Projectile Motion

- One scientific topic of great interest in the Renaissance was motion.
- All kinds of motion were of interest, but a particular problem was the explanation of projectile motion – objects flying through the air.
- Ancient science tried to explain all motion as either
  - Some sort of stability, e.g. focus on planetary orbits rather than on the moving planet itself.
  - Or, as the result of direct contact with a motive force.

Aristotle on motion

- Aristotle divided motions on Earth into three categories:
  - Natural motion – objects seeking their natural place.
  - Forced motion – objects being pushed or pulled.
  - Voluntary motion – objects moving themselves, e.g. animals.
- These reasonable categories ran into difficulty explaining projectile motion.
Aristotle’s Antiperistasis

- A projectile, e.g., an arrow, was not deemed capable of voluntary motion. Therefore its motion must be either natural or forced (or a combination).
- Natural motion would take the (heavy) arrow down to the ground. Forced motion required direct contact.
- Solution: Antiperistasis. The flying arrow divided the air before it, which rushed around to the back of the arrow and pushed it forward.

The Search for the Aristotelian Explanation

- Aristotle’s explanation was so unsatisfactory that Scholastic philosophers through the Middle Ages tried to find a better explanation.
- Impetus theory.
  - The idea that pushing (or throwing, shooting, etc.) an object imparted something to it that kept it moving along.

Niccolo Tartaglia

- 1500-1550
- Mathematics teacher
- Wrote The New Science
  - Analyzed the path of cannon balls
  - Found that a cannon will shoot farthest aimed at 45 degrees
- Translated Euclid and Archimedes in 1543
  - That date again. (Copernicus, Vesalius)
- Was the teacher of Galileo’s mathematics teacher
The Goal of Science: How, not Why

- Aristotelian philosophy had as its goal to explain everything.
  - To Aristotle, a causal explanation was not worth much unless it could explain the purpose served by any thing or action.
    - E.g. A heavy object fell in order to reach its natural place, close to the centre of the universe.
- Galileo argued for a different goal for science:
  - Investigate *How* phenomena occur; ignore *Why*.

Galileo on Falling Bodies

- The Leaning Tower demonstration showed that Aristotle was wrong in principle about heavier bodies falling faster than lighter ones.
  - Actually, they did, but only slightly.
- Galileo applied Archimedes' hydrostatic principle to motion.
  - Denser objects fall faster because less buoyed by air.
  - Hypothesis: In a vacuum a feather would fall as fast as a stone.
- How could this be tested?

The Idealized Experiment

- Problem of testing nature:
  - Getting accurate measurements.
  - Nature's imperfections interfere with study of natural principles.
- Solution:
  - Remove imperfections to the extent possible
  - Make a nearly perfect model on a human scale (to aid measurement).
Galileo's inclined plane: The first scientific laboratory instrument

- To study falling bodies, Galileo invented a device that would slow the fall enough to measure it.
- Polished, straight, smooth plane with groove, inclined to slow the downward motion as desired.
- Smooth, round ball, as perfectly spherical as possible.

Galileo's inclined plane experiment

- Roll a smooth, round ball down a polished, straight, smooth path. Incline the path as desired to slow or speed up the fall of the ball.
- Results: A fixed relationship between distance rolled and time required.

The amazing results

- What astounded Galileo was that he found a simple numerical relationship between the distance the ball rolled down the plane and the time elapsed.

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>Distance rolled in interval $n$</th>
<th>Total Distance</th>
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<tbody>
<tr>
<td>$1^{st}$</td>
<td>1d</td>
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<td>$3^{rd}$</td>
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<td>7d</td>
<td>16d</td>
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<td>$5^{th}$</td>
<td>9d</td>
<td>25d</td>
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$n$th odd number $\times d$ $\times n^2 \times d$
The amazing results

- No matter how steep or not the inclined plane was set and no matter whether the ball rolled was heavy or light, large or small, it gained speed at the same uniform rate.
- Also the total distance travelled was always equal to the distance travelled in the first time interval times the square of the number of time intervals.

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Galileo’s Law of Uniform Acceleration of Falling Bodies

- By concentrating on measuring actual distances and time, Galileo discovered a simple relationship that accounted for bodies “falling” toward the Earth by rolling down a plane.
- Since the relationship did not change as the plane got steeper, Galileo reasoned that it held for bodies in free fall.

Galileo’s Law of Uniform Acceleration of Falling Bodies, 2

- The law states that falling bodies gain speed at a constant rate, and provides a formula for calculating distance fallen over time once the starting conditions are known.
- Nowhere does the law attempt to explain why a heavy body falls down.
- The law specifies how a body falls, not why.
Examples:

1. A ball is rolled down a plane and travels 10 cm in the first second. How far does it travel in the third second?
   - Answer: It travels 5 x 10 cm = 50 cm in the third second.
   - 5 is the third odd number. 10 cm is the original distance in the first unit of time, which happens to be one second in this case.

2. A stone is dropped off a cliff. It falls 19.6 meters in the first 2 seconds. How far does it fall altogether in 6 seconds?
   - 19.6 meters is the unit of distance. The unit of time is 2 seconds. Six seconds represents 3 units of time.
   - The total distance fallen is 3^2 x 19.6 meters = 9 x 19.6 meters = 176.4 meters.

3. Aristotle knew that bodies fall faster and faster over time, but how much faster he could not determine.
   - If an object falls 16 feet in the first second after it is released, how much speed does it pick up as it falls?
     - Answer: Every time second, the object adds an additional 2 x the original distance travelled in the first second to that travelled in the second before it (1d, 3d, 5d, etc.) So in this example, the object accelerates at 2 x 16 = 32 feet per second.
What about projectiles?

- Galileo had devised an apparatus to study falling bodies, based on the assumption that whatever it was that made bodies fall freely through the air also made them roll downhill.
- How could he make comparable measurements of a body flying through the air?

Solution: Use the inclined plane again

- Since Galileo could measure the speed that a ball was moving when it reached the bottom of his inclined plane, he could use the plan to shoot a ball off a table at a precise velocity.
- Then he could measure where it hit the floor when shot at different speeds.

Galileo’s trials and calculations

- Galileo’s surviving notebooks show that he performed experiments like these again and again looking for the mathematical relationship he thought must be there.
Galileo’s Law of Projectile Motion

- Finally he found the key relationship:
- A projectile flying through the air has two distinct motions:
  - One is its falling motion, which is the same as if it had been dropped. (Constantly accellerating.)
  - The other is the motion given to it by whatever shot it into the air. This remains constant until it hits the ground.

Another simple solution

- The falling speeds up constantly, the horizontal speed remains the same.
- Shoot a bullet horizontally at a height of 4.9 meters from the ground and at the same time, drop a bullet from the same height.
  - They both hit the ground at the same time – one second later.

Many questions answered here

- Galileo’s fellow mathematician/engineers were losing a lot of sleep trying to figure out how a cannon fires, how to aim it, etc.
- Galileo’s Law of Projectile Motion provides a way to solve their problems.
Example

- From the top of a cliff, 78.4 meters high, a cannon is shot point blank (horizontally) off the cliff. In the first second it drops 4.9 meters vertically and travels 100 meters horizontally. How far from the base of the cliff will it land?
  - First figure when it will land. How long will it take to fall 78.4 meters?
    - 78.4 meters = \(4^2 \times 4.9\) meters. This indicates that it will take 4 seconds to hit the ground.
  - In 4 seconds, the bullet will travel \(4 \times 100\) meters horizontally.
  - It will therefore hit the ground 400 meters from the base of the cliff.

Galileo’s Two New Sciences

- Galileo’s work on the science of motion was published in 1638, while under house arrest, and blind.
- The title was Discourses and Mathematical Demonstrations on Two New Sciences.
  - One science was motion of bodies (free fall and projectile).
  - The other was strength of materials (an engineering topic).
- The book became a model treatise for how to do science. It is the first important work in physics as we know it today.

Galileo’s Scientific Method

1. Examine phenomena.
2. Formulate hypothesis about underlying structure.
3. Demonstrate effects geometrically.
   - I.e., give a mathematical account of the phenomena, or “save the phenomena”
4. Calculate the effects expected.
5. [Implied.] Compare calculated effects with observed effects.
Galileo’s use of mathematics in scientific investigation is different from his predecessors and contemporaries.

For Pythagoras, and by analogy, for Plato, Copernicus, and Kepler, mathematics is the secret of nature. To discover the mathematical law is to know what there is to know.

For Galileo, mathematics is merely a tool, but an essential one. Nature, he believed, operated in simple relationships that could be described in concise mathematical terms. Mathematics is the Language of Nature.