Tutorial Problems

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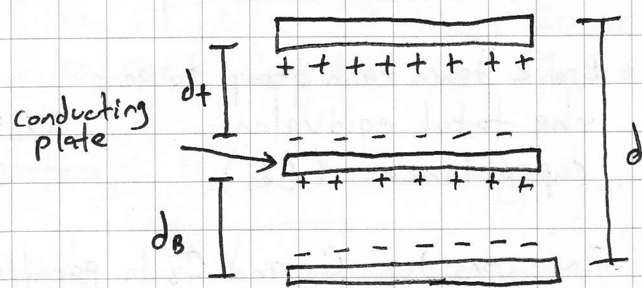
Ex Dielectrics are insulators. But why not use a conductor?
(see Giordano fig. 18.25)

- No plate inbetween:
  $$C_0 = \epsilon_0 A/d$$



$$d_t + d_b = d$$

- Stick plate in middle
  → two capacitors in series!

$$C_t = \frac{\epsilon_0 A}{d/2} = 2\frac{\epsilon_0 A}{d} = C_b = 2C_0$$

but $$C_{eq} = \frac{C_t C_b}{C_t + C_b} = \frac{4C_0^2}{4C_0} = C_0$$

→ same capacitance!!

NOTE: $\bar{E}$ field inside middle plate is zero (see 1/22 notes for reference)

- what if we vary d_t and d_b?

$$C_t = \frac{\epsilon_0 A}{C_t}, \quad C_b = \frac{\epsilon_0 A}{C_b}$$

$$C_{eq} = \frac{C_t C_b}{C_t + C_b} = \frac{\frac{(\epsilon_0 A)^2}{d_t d_b}}{\epsilon_0 A\left(\frac{1}{d_t} + \frac{1}{d_b}\right)} = \epsilon_0 A \frac{\frac{1}{d_t d_b}}{\frac{d_b + d_t}{d_t d_b}} = \frac{\epsilon_0 A}{d}$$

→ so we don't gain (or lose) anything by sticking the conducting plate in there. However, if you consider the thickness of that middle plate (i.e. it takes up room and could effectively reduce d), you may be able to get a bit of a boost in C_{eq} ...

$\therefore$

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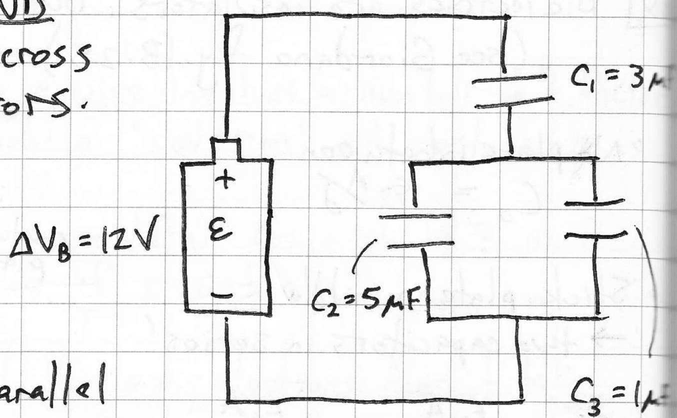
ex. Find the charge on AND the potential difference across each of the three capacitors.



- Break down each group to get the total equivalent capacitance (C_{eq})

- First consider C_2 and C_3 in parallel

$$C_{eq23} = C_2 + C_3 = 6\ \mu F$$

(C_{23} is the equiv. capacitance across C_2 and C_3)

- Now we have two in series : $C_{eq} = \dfrac{C_1 C_{eq23}}{C_1 + C_{eq23}}$

$$= \frac{(3)(6)}{6+3} = \frac{18}{9} = 2\ \mu F$$

⇝ so the total equivalent capacitance is $C_{eq} = 2\ \mu F$. But now what?

- $\Delta V_c = \Delta V_B = 12\ V$ (this is the potential drop across the equivalent capacitor)

$Q = C \Delta V_c = 24\ \mu C$ (total charge across 'equiv. capacitor')

⇝ now we go in reverse!

- Capacitors in series have the same charge on them, so

$$Q_1 = Q_{23} = 24\ \mu F$$

(Q_{23} is charge on that equiv. capacitance C_{23})

Since $Q = C\Delta V$, $\begin{cases} Q_1 = C_1 \Delta V_1 \longrightarrow \Delta V_1 = 8\ V \\ Q_{23} = C_{23} \Delta V_{23} \longrightarrow \Delta V_{23} = 4\ V \end{cases}$

ex (cont.) 1/29/13

- Now, capacitors in parallel have the same potential difference across them, so:

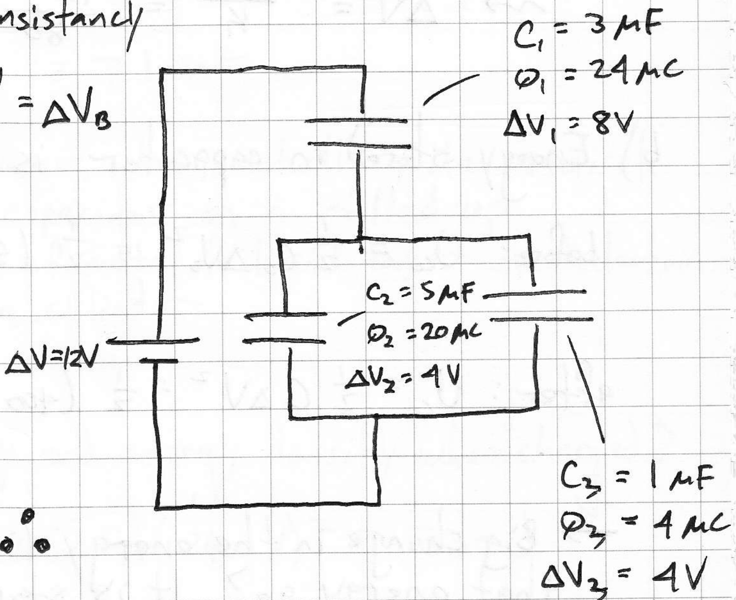
$\Delta V_2 = \Delta V_3 = 4\,V \rightarrow \begin{cases} Q_2 = C_2 \Delta V_2 \rightarrow Q_2 = 20\,\mu C \\ Q_3 = C_3 \Delta V_3 \rightarrow Q_3 = 4\,\mu C \end{cases}$

- Two important checks re consistancy

1) $\Delta V_1 + \Delta V_{23} = 8 + 4 = 12\,V = \Delta V_B$

2) $Q_2 + Q_3 = 20 + 4 = 24\,\mu C$

→ so our found values for each **has to be** consistent w/ our total 'equivalent' circuit



ex A 5.0 nF parallel-plate capacitor is charged to 160 V. It is then disconnected from the battery and immersed in distilled water.

a) What is the capacitance and voltage of the water-filled capacitor?

b) What is the energy stored before and after immersion?

- Distilled water has no (conducting) ions, so it is a good insulator. Thus, the water between the plates will act as a dielectric.

$K \simeq 80$ for distilled water (see table 18.1 of Giordano)

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ex (cont.)

a) • change in capacitance: $C = K C_0 = 80 \cdot (5.0 \times 10^{-9} F) = 400\ nF$

• change in voltage: $\rightarrow$ Q stays const. and $\Delta V = Q/C$

$$\rightsquigarrow \Delta V = \frac{\Delta V_0}{K} = \frac{160 V}{80} = 2.0 V$$

b) Energy stored in capacitor is $U_c = \frac{Q^2}{2C} = \frac{1}{2} C \Delta V^2$

before: $U_{C_0} = \frac{1}{2} C_0 \Delta V_0^2 = \frac{1}{2}(5.0 \times 10^{-9} F)(160 V)^2 = 6.4 \times 10^{-5} J$

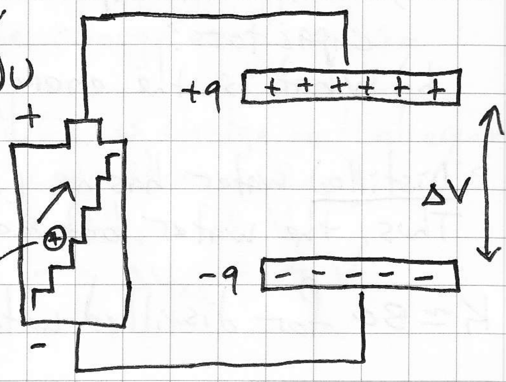
after: $U_c = \frac{1}{2} C \Delta V^2 = \frac{1}{2}(400 \times 10^{-9} F)(2 V)^2 = 8.0 \times 10^{-7} J$

$\rightarrow$ Big change in the energy (it decreased!). Where did that energy go? It is stored in the electric field that is polarizing the dielectric!!
(so energy is conserved, it is just now be stored in a different fashion)

$\therefore$

ASIDE Energy stored in capacitor

• think of battery as a 'charge ~~escalator~~ escalator' that does work $dq \Delta V = dU$ to move dq from negative to positive plate



$$dU = dq \Delta V = \frac{q\,dq}{C}$$

$$\rightarrow U_c = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$

ex A defibrillator unit contains a 150 μF capacitor charged to 2100 V. The plates are separated by a 0.05 mm thick insulator w/ dielectric const. $\kappa = 80$.

a) What is the area of the plates?

$$C = \kappa \frac{\epsilon_0 A}{d} \longrightarrow A = \frac{cd}{\kappa \epsilon_0} = \frac{(150\times10^{-6}\,F)(5.0\times10^{-5}\,m)}{(120)(8.85\times10^{-12}\,C^2/N\cdot m^2)}$$

$$= 7.1\ m^2$$

→ this is really big! However, this is actually quite practical as one can make the capacitor as a **rolled-up 'sandwich'** (sheet of metal foil and layer of dielectric atop rolled up into a cylinder

b) what is the stored energy and energy density when charged?

$$U_c = \tfrac{1}{2} C \Delta V^2 = \tfrac{1}{2}(150\times10^{-6}\,F)(2100\,V)^2 = 330\,J$$

- Electric field strength: $E = \frac{\Delta V}{d} = \frac{2100\,V}{5.0\times10^{-5}\,m} = 4.2\times10^{7}\,\frac{V}{m}$
- energy density $\left(u_E \equiv \frac{\text{energy stored}}{\text{volume stored}} = \kappa \frac{U_c}{A d} = \tfrac{1}{2}\kappa \epsilon_0 E^2\right)$

$$u_E = \tfrac{1}{2}(120)(8.85\times10^{-12}\,\tfrac{C^2}{N\cdot m^2})(4.2\times10^{7}\,V/m)^2$$

$$= 9.4\times10^{5}\ J/m^3$$

→ energy is delivered quickly to patient's chest (to 'jumpstart' the heart) when the capacitor is discharged