every year.
  - The entire cycle takes about 26,000 years to complete.
- Hipparchus was able to discover this shift and to calculate its duration accurately, but the ancients had no understanding what might be its cause.

The Problem of the Planets, again

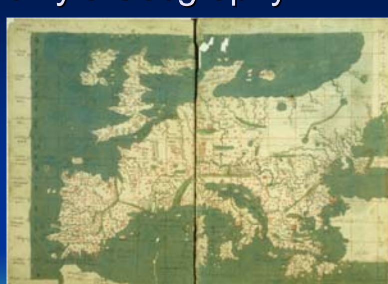
- 300 years after Hipparchus, another astronomer uses his calculating devices to create a complete system of the heavens, accounting for the weird motions of the planets.
- Finally a system of geometric motions is devised to account for the positions of the planets in the sky mathematically.

Claudius Ptolemy

- Lived about 150 CE, and worked in Alexandria at the Museum.

Ptolemy's Geography

- Like Eratosthenes, Ptolemy studied the Earth as well as the heavens.
- One of his major works was his *Geography*, one of the first realistic atlases of the known world.



The Almagest

- Ptolemy's major work was his *Mathematical Composition*.
- In later years it was referred as *The Greatest* (*Composition*), in Greek, *Megiste.*
- When translated into Arabic it was called *al Megiste.*
- When the work was translated into Latin and later English, it was called *The Almagest.*

The Almagest, 2

- *The Almagest* attempts to do for astronomy what Euclid did for mathematics:
  - Start with stated assumptions.
  - Use logic and established mathematical theorems to demonstrate further results.
  - Make one coherent system
- It even had 13 books, like Euclid.

Euclid-like assumptions

1. The heavens move spherically.
2. The Earth is spherical.
3. Earth is in the middle of the heavens.
4. The Earth has the ratio of a point to the heavens.
5. The Earth is immobile.

Plato versus Aristotle

- Euclid's assumptions were about mathematical objects.
  - Matters of definition.
  - Platonic forms, idealized.
- Ptolemy's assumptions were about the physical world.
  - Matters of judgement and decision.
  - Empirical assessments and common sense.

Ptolemy's Universe

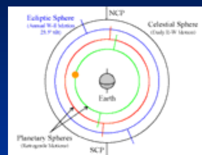
- The basic framework of Ptolemy's view of the cosmos is the Empedocles' two-sphere model:
  - Earth in the center, with the four elements.
  - The celestial sphere at the outside, holding the fixed stars and making a complete revolution once a day.
- The seven wandering stars—planets—were deemed to be somewhere between the Earth and the celestial sphere.

The Eudoxus-Aristotle system for the Planets

- In the system of Eudoxus, extended by Aristotle, the planets were the visible dots embedded on nested rotating spherical shells, centered on the Earth.

The Eudoxus-Aristotle system for the Planets, 2

- The motions of the visible planet were the result of combinations of circular motions of the spherical shells.
  - For Eudoxus, these may have just been geometric, i.e. abstract, paths.
  - For Aristotle the spherical shells were real physical objects, made of the fifth element.

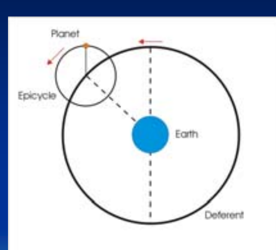


The Ptolemaic system

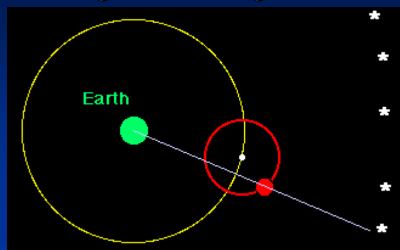
- Ptolemy's system was purely geometric, like Eudoxus, with combinations of circular motions.
  - But they did not involve spheres centered on the Earth.
  - Instead they used a device that had been invented by Hipparchus 300 years before: Epicycles and Deferents.

Epicycles and Deferents

- Ptolemy's system for each planet involves a large (imaginary) circle around the Earth, called the *deferent*, on which revolves a smaller circle, the *epicycle.*
- The visible planet sits on the edge of the epicycle.
- Both deferent and epicycle revolve in the same direction.



Accounting for Retrograde Motion



- The combined motions of the deferent and epicycle make the planet appear to turn and go backwards against the fixed stars.

Saving the Appearances

- An explanation for the strange apparent motion of the planets as “acceptable” motions for perfect heavenly bodies.
  - The planets do not start and stop and change their minds. They just go round in circles, eternally.

How did it fit the facts?

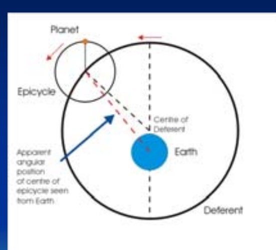
- The main problem with Eudoxus’ and Aristotle’s models was that they did not track that observed motions of the planets very well.
- Ptolemy’s was much better at putting the planet in the place where it is actually seen.

But only up to a point….

- Ptolemy's basic model was better than anything before, but still planets deviated a lot from where his model said they should be.
- First solution:
  - Vary the relative sizes of epicycle, deferent, and rates of motion.

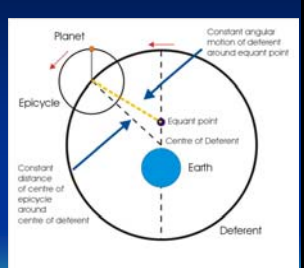
Second solution: The Eccentric

- Another tack:
- Move the centre of the deferent away from the Earth.
- The planet still goes around the epicycle and the epicycle goes around the deferent.



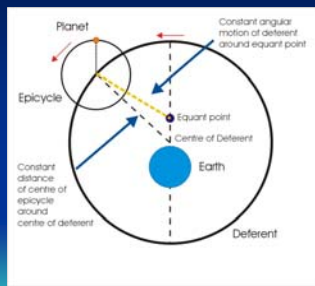
Third Solution: The Equant Point

- The most complex solution was to define another "centre" for the deferent.
- The equant point was the same distance from the centre of the deferent as the Earth, but on the other side.



Third Solution: The Equant Point, 2

- The epicycle maintained a constant *distance* from the physical centre of the deferent, while maintaining a constant *angular motion* around the equant point.



Ptolemy's system worked

- Unlike other astronomers, Ptolemy actually could specify where in the sky a star or planet would appear throughout its cycle – within acceptable limits.
- He "saved the appearances."
  - He produced an abstract, mathematical account that explained the sensible phenomena by reference to Platonic forms.

But did it make any sense?

- Ptolemy gave no reasons why the planets should turn about circles attached to circles in arbitrary positions in the sky.
- Despite its bizarre account, Ptolemy's model remained the standard cosmological view for 1400 years.