The Newtonian Synthesis

The Mathematical Principles of Natural Philosophy

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The Falling Apple

- According to Newton, it was while he was in the orchard at Woolsthorpe during the plague years of 1665-1666 that he noticed an apple fall and realized that whatever made it fall also kept the Moon in its orbit around the Earth.

The orchard at Woolsthorpe Manor.

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From Falling Apple to *Principia*

- The falling apple insight started Newton on the path that brought together the insights of Renaissance astronomy and physics into a comprehensive system.
- It took another 20 years before he was ready to put it all together in *Principia Mathematica – The Mathematical Principles of Natural Philosophy.*

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Concepts considered by Newton

- *Kepler's Laws:*
- 1. Elliptical orbits of planets.
- 2. Planets sweep out equal areas in equal times.
- 3. Harmonic law: $D^3/T^2 = K$, providing a formula that relates the period of revolution of a planet, *T*, to its distance from the Sun, *D*.

Concepts considered by Newton, 2

- *Galileo's findings:*
- 1. Times square law for falling bodies.
- 2. Projectiles in parabolic path.
- 3. Galilean relativity.

Concepts considered by Newton, 3

- *Descartes' Principles:*
- 1. Motion is natural.
- 2. Inertia: Bodies in motion tend to stay in motion in a straight line — unless forced from it.
- 3. All motion due to impact.
  - Forces are occult – i.e., forbidden in a mechanical system.

Aristotle's philosophical approach to physics

- 1. Two separate realms:
  - The heavens and the earth.
- 2. Heavenly motions:
  - Eternal, changeless, and always circular.
- 3. Earthly motions:
  - Either natural or forced.
  - Natural motion either up (light things) or down (heavy things) – bodies seek their natural places.
  - Forced motions caused by pushes – Cannot occur "naturally."

Euclid's Mathematical approach to certain knowledge

- Axiomatic Structure:
  - Definitions
  - Axioms & Postulates
  - Rules of reasoning
- Begin from reasonable assumptions and through logic and other strict rules of inference, build up a body of knowledge.

The Lucasian Professor of Mathematicks

- Newton returned to Cambridge after the plague.
- After a few years his former mathematics professor, Isaac Barrow, resigned his position, and recommended that Newton be his replacement.
- Newton became the 2nd Lucasian Professor of Mathematicks, a position he held for 27 years.

Newton's sporadic output

- Over the next 15-20 years, Newton published a work on the calculus, the ideas of which he was accused of stealing from Leibniz, and some of his work on light, which Robert Hooke claimed he had conceived of first.
- Newton, disgusted, retreated into his own studies, publishing nothing.

Edmund Halley's Visit

- One of Newton's few friends was the astronomer, Edmund Halley.
- In 1684, Halley and architect Christopher Wren, speculated that the force that held the planets in their orbits must be inversely related to their distance from the sun.
- Halley thought Newton might be able to settle the matter.

Halley's question

- Instead of asking Newton what kind of force would hold the planets in their orbits, Halley asked Newton what curve would be produced by a force of attraction that diminished with the square of the distance.
- Newton replied immediately, "An Ellipse."
  - Halley asked for the proof, but Newton could not find it, and promised to send it to him.

Newton's first draft

- Newton sent Halley a nine page proof three months later.
- Halley urged Newton to publish it, but Newton refused, realizing that the consequences were far greater than the solution to that problem.
- For 18 months, Newton developed the theory farther.

The Principia

- Finally, three years after Halley's visit, Newton's results were published—at Halley's expense—in the single most important work in the history of science:
- *Philosophiæ Naturalis Principia Mathematica,* translated as *The Mathematical Principles of Natural Philosophy,* published in 1687.

The title tells all…

- Descartes' attempt at a new system of philosophy was *The Principles of Philosophy.*
- *Newton adds two words:*
  - Natural – referring to the physical world only, not to *res cogitans.*
  - Mathematical – perhaps not *all* of the principles of philosophy, just the mathematical ones.

The Axiomatic Structure of Newton's *Principia*

- Definitions, axioms, rules of reasoning, just like Euclid.
- Examples:
  - *Definition*
  - 1. The quantity of matter is the measure of the same, arising from its density and bulk conjunctly.
    - How Newton is going to use the term "quantity of matter."

Rules of Reasoning

- 1. We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances.
  - This is the well-known Principle of Parsimony, also known as Ockham's Razor. In short, it means that the best explanation is the simplest one that does the job.

The Axioms

- 1. Every body continues in its state of rest of or uniform motion in right line unless it is compelled to change that state by forces impressed upon it.
  - This is Descartes' principle of inertia. It declares that straight-line, constant speed, motion is the natural state. Force is necessary to change that motion.
  - Compare this to Aristotle's need to explain motion.

The Axioms, 2

- 2. The change in motion is proportional to the motive force impressed and is made in the direction of the right line in which that force is impressed.
  - A force causes a change in motion, and does so in the direction in which the force is applied.

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The Axioms, 3

- 3. To every action there is always opposed an equal reaction; or, the mutual actions of two bodies upon each other are always equal and directed to contrary parts.
  - Push against any object; it pushes back at you. This is how any object is held up from falling, and how a jet engine works.

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Known Empirical Laws Deduced

- Just as Euclid showed that already known mathematical theorems follow logically from his axioms, Newton showed that the laws of motion discerned from observations by Galileo and Kepler followed from his axiomatic structure.

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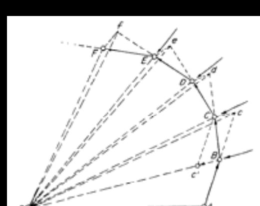
Galileo's Laws

- Galileo's laws about physics on Earth:
  - The law of free fall.
    - Galileo asserts that falling bodies pick up speed at a uniform rate.
    - Newton shows that a constant force acting in line with inertial motion would produce a constant acceleration. This is implied by his first 2 axioms.
  - The parabolic path of a projectile.
    - Likewise, if a body is initially moving inertially (in any direction), but a constant force pushes it downwards, the resulting path will be a parabola.

Kepler's Laws

- Newton's very first proposition is Kepler's 2nd law (planets sweep out equal areas in equal times).
- It follows from Newton's first two axioms (inertial motion and change of motion in direction of force) and Euclid's formula for the area of a triangle.

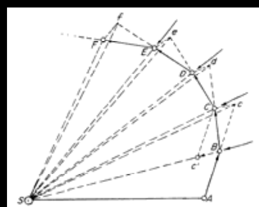
Kepler's 2nd Law illustrated



- In the diagram, a planet is moving inertially from point *A* along the line *AB*.
- *S is the Sun.* Consider the triangle *ABS* as "swept out" by the planet.
- When the planet gets to *B*, Newton supposes a sudden force is applied to the planet in the direction of the sun.
- This will cause the planet's inertial motion to shift in the direction of point C.

Kepler's 2nd Law illustrated, 2

- Note that if instead of veering off to C, the planet continued in a straight line it would reach c (follow the dotted line) in the same time.
- Triangles ABS and BcS have equal area.
  - Equal base, same height.



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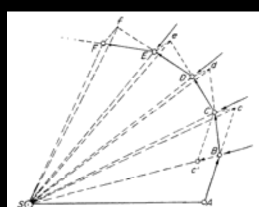
Kepler's 2nd Law illustrated, 3

- Newton showed that triangles BCS and BcS also have the same area.
  - Think of BS as the common base. C and c are at the same height from BS extended.



- Therefore ABS and BCS are equal areas.
  - Things equal to the same thing are equal to each other.

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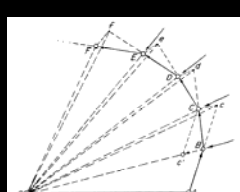
Kepler's 2nd Law illustrated, 4

- Now, imagine the sudden force toward the sun happening in more frequent intervals.
  - The smaller triangles would also be equal in area.



- In the limiting case, the force acts continuously and any section taking an equal amount of time carves out an equal area.

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The Same Laws of Motion in the Heavens and on Earth

- Newton's analysis showed that from the same assumptions about motion, he could account for the parabolic path of a projectile on Earth and for a planet (or the Moon) in orbit.

Newton's illustration of the relationship between a projectile and an object in orbit.

A Mechanical system

- Newton's axiomatic "principles" implied a mechanistic model of the universe.
  - This was all that made sense to Newton.
- The Clockwork Universe
  - God makes clock and winds it up.

Universal Gravitation

- A deduced effect
  - That which makes apples fall and the moon stay in orbit.
  - And the planets, and projectiles, etc.
- The gravitational force:

  $$G = g(M_1M_2/d^2)$$

  - The force varies inversely with square of distance.
    - It gets much weaker as the distance between objects is greater, but never disappears entirely.

Action at a Distance

- Gravity, and magnetism too, operate over apparently empty space.
  - Is this an occult force?
- Newton postulates an "Aether" to transmit gravity, magnetism, etc.
  - Makes empty space no longer empty.
    - Note the return to Parmenides' and Aristotle's denial of the existence of "nothing."

Hypotheses non fingo

- Unlike Aristotle (but like Galileo), Newton did not claim to have an explanation for everything.
  - For example, he described how gravity works, on the basis of the effects seen. He does not say what gravity is.
  - On this an other mystery subjects, Newton said that he "frames no hypotheses."

The Newtonian Model for true knowledge

- Axiomatic presentation.
- Mathematical precision and tight logic.
  - With this Euclidean style, Newton showed that he could (in principle) account for all observed phenomena in the physical world, both in the heavens and on Earth.
- Implication: All science should have this format.
  - This became the model for science.

Newtonianism

- The application of the Newtonian model beyond physics, e.g. in philosophy, psychology, sociology, economics.
- John Locke, *Essay on Human Understanding*
- Benedict Spinoza, *Tractatus Theologia*
- Adam Smith, *Wealth of Nations*