Electric Potential + Energy

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- Consider a **battery**: When you hook it up to something it can do work (e.g. flashlight, a pump for an air mattress)
  → so somehow there is a deep connection between 'charge' (and an associated electric field) and energy

- Quick review of electric fields
  - point charge Q ($\frac{kQ}{r^2}$, radial direction)
  - line of charge w/ density $\lambda = Q/L$ ($k\lambda/x$, $\perp$ to line)
  - sheet of charge (i.e. a plate) w/ density $\sigma = Q/A$ ($\frac{\sigma}{2\epsilon_0}$, $\perp$ to plate)
  - parallel sheets ($\frac{\sigma}{\epsilon_0}$, $\perp$ to plates, but only inbetween)

- An electric 'field' ($\bar{E}$) is analogous to a gravitation 'field' ($\bar{g}$) at earth's surface (it pointing radially towards the center of the earth, call it $\hat{r}$)

Gravity: $\bar{W} = m\bar{g}$ (weight) — definition of gravitational force in terms of a 'test mass'

$$\bar{F}_G(\bar{r}) = m\left(-\frac{GM}{r^2}\right)\hat{r} \qquad (\text{call } \bar{r} = r\hat{r})$$

Electro-statics: $\bar{F}_E = q\bar{E}$ ← definition of electric force (or in turn, the electric field) in terms of a 'test charge'

- Let's revisit a key concept: **Work** (initially in the context of gravity)

work — $W \equiv \bar{F} \cdot d\bar{x}$ (in simpler terms, $W = F\Delta x$)

  - units of W are J(oules) (energy!!) and work is a **scalar**
  - discussed in the context of doing work w/ regard to changes in energy due to gravity

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- We used the notion of work to define the concept of **potential energy** (U; Note that the book uses PE, but we'll use the more common notation of U):

$$U = -W$$

→ the 'potential' was **energy stored** due to work haven been done in some fashion

- We then used **conservation of energy** in a wide range of contexts to solve 1-D or 2-D mechanics problems

$$K + U = \text{const.} \longleftrightarrow (K_1 + U_1) = (K_2 + U_2)$$

Kinetic energy ($= \frac{1}{2}mv^2$)

→ this is, energy is just transferred back and forth between kinetic and potential forms

- In electrostatics, we will deal w/ charged particles (electrons usually) moving in const. electrical fields (analogous to free-fall in $\bar{g}$)

$$\Delta U_E = -W_E = -\bar{F}_E \cdot \Delta\bar{x} = -q\bar{E} \cdot \Delta\bar{x}$$

Note that the way we express it here is as a potential **energy difference**

→ depending upon the geometry, in many problems this simply reduces to:

$$\Delta U_E = -qE\Delta X$$

NOTE: A 'difference' implies a reference. In many cases, that ref. point is taken to be one infinitely far away (see ex. 18.2)

⟹ So the electrical potential energy is ultimately defined in terms of **work** being done due to an **electric field** (or put another way, electrical potential energy provides us a way to describe the 'strength' of an electric field)

- Can discuss the potential energy due to two point charges separated by a distance r }:

$$U_E = \frac{k q_1 q_2}{r}$$

$(k = \frac{1}{4\pi\epsilon_0})$

→ think of this as the energy needed to keep those two charges in that configuration (or better yet, what it took to get them there in the first place); possible to get that energy back (hence 'stored energy')

- Something to think about re Giordano ex. 18.1 ('potential energy of a hydrogen atom')

→ Why doesn't the electron just get sucked into the proton?

[there is a lot more to this story ....]

- An important point to emphasize here is highlighted in Giordano ex. 18.2 ('Moving charges around') and in Fig. 18.6 in that the path taken doesn't matter, only the initial and final point

→ Think carefully about why this is

essentially amounts to the notion of conservative forces and that any energy lost/gained due to an extra excursion will get accounted for on the return path (think of a roller coaster!)

$\boxed{ex}$ Two protons in a nucleus of U^{238} are $\sim 6.0 \times 10^{-15}$ m apart. What is their mutual potential energy?

$$U = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = \left(9.0\times10^{9}\ \frac{N\cdot m^2}{C^2}\right)\frac{(1.6\times10^{-19}\ C)^2}{(6.0\times10^{-15}\ m)} = 3.8\times10^{-14}\ J$$

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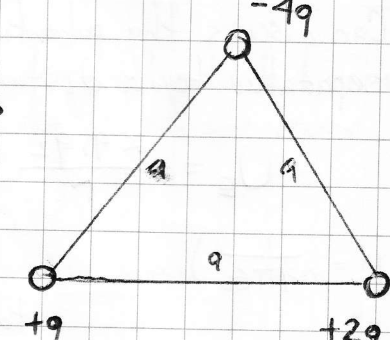
ex Three charges are arranged as shown. What is their mutual potential energy? Assume $q = 1.0 \times 10^{-7}$ C and $a = 10$ cm.



→ the total energy is the sum (or in Giordano term, the 'superposition') of the energies from each pair:

$$U = U_{12} + U_{13} + U_{23}$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{(+q)(-4q)}{a} + \frac{(+q)(+2q)}{a} + \frac{(-4q)(+2q)}{a}\right]$$

$$= -\frac{1}{4\pi\epsilon_0}\frac{10q^2}{a} = -9.0 \times 10^{-3}\ \text{J}$$

⇒ The fact that the potential energy is negative means that 'negative work' would have been done to assemble this structure. Put another way, 9.0×10^{-3} J worth of work would need to be done to dismantle this. Put yet another way, if these charges were released they'd move towards one another.

☐ In common practice (as noted earlier), infinity is taken as the zero of electric potential (a notion we will define next!), a positive potential energy ~~means~~ corresponds to repulsive electric forces and a negative potential energy corresponds to attractive forces.