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ex Two 1.0 x 2.0 cm rectangular metal plates are placed 1.0 mm apart. What charge must be placed on each plate to create a uniform electric field of strength 2.0×10^{6} N/C between them?

→ parallel-plate capacitor!

$$E = \frac{Q}{\epsilon_0 A} \quad \text{so} \quad Q = \left(8.85\times10^{-12}\ \frac{C^2}{Nm^2}\right)\left(2.0\times10^{-4}\ m^2\right)\left(2.0\times10^{6}\right.$$

$$= 3.5 \times 10^{-9}\ C = 3.5\ nC$$

(positive plate is charged to +3.5 nC, while the other is charged to −3.5 nC)

→ In terms of the # of electrons, this corresponds to

$$N = \frac{Q}{e} = \frac{3.5\times10^{-9}\ C}{1.6\times10^{-19}\ C/\text{electron}} = 2.2\times10^{10}\ \text{electrons}$$

→ so 2.2×10^{10} electrons were moved from one plate to the other (but note that capacitor as a whole has no net charge!)

ex The cell wall of a neuron typically has an electric field strength of $\sim 1.0\times10^{7}$ N/C (established because the inside of the cell is negative re the outside). What is the typical surface charge density of the cell wall?

→ assume any curvature is negligible, thus we can treat the membrane like a parallel-plate capacitor

$$E = \frac{\sigma}{\epsilon_0} \quad \text{where } \sigma = \text{surface charge density}$$

$$\sigma = \epsilon_0 E = \left(8.85\times10^{-12}\ \frac{C^2}{Nm^2}\right)\left(1.0\times10^{7}\ N/C\right)$$

$$= 8.9\times10^{-5}\ C/m^2$$

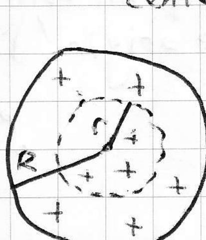
→ though σ is somewhat large, cells are fairly small. If we assume a cell is spherical (in contrast to our negligible curvature assumption!) and $\simeq 10\ \mu m$ in diameter, the surface area is $\simeq 3\times10^{-10}\ m^2$. So the total charge on the outer surface is $\sim 3\times10^{-14}\ C$, or ~ 200000 ions

∴

Ex What is the electric field inside a uniformly charged sphere of radius R?

- Does E increase or decrease as we move away from the center?
- Symmetry indicates E must point either radially inwards or outwards
- Similarly, it must only depend upon r (the radial distance away from the center)

→ Use Gauss' Law! (Gaussian surface w/ $r \leq R$ centered about the charged sphere)



$$\Phi_E = E A_{sphere} = 4\pi r^2 E = \frac{Q_{in}}{\epsilon_0}$$

So what is Q_{in}?
(just the charge inside the Gaussian surface; we will make use of the total sphere's **uniformity**)

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ex. (cont.)

total sphere's volume charge density:

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3} \quad (Q\text{ - total charge of sph}$$

- the charge enclosed in our Gaussian surface is a fraction of that:

$$Q_{in} = \rho V_r = \left(\frac{Q}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) = \frac{r^3}{R^3} Q$$

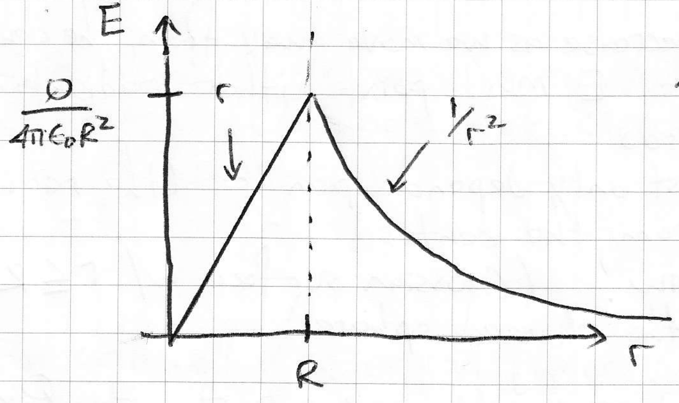
→ so the amount of enclosed charge increases as the cube of r!

- Back to Gauss' Law:

$$4\pi r^2 E = \frac{(r/R)^3 Q}{\epsilon_0}$$

$$\rightarrow \quad E_{inside} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^3} r$$

so the field strength increases linearly w/ r inside the sphere (but will decrease as $1/r^2$ outside it!)



→ at the surface, the electric field is

$$E = k\frac{Q}{R^3}R = k\frac{Q}{R^2}$$

(like a point charge at the origin!)

∴

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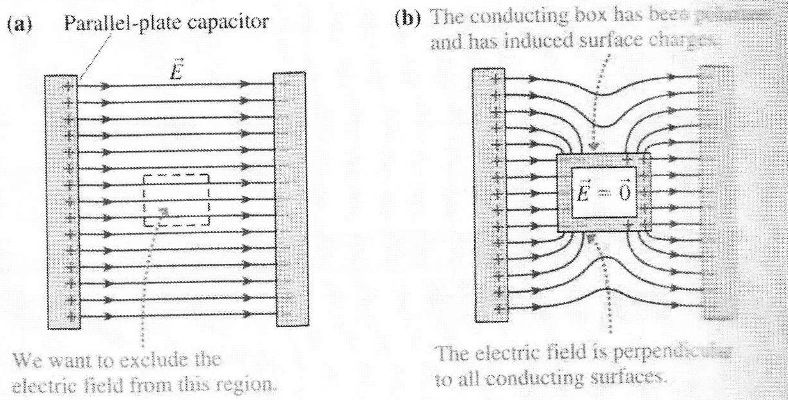
ex Devise a means to create a 'shielded' region of zero electric field inside the field of a uniform parallel-plate capacitor.

→ metal box!

FIGURE 27.32 The electric field can be excluded from a region of space by surrounding it with a conducting box.



→ some further figures motivating how to think about electric fields **inside** conducting materials

FIGURE 27.31 A Gaussian surface surrounding a hole inside a conductor in electrostatic equilibrium.

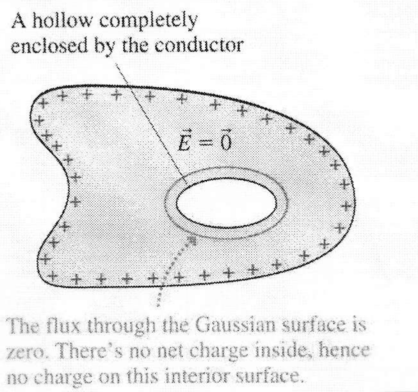
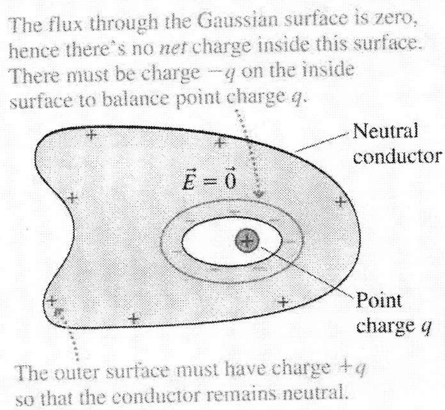


FIGURE 27.33 A charge in the hole causes a net charge on the interior and exterior surfaces.



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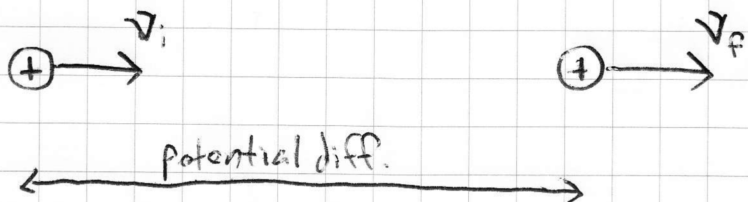
ex A proton w/ a speed 2.0×10^5 m/s enters a region of space where an electric potential exists. What is the proton's speed after it moves through a region where $\Delta V = 100$ V? What if it were an electron instead?

velocity (not voltage!)



$$\Delta V = V_f - V_i$$

- Energy is conserved. The electric potential has potential energy that can be transferred to the proton and thereby affect its kinetic energy.
- A positive charge slows down as it moves into a space of higher potential $(K \to U)$. An electron speeds up $(U \to K)$
- potential energy of charge is $U = qV$. Conservation of energy implies:

$$K_f + qV_f = K_i + qV_i \longrightarrow K_f = K_i - q\Delta V$$

so $\frac{1}{2} m v_f^2 = \frac{1}{2} m v_i^2 - q\Delta V$

$$\longrightarrow v_f = \sqrt{v_i^2 - \frac{2q}{m}\Delta V}$$

⇝ for our proton, we'll have $1.4 \times 10^5 \ \frac{m}{s} = v_f$ (the electron picks up speed to 5.9×10^6 m/s)