Electric Potential

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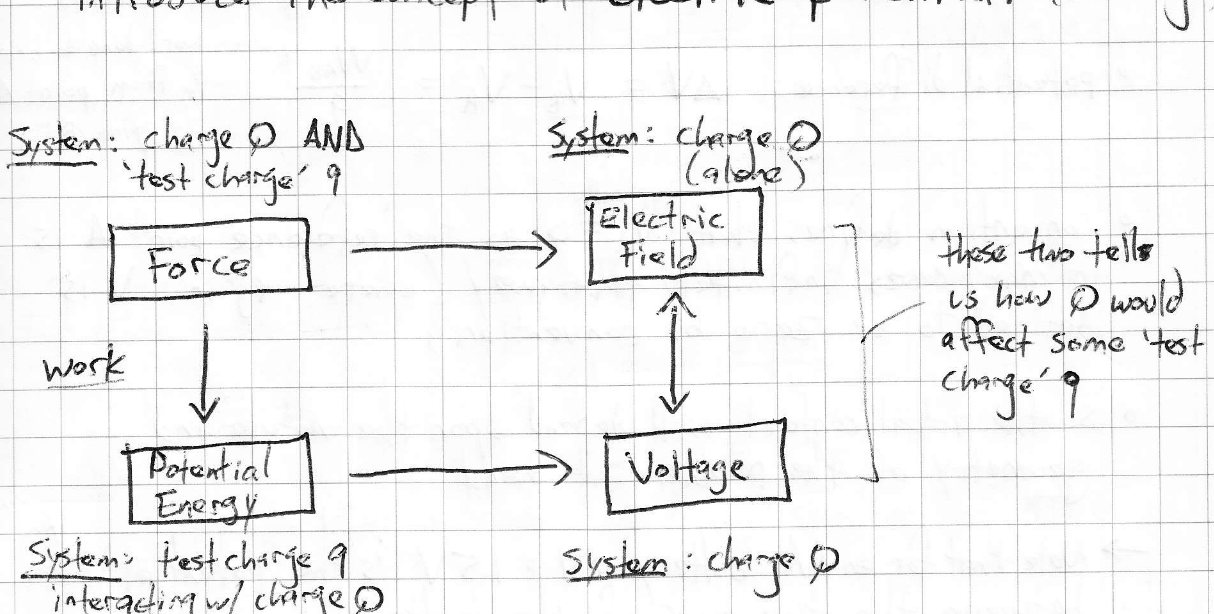
CAUTION! there is a distinction between 'electric potential' and 'electric potential energy' and 'potential difference'

though this last one is equivalent to **voltage**

Basic Review/Overview

0. Charge as a Fundamental property
1. charges can cause forces on other charges (Coulomb's Law)
   → motivates the idea that work can be done!
2. Notion of an electric field $\bar{E}$ (analogous to a gravitational field)
   → motivates notion of a 'test charge'
3. to characterize $\bar{E}$, we introduced the notion of flux (Φ) through a surface and Gauss' Law
4. We firmed up the notion that there is energy (as potential energy) associated w/ an electric field (due to the charge creating the field!)

→ We now take one last step to build off of #4 to introduce the concept of **electric potential** (i.e. 'voltage')



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- So we define the electric potential as follows:

$$V(\bar{r}) = \frac{U(\bar{r})}{q_0} = -\frac{W(\bar{r})}{q_0}$$

- 'electrical potential' = 'voltage'
- while V depends upon position, it is a scalar quantity
- has units of volts ($= J/C = N\cdot m/C$)
- tells us the energy of 'test charge' q_0 at some position due to the electric potential energy created by some system of charge
- another unit for energy is an electron-Volt (eV): $1\,eV = e\Delta V = (1.6\times10^{-19}\,C)(1\,V) = 1.60\times10^{-19}\,J$

- 'Electric potential' (V) $\boxed{VS}$ 'potential difference' (ΔV)

→ this deals w/ both convention and a chosen reference

- electric potential: $V = \frac{W}{q_0}$ ← work done to bring q_0 to a point in space

- potential difference: $\Delta V = V_B - V_A = \frac{W_{AB}}{q_0}$ ← work done to bring q_0 from point A to point B

- Convention defines that $V_A = 0$ typically as the 'reference point A is in many cases infinitely far away (where, again, V is defined to be zero by convention)

- So the actual context will depend upon the nature and geometry of the problem at hand

→ Note that for an AA battery $\Delta V = 1.5\,V$ is the potential diff. between the two ends of the battery

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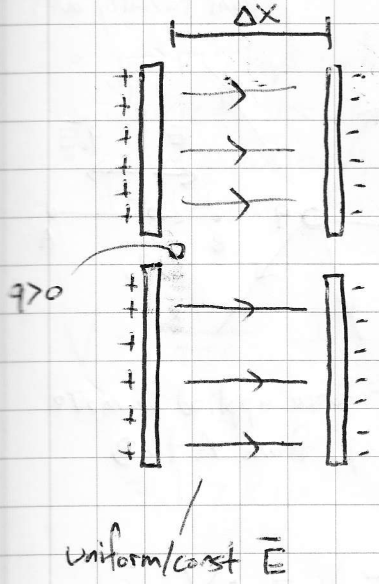
□ Electrostatics $\longleftrightarrow$ Gravity: Analogy Revisited

$$|F_E| = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} \qquad |F_G| = G\frac{m_1 m_2}{r^2}$$

→ gravitational potential: $V_G = \frac{W}{m}$ ← work to move m from infinity to the point in question V_G is evaluated at

(m ← test mass)

ex Parallel-plates of charge (see Giordano Fig. 18.8)



- $\bar{E}$ accelerates test charge to right
- test charge gains energy:

$$U = -W = -qE\Delta x$$

- Divide out q: potential @ + plate → $V(0) = 0$ (we define as)

  potential @ − plate → $V(\Delta x) = -E\Delta x$

- potential between plates: ~~$\Delta V = -E\Delta x$~~

$$\Delta V = V(\Delta x) - V(0) = -E\Delta x$$

- Note then that $E = -\frac{\Delta V}{\Delta x}$

□ We can generalize these statements further (to make them a bit more powerful):

$$E_x(x) = -\frac{dV(x)}{dx}$$

$$\Delta V = V_B - V_A = \frac{W_{AB}}{q_0} = -\int_A^B \bar{E}\cdot d\bar{r}$$

$$V = -\int_\infty^B \bar{E}\cdot d\bar{r}$$

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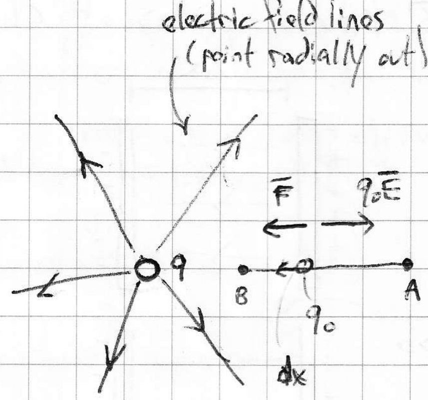
- Note that when $q>0$ goes from a region of high potential to low potential, it converts electric potential energy into kinetic energy. For $q<0$ (e.g. an electron, it goes the other way around

  → So there is a direct relationship between 'volts' and how much energy one can take out of the electric field (think back to a **battery**!)



$\bar{F}$ - force applied to move q_0 from A to B

- Potential Due to a Point Charge
  (derivation more 'complete' re the book)

$$\bar{E}\cdot d\bar{x} = -E\,dx = E\,dr$$

r is meas. re to origin

$$V_B - V_A = -\int_A^B \bar{E}\cdot d\bar{r}$$

$$= -\int_{r_A}^{r_B} E\,dr \qquad \left(E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\right)$$

$$= -\frac{q}{4\pi\epsilon_0}\int_{r_A}^{r_B}\frac{dr}{r^2} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right) = \Delta V$$

- If $r_A \to \infty$, the $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$

- Conversely, since we know $E = \frac{kq}{r^2}$ and $E = \frac{dV}{dr}$, then we must have $V = k\frac{q}{r}$
  → Put another way, the potential tells us what the (vector) electric field is!

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ex What must the magnitude of an isolated positive point charge be for the electric potential at 10 cm from the charge to be + 100 V?

$$q = V\,4\pi\epsilon_0 r = (100\text{ V})(4\pi)\left(8.9\times10^{-12}\ \frac{C^2}{Nm^2}\right)(0.1\text{ m})$$

$$= 1.1\times10^{-9}\ C$$

∴

ex What is the electric potential at the surface of a gold nucleus?

- gold nucleus radius is 6.6×10^{-15} m
- atomic # is $Z = 79$ (i.e. this is the # of protons)
- assume nucleus is spherically symmetric and can be treated as a point charge

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} = \left(9.0\times10^{9}\ \frac{N\,m^2}{C^2}\right)\left(79\cdot1.6\times10^{-19}\ C\right)\Big/\left(6.6\times10^{-15}\text{ m}\right)$$

$$= 1.7\times10^{7}\ V$$

∴