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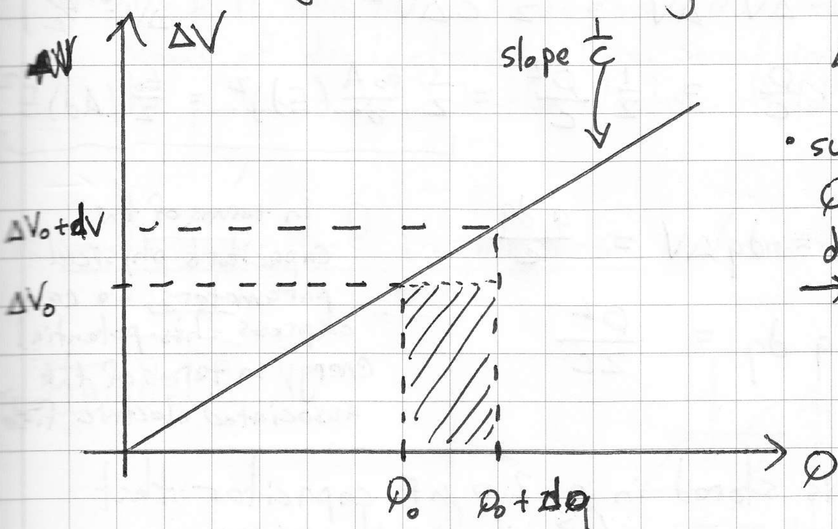
Energy Storage in Capacitors

- Imagine two (initially uncharged) parallel plates and we start moving electrons from one plate to the other.

- This will set up an electric field and we will need to increasingly do more and more work to get those electrons across

- Thus, the plates (i.e. a parallel-plate capacitor) is essentially storing up the energy that we did work on to move those charges to create the electric field
  → analogous to compressing a spring!



$$\Delta V = \frac{1}{C} Q$$

- suppose the plates have charge Q_0 and thereby a potential difference ΔV_0.
  → How much energy is required to charge up the capacitor by an extra amount of charge dq?

→ essentially we have to move charge dq across the plates through the electric field $E = \Delta V_0 / d$

Since $U = qV$, the $\Delta U \simeq dq\, \Delta V_0$

change in potential energy

only approx. unless dq is small!

area of shaded rectangle

NOTE: Unless dq is small, we neglect the triangle!

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- So the change in energy is ΔU, that is we did work to move dq such that the capacitor changes its potential from ΔV_0 to $\Delta V_0 + dV$ (i.e. the capacitor stored the energy ΔU)

- So we could add up all the contributions along these lines, which essentially means determining the area under the ΔV vs. Q curve (just a simple triangle!)

$$U_{capacitor} = \frac{1}{2} Q_0 \Delta V_0 \longrightarrow \frac{1}{2} Q \Delta V$$

call this U_c

$$= \frac{1}{2} C \cdot \Delta V \cdot \Delta V = \frac{1}{2} C \Delta V^2 \quad \left(\text{since } \Delta V = Q/C\right)$$

$$= \frac{1}{2} Q \cdot \frac{Q}{C} = \frac{1}{2} \frac{Q^2}{C} = \underbrace{\frac{1}{2} \frac{\epsilon_0 A}{d} (Ed)^2 = \frac{\epsilon_0}{2} (Ad) E^2}$$

in terms of the capacitor's physical parameters, we can express this potential energy in terms of the associated electric field

Calculus Aside

$$dU = dq \Delta V = \frac{q\,dq}{C}$$

$$U_c = \frac{1}{C} \int_0^Q q\,dq = \frac{Q^2}{2C}$$

$\boxed{\text{ex}}$ How much energy is stored in a 2.0 μF capacitor that has been charged to 5000 V? What is the average power dissipation if its discharged in 10 μs?

$$U_c = \frac{1}{2} C \Delta V^2 = \frac{1}{2} (2.0 \times 10^{-6}\, F)(5000\, V)^2 = 25\, J$$

$$P = \frac{\Delta E}{\Delta t} = \frac{25\, J}{1.0 \times 10^{-5}\, s} = 2.5 \times 10^6\, W = 2.5\, MW$$

(equivalent to dropping a 1 kg mass 2.5 m)

→ this is the basic idea underlying a **defibrillator** (which stores ~300 J to release over ~2 ms)

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$\boxed{ex}$ The spacing between the plates of a 1.0 μF capacitor is 0.050 mm

- what is the surface area of the plates?

$$C = \frac{\epsilon_0 A}{d} \rightarrow A = \frac{dC}{\epsilon_0} = 5.65\ m^2$$ → this is an enormous surface area!

definition of capacitance!

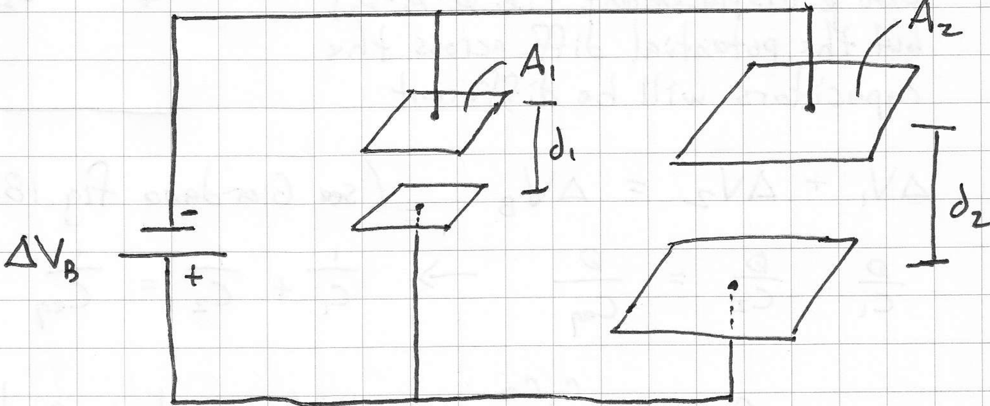
- If attached to a 1.5 V battery, how much charge is on the plates?

$$Q = C\Delta V = 1.5 \times 10^{-6}\ C = 1.5\ \mu C$$

→ we will soon see how we can reduce this surface area by means of **dielectrics** (i.e. something to sandwich between the plate to affect the electric field)

Combinations of Capacitors (series vs. parallel)

$\boxed{\text{Parallel}}$



→ the same potential difference (ΔV_B) appears across both capacitors, but a different amount of charge is displaced (to see *why* ΔV_B is the same for both, think about the way the are connected to one another → charge would move to equilibrate any

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Parallel (cont.)

$$Q_1 = C_1 \Delta V_B \quad \text{and} \quad Q_2 = C_2 \Delta V_B$$

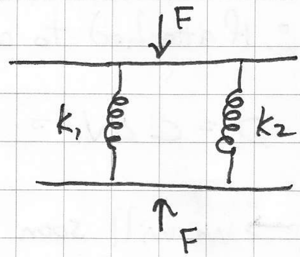
$$Q_{total} = Q_1 + Q_2 = C_1 \Delta V_B + C_2 \Delta V_B = (C_1 + C_2)\Delta V_B$$

$$= C_{eq} \Delta V_B$$

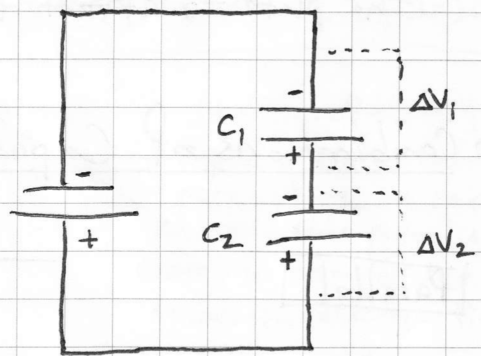
[C_{eq} is the 'equivalent' capacitance if you were to consider the 'whole' as one capacitor

→ the combined system acts like one capacitor with a (bigger!) capacitance $C_{eq} = C_1 + C_2$

⇒ If this seems a bit odd/unintuitive, think about what happens when you place two springs between two boards and try to compress them



Series

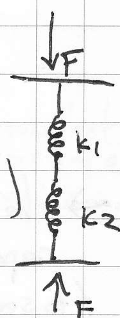


- this case is a bit trickier, as the two now have the same charge displacement (i.e. $Q = Q_1 = Q_2$) but the potential diff. across the capacitors will be different

$$\Delta V_1 + \Delta V_2 = \Delta V_B$$ (see Giordano fig. 18.25)

$$\frac{Q}{C_1} + \frac{Q}{C_2} = \frac{Q}{C_{eq}} \longrightarrow \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_{eq}}$$

or $$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$ (again think of the mechanical spring analogy)



→ very different re 'parallel' case in that if $C_1 >> C_2$, C_{eq} is small (i.e. since C_2 can't hold much charge, the combined system is limited)