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ex Consider a cylinder of radius R placed into a uniform electric field $\overline{E}$ such that the cylinder axis is parallel to the field. What is the flux Φ_E for this 'closed surface'?

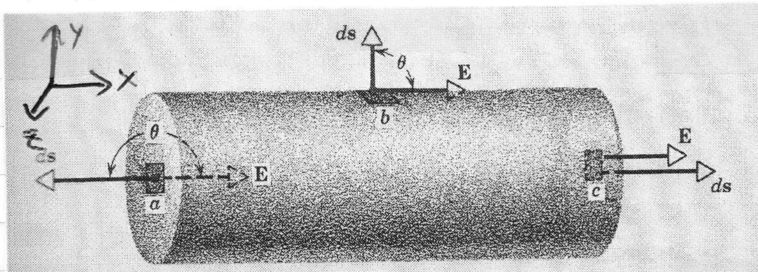

Fig. 28-3 Example 1. A cylindrical surface immersed in a uniform field E parallel to its axis.

⇝ we can break the calculation for Φ_E down into three pieces: what goes through the left end (Φ_L), the right end (Φ_R) and the cylindrical surface (Φ_C)

uniform field strength

$$\overline{E} = E_0 \hat{x}$$

$$A = \pi R^2$$

$$\Phi_E = \Phi_L + \Phi_R + \Phi_C$$

$$= \overline{E} \cdot \overline{A}_{endL} + \overline{E} \cdot \overline{A}_{cyl.} + \overline{E} \cdot \overline{A}_{endR}$$

$$= [E_0 \hat{x} \cdot -A\hat{x}] + \underbrace{[E_0 \hat{x} \cdot A_{body}(\hat{y}, \hat{z})]}_{\text{this is zero because the surface is always orthogonal to } \overline{E}!} + [E_0 \hat{x} \cdot A\hat{x}]$$

negative because end's 'normal' points in negative x direction

$$= -E_0 A + E_0 A = 0$$

→ the total flux through the cylinder is zero!

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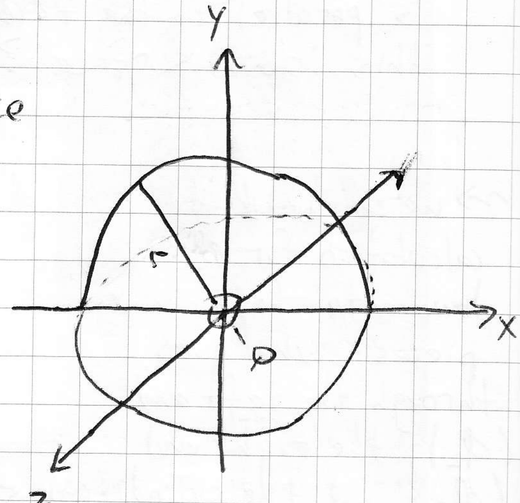
ex Consider (in 3-D) a charge Q located at the origin and surrounded by a sphere of radius r. What is the total flux through the surface of the sphere?



- the electric field due to Q points in a radial direction (with magnitude E_0)
- the direction of the sphere's surface area also points radially (outward) $\rightarrow \bar{E} \cdot \bar{A} = |E||A|\cos\theta = |E||A|$ (since $\theta = 0$)
- due to symmetry, this is uniform everywhere on the surface of the sphere!

surface area of a sphere

so $\Phi = \bar{E} \cdot \bar{A} = E_0 \cdot 4\pi r^2$

but from Coulomb's Law: $E_0 = K \frac{Q}{r^2}$

$\rightsquigarrow \Phi = K \frac{Q}{r^2} \cdot 4\pi r^2 = \frac{1}{4\pi\epsilon_0} \cdot 4\pi Q = \frac{Q}{\epsilon_0}$

- So the flux does not depend upon the radius of the sphere (i.e. we can make it really big or really small and Φ is unchanged)

- Note that if Q flips sign, so does Φ (i.e. a negative charge makes the flux inwards)

→ So there seems a deep connection between the 'surface' and the charge therein...

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Gauss' Law

- Pulling together intuitively the pieces gathered from the previous examples, we can state Gauss' Law:

→ The electric flux **through** any **closed surface** is proportional to the **total** charge q **inside** the surface (the const. of proportionality being $1/\epsilon_0$)

$$\Phi_E = \frac{q}{\epsilon_0}$$

total flux **through** a closed surface

total charge inside a **closed surface** (sometimes called a Gaussian surface)

permitivity of free space ($8.85 \times 10^{-12}\ C^2/N \cdot m^2$)

- while a bit abstract, a key physical concept here is that of a **surface** that we consider the flux as passing through (it takes a bit of time to get your head wrapped around this)

- Note that Φ_E is the **total** flux passing through that surface **AND** that the surface is **closed** such that it contains a total amount of charge q

- Typically to make Gauss' Law useful for a given problem at hand, you want to make a clever choice for the surface (e.g. sphere, cylinder, ring) that exploits symmetry in the problem

- A 'closed surface' (in 3-D) is one where if you start on the outside, you stay on the outside and vice versa (e.g. a sphere, toroid); an open surface (in 3-D) would be a flat disc or a 'bowl'

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Calculus Aside: A more effective/complete way to write Gauss' Law is as a **surface integral**

$$\oint \bar{E} \cdot d\bar{A} = \frac{q}{\epsilon_0}$$

integral over a closed surface (that's what the o indicates)

dot product of electric field and all the **surface** area vectors (see 1/11 notes, Fig. 28-2)

NOTE: Remember that the integral sign is just a long S to indicate a **SUM**

$$\int_a^b f(x)\,dx = \lim_{\Delta x \to 0} \sum_{x=a}^{x=b} f(x)\,\Delta x$$

(i.e. an integral is just a sum of a LOT of little things!)

- Gauss' Law, w/ a good choice for a closed surface, gives up a powerful tool for determining the electric field associated with a lot of different geometric charge distributions (e.g. line of charge, sheet(s) of charge, etc....)
- Considering these different geometries is an important piece of the story as it will relate to some key concepts we'll build off soon (e.g. thinking of a line of charge is useful for considering electric wires, or two parallel plates for considering a capacitor, a key component in most electric circuits)

  → so while a bit abstract, this is a key foundational concept

- Make sure to carefully read/review the examples described in the book (i.e. ex. 17.6–17.8)
- In some examples, the geometry is 2-D while in others it is explicitly 3-D (so think carefully about this!)