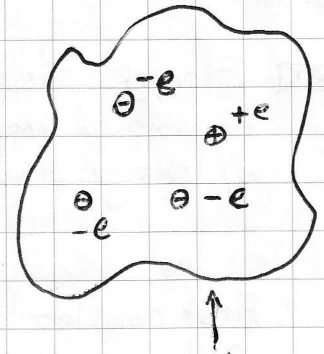
ex Determine the flux through the surface to the right, which encloses three electrons and one proton.



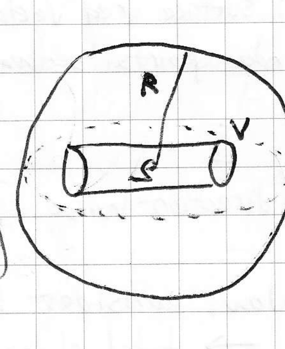
- total charge is $-3e + e = -2e$
$$= -3.2 \times 10^{-19}\ C$$
$$= q$$
- though the surface shape is irregular, that doesn't matter: Gauss' Law tells us that the flux through the **ENTIRE** surface depends upon the total charge therein

$$\Phi_E = \frac{q}{\epsilon_0} = -\frac{3.2 \times 10^{-19}\ C^*}{8.85 \times 10^{-12}\ C^2/N \cdot m^2} = -3.6 \times 10^{-8}\ \frac{N \cdot m^2}{C}$$

$\rightsquigarrow$ this means that the net flux points inwards (though we don't know from Gauss' Law what it is at any particular location on the surface)

$\therefore$

ex A cylinder has volume V [m^3] and charge density ρ [C/m^3]. What is the electric flux through a sphere of radius R that completely encloses the cylinder?



$q = \rho V$ [C] (total charge of the cylinder)

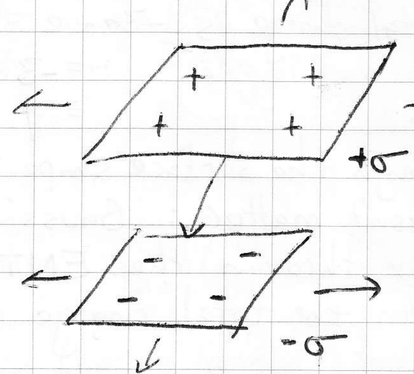
$$\Phi_E = \frac{q}{\epsilon_0} = \frac{\rho V}{\epsilon_0}$$

$\rightsquigarrow$ Note that as long as the sphere completely encloses the cylinder, we can make it as big as we want

$\rightsquigarrow$ What if the sphere only contains part of the cylinder?

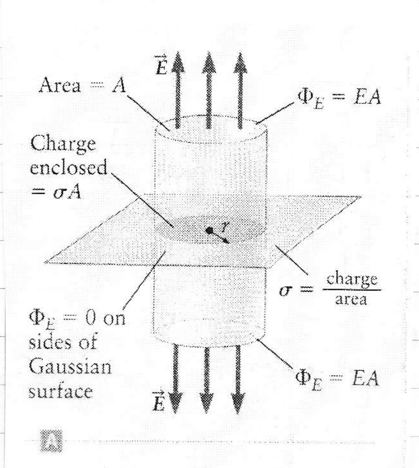
1/15/13

ex Electric field between two sheets of charge (ex. 17.8; this is such an important example, we'll discuss here) w/ charge density σ [C/m^2]



charge sheets extend at indefinite (i.e. no edge)

- First consider a single sheet of charge and choose the best 'Gaussian surface' $\rightarrow$ cylinder (see Fig. 17.32)



[single-sheet]

- 'caps' of cylinders have area A [m^2]
- no flux through 'body' of cylinder, caps only
- total charge enclosed is $\sigma A = q$ (since $[\sigma] = C/m^2$)
- symmetric about both caps:

$$\Phi_E = (\bar{E} \cdot \bar{A})_{top} + (\bar{E} \cdot \bar{A})_{bottom}$$

$$= 2|E|A = \frac{q}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

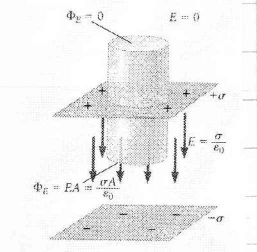
NOTE: surface area vector of cylinder points 'outwards'

$$\longrightarrow |E| = \frac{\sigma}{2\epsilon_0}$$

(direction-wise, $\bar{E}$ points away from the sheet)

- Now, consider both sheets w/ cylinder extending through both $\rightarrow$ total charge contained inside is zero, so $\Phi_E = 0$ (a surprising result!)

- Now consider both sheets, but cylinder only passing through one of them (Giordano fig. 17.34)



▲ **Figure 17.34** Example 17.8. Using Gauss's law to check the result for the electric field of two charged planes.

$\boxed{\text{ex (cont.)}}$ $\boxed{1/15/13}$

- total charge contained is $+\sigma A$
- but what about $\bar{E}$? We also need to consider the other sheet too here!
- other sheet will make $\bar{E} = 0$ through the top cap (since the two cancel one another out, see Giordano fig. 17.33 for ref.)
- so only through the bottom cap will $\bar{E}$ be non-zero (and will point downwards)

$$\Phi_E = \bar{E} \cdot \bar{A} + 0 = |E| A = \frac{q}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$\rightarrow |\bar{E}| = \frac{\sigma}{\epsilon_0}$$

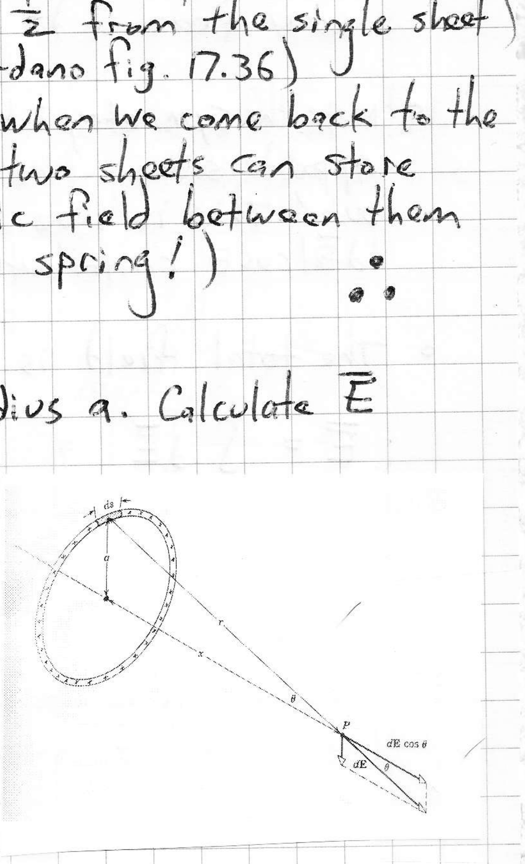
⇝ this is only non-zero between the plates and the direction is from + toward − (another way to think about it is that the two sheets both contribute equally in-between, thus cancelling out the factor of $\frac{1}{2}$ from the single sheet) (see Giordano fig. 17.36)

⇒ this idea here will be important when we come back to the notion of a **capacitor** and how two sheets can store energy by virtue of the electric field between them (in direct mechanical analogy to a spring!) ∴

$\boxed{\text{ex}}$ Consider a ring of charge q with radius a. Calculate $\bar{E}$ for points on the axis of the ring at a distance x from its center.

- Trickier, but if we
  - draw a clear diagram
  - note the symmetries
  - use a bit of calculus

⇝ this problem is readily solvable



11/15/13 $\boxed{\text{ex. (ring of charge, cont.)}}$

- Consider a small element of the ring (ds) and it's effect at point P (see figure)

  - it is at a distance r from P (where $r = \frac{x}{\cos\theta} = \sqrt{a^2+x^2}$)
  - the element ds has the amount of charge

$$dq = \underset{\uparrow}{q} \underbrace{\frac{ds}{2\pi a}}$$

total charge (assumed uniformly distributed)

fraction of the ring ds comprises (the denominator is the total circumfrence of the ring)

  - $d\bar{E}$ is the vector contribution to the electric field that ds has at point P; it has two components: $|d\bar{E}|\sin\theta$ and $|d\bar{E}|\cos\theta$ (this latter one pointing along the ring's axis away from it)

- Note the **symmetry**: When considering two small elements on opposite sides, the $|d\bar{E}|\sin\theta$ contributions will cancel out! This is true all around the circle, leaving only the $|d\bar{E}|\cos\theta$ contribution to add up

- The total field is given by

$$\bar{E} = \int d\bar{E} = \lim_{ds\to 0} \sum_{\substack{\text{around}\\ \text{circle}}} d\bar{E}$$

integrate around the ring

contribution of ds at point P, i.e. you are just summing around the circle for LOTS of really small elements ds!

  - because of symmetry, we can consider this as a scalar (noting that $\bar{E}$ points away from the ring along its axis)

11/15/13

$$E = \int_0^{2\pi a} dE\cos\theta = \int_0^{2\pi a} \underbrace{\frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}}_{=dE} \underbrace{\frac{x}{\sqrt{a^2+x^2}}}_{\cos\theta}$$

(a very general expression due to the charge dq)

$\cos\theta$ (just re-written via a bit of trig.)

$$= \int_0^{2\pi a} \frac{1}{4\pi\epsilon_0} \underbrace{\frac{q}{2\pi a}}_{dq} \underbrace{\frac{1}{a^2+x^2}}_{1/r^2} \underbrace{\frac{x}{\sqrt{a^2+x^2}}}_{\cos\theta} ds$$

$$= \frac{1}{4\pi\epsilon_0}\frac{qx}{(2\pi a)(a^2+x^2)^{3/2}} \int_0^{2\pi a} ds$$

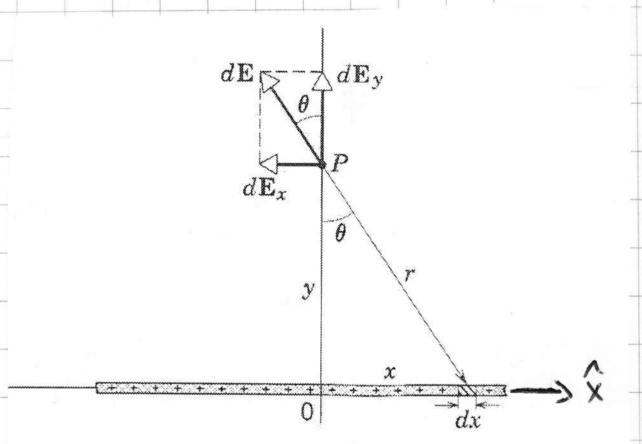
this is simply just $2\pi a$!

$$= \frac{1}{4\pi\epsilon_0}\frac{qx}{(a^2+x^2)^{3/2}} = E$$

- Note that when $x \approx 0$, $E \simeq 0$ (which makes sense) and that when $x \gg a$, $E \simeq \frac{1}{4\pi\epsilon_0}\frac{q}{x^2}$ (i.e. the ring of charge acts like a point charge, which also makes sense!)

☐ A similar approach for a 'line of charge' (w/ charge density λ [C/m]) yields a result briefly alluded to in Giordano's book (fig. 17.31) which was determined via Gauss' Law:

$$E = \frac{\lambda}{2\pi y\epsilon_0}$$



Example 6. A section of an infinite line of charge.