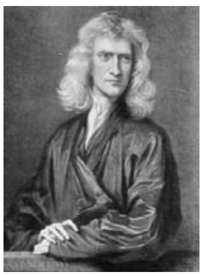
Isaac Newton

Nature, and Nature's Laws lay hid in Night.
God said, Let Newton be! and All was *Light.*

Isaac Newton, 1642-1727

- Born December 25, 1642
  - by the Julian Calendar
  - or January 4, 1643 by the Gregorian Calendar.
- From a family of yeomen farmers.
  - His father died some months before Newton was born (a "posthumous" child).
  - His mother remarried and left Newton to be raised by her mother.

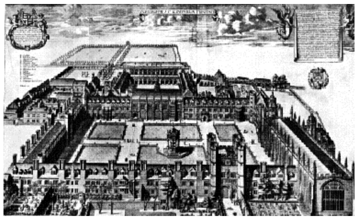


Woolsthorpe Manor

- Newton was slated to take over family estate and manage a farm, but was obviously too interested in books and study.

Newton's birthplace, Woolsthorpe Manor, in Lincolnshire.

Trinity College, Cambridge



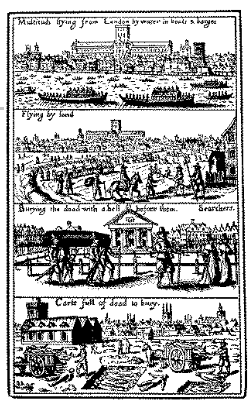
- Sent to Trinity College, University of Cambridge in 1661. Took an ordinary BA in 1665.

Newton discovered the world of mathematics and natural philosophy

- After finishing his BA, Newton planned to stay on at Cambridge for further study.
  - In his last years at Cambridge as an undergraduate, Newton had become very interested in the new mechanical philosophy (Descartes, etc.), in mathematics, and in Copernican astronomy.

The Plague hits England

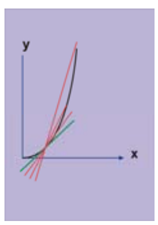
- An outbreak of the plague hit England in 1666.
- Cambridge closed for 18 months.
- Newton returned to Woolsthorpe to wait it out.



Newton's Annus Mirabilis

- Newton's "miracle year," **1666**
  - **A date to remember. This is the fifth date you must remember in this course.**
- During the plague, Newton returned home, thought about his new interests and made his most original insights.

3 Major Insights of 1666

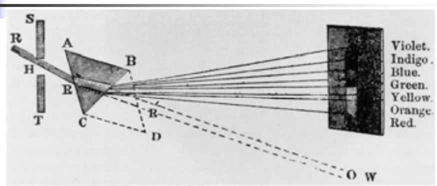


- Light Calculus The falling apple

Light

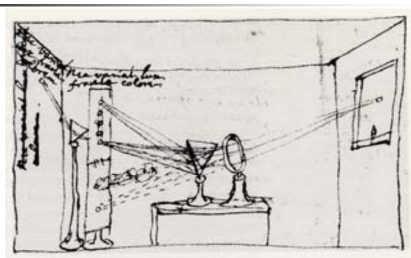
- Light one of the great *mysteries.*
- Magical nature
  - Connected with the Sun, with fire, with vision.
- Colour thought of as a *quality*, e.g. blueness.
- Mathematical or mechanical treatment seemed *impossible.*
  - Descartes believed colours due to the spin of light particles.

Newton investigates light



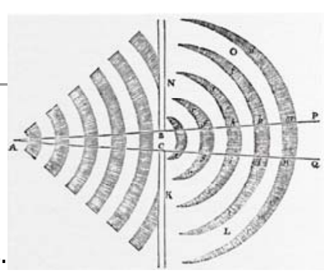
- Newton undertook to investigate using a *triangular prism*
  - Concluded that *coloured light is simple*
  - There are particles of blue light, particles of red light, etc.
  - *White light is a mixture* of coloured lights

The Crucial Experiment



- *The crucial experiment*--on light of a single colour

Light as Particles



- *Particles*, not waves because light rays move in straight lines.
  - The diagram is Newton's illustration of what light *waves* would do, passing through a small opening.

Letter to *Royal Society*, 1672

A Letter of Mr. Isaac Newton, Professor of the Mathematicks in the University of Cambridge; containing his New Theory about Light and Colors: sent by the Author to the Publisher from Cambridge, Febr. 6. 1671/72; in order to be communicated to the R. Society.

Sir,

To perform my late promise to you I shall without further ceremony acquaint you that in the beginning of the Year 1666 (at which time I applyed my self to the grinding of Optick glasses of other figures than Spherical) I procured me a Triangular glass-Prisme, to try therewith the celebrated Phænomena of Colours.And in order thereto having darkened my chamber, and made a small hole in my window-shuts, to let in a convenient quantity of the Sun's light, I placed my Prisme at his entrance, that it might be thereby refracted to the opposite wall. It was at first a pleasing divertisement, to view the vivid and intense colours produced thereby; but after a while applying my self to consider them more circumspectly, I became surprised to see them in an oblong form; which, according to the received laws of Refraction, I expected should have been circular….

The Calculus

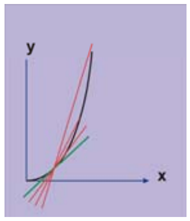
- Concerns quantities that change over time (or space).
  - Example: If the speed of a falling body changes constantly, how fast is it going at any given moment?
- According to Zeno, it is not going anywhere in a fixed instant.
  - speed = velocity = (distance travelled)/(time used) = d/t
  - In any instant, t = o, d = o
- **What is 0 / 0?**

Average Velocity

- Can be expressed as the total distance divided by the total time.
- Example:
  - A car trip from Toronto to Kingston takes 3 hours and the distance is 300 km.
  - The average speed of the car is 300 km/3 hrs = 100 km/hr.
  - This is the average speed, though the car may have sped up and slowed down many times.

Instantaneous velocity

- What is the car's speed at any given moment?
- What does that number on the speedometer really mean? Newton's definition:
  - Instantaneous Velocity = the Limit of Average Velocity as the time interval approaches zero.



SC/NATS 1730, XVII 16

Smaller and smaller time intervals

- Suppose on a trip to Kingston along the 401, the car went 50km in the 1st hour, 100 km in the 2nd hour, and 150 km in the 3rd hour.
- The total time remains 3 hours and the total distance remains 300 km, so the average speed for the trip remains 100 km/hr.
- But the average speed for the 1st hour is 50 km/hr; for the 2nd is 100 km/hr; and for the third is 150 km/hr.

SC/NATS 1730, XVII 17

Still, the problem of 0/0

- As the time intervals get smaller, a closer approximation to how fast the car is moving at any time is still expressible as distance divided by time.
- But if you get down to zero time, there is zero distance, and Zeno's objections hold.

SC/NATS 1730, XVII 18

Newton's clever way to calculate the impossible

- Newton found a way to manipulate an equation so that one side of it provided an answer while the other side seemed to defy common sense.
- For example, Galileo's law of falling bodies, expressed as an equation in Descartes' analytic geometry.

Deriving a meaning for 0/0

- The equation for the position of a falling body near the surface of the Earth is:

$$s = 4.9t^2$$

Where
  - s is the total distance fallen,
  - 4.9 is the distance the object falls in the 1st second, expressed in meters,
  - and t is the time elapsed, in seconds.

Calculation of the Limit of Average Velocity

- The average velocity of a falling object over any interval of time during its fall is d/t, that is, distance (during that interval), divided by the time elapsed.
  - For example, by Galileo's Odd-number rule, if an object falls 4.8 meters in the 1st second, in the 3rd second it will fall 5x4.8 meters = 24 meters.
  - Its average velocity during the 3rd second of its fall is 24 meters per second.

But the object speeds up constantly

- The smaller the time interval chosen, the closer will the average velocity be to the velocity at any moment during that interval.
- Suppose one could take an arbitrarily small interval of time.
  - Call it Δt. Call the distance travelled during that small interval of time Δs.

Approximating the instantaneous velocity

- The average velocity of a moving object is distance divided by time.
- The average velocity during the arbitrarily small interval Δt is therefore

$$\Delta s / \Delta t$$

- The smaller Δt is, the closer will be $\Delta s / \Delta t$ to the "real" velocity at time t.

Working with the Galileo/Descartes equation

- 1. $s = 4.9t^2$
- 2. $s + \Delta s = 4.9(t+\Delta t)^2$
- 3. $s + \Delta s = 4.9(t^2+2t\Delta t + [\Delta t]^2)$
- 4. $s + \Delta s = 4.9t^2+9.8t\Delta t + 4.9[\Delta t]^2$
  - *Now, subtract line 1. from line 4. (Equals subtracted from equals.)*
- 5. $(s + \Delta s) - s = (4.9t^2+9.8t\Delta t + 4.9[\Delta t]^2) - 4.9t^2$
- 6. $\Delta s = 9.8t\Delta t + 4.9(\Delta t)^2$
  - *Line 6 gives a value for the increment of distance in terms of the total time, t, and the incremental time, Δt.*

Working with the Galileo/Descartes equation, 2

- 6. $\Delta s = 9.8t\Delta t + 4.9(\Delta t)^2$
  - *Now, divide line 6. by the time increment, Δt.*
- 7. $\Delta s/\Delta t = [9.8t\Delta t + 4.9(\Delta t)^2]/\Delta t$
  - Which simplifies to
- 8. $\Delta s/\Delta t = 9.8t + 4.9\Delta t$
  - What happens to $\Delta s/\Delta t$ when Δt (and Δs) *go to zero*?
  - We don't know because 0/0 is not defined.

Working with the Galileo/Descartes equation, 4

- Newton's solution:
  - Forget about the left side of the equation:
- 8. $\Delta s/\Delta t = 9.8t + 4.9\Delta t$
  - Just look at what happens on the right side.
  - As Δt becomes smaller and smaller,
- 9. $9.8t + 4.9\Delta t$ becomes
  $9.8t + 4.9(0) = 9.8t$

Working with the Galileo/Descartes equation, 5

  - As the time increment Δt gets closer and closer to zero,
- 10. $\Delta s/\Delta t$ gets closer to $9.8t$.
  - Since the left side of the equation must equal the right side and the left side is the velocity when Δt goes to zero, then *9.8 t is the instantaneous velocity* at time *t*.
- Example: After 3 full seconds of free fall, the object falling will have reached the speed of 9.8 x 3 = 29.4 meters per second.

Newton's breakthrough

- Newton's genius in the calculus was to find a way to get around the static definitions which ruled out such calculations.
- He was willing to entertain the "impossible" idea of an object moving in an instant of time, and found an answer.