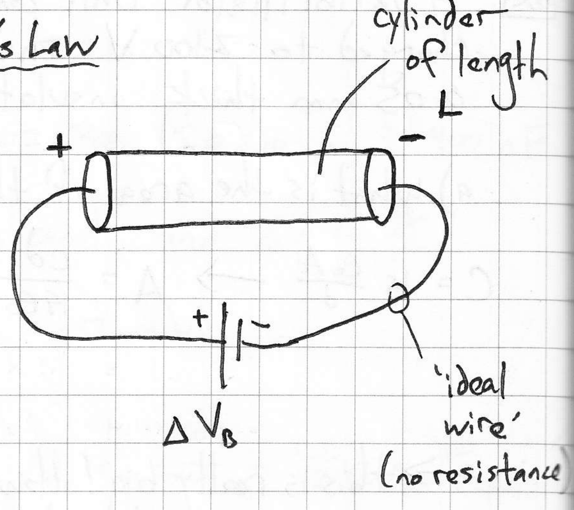
1/30/13

Current and Resistance → Ohm's Law



- Suppose a battery (or charged capacitor) tries to push charge through a 'wire'.

- We assume the 'wire' is a cylinder of length L and contains conduction electrons

→ battery creates a potential differen ΔV_R across the cylinder where $\Delta V_B = \Delta V_R$ (i.e. the battery creates the voltage drop across the cylinder)

- An electric field of strength $E = \frac{\Delta V_R}{L}$ is set up (i.e. since the cylinder ~~is a conductor~~ can have properties of both a conductor and an insulator, we can consider a possible electric field set up inside the wire)

→ this $\bar{E}$ field would push positive charges from left to right (or equivalently, conduction electrons from right to left)

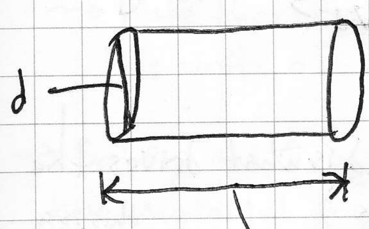
- $\bar{E}$ continuously accelerates electrons, but they will quickly reach a terminal velocity due to the metal lattice ions deflecting them (they absorb a bit of energy and thereby create a drag force) **on average**

v_d (**drift velocity**)

→ notion of a '**resistance**'

⇒ we end up w/ a constant current (through a given cross-sectional area of the wire)

- Consider a segment of the cylinder w/ diameter d



cross-sectional area : $A = \frac{\pi}{4} d^2$

Volume $= \pi \frac{d^2}{4} \cdot v_d \Delta t \quad (\equiv \mathcal{V})$

average distance traveled by a drifting e^- in time Δt $(= v_d \Delta t)$

- This volume contains the electrons that will cross the front surface in time Δt. Our job now is to estimate how much charge that is (i.e. how much current there is)

→ Need to know the density of conduction electrons (n_e)

This is a material property

Metal	n_e $\left(\frac{\#}{m^3} = m^{-3}\right)$
Al	6.0×10^{28}
Cu	8.5×10^{28}
Fe	8.5×10^{28}
Au	5.9×10^{28}
Ag	5.8×10^{28}

- # of charge carriers: $N \equiv n_e \mathcal{V} = n_e \left(\frac{\pi}{4} d^2 \cdot v_d \Delta t\right)$

- amount of charge passed: $\Delta q \equiv N(-e) = -e n_e \left(\frac{\pi}{4} d^2 v_d \Delta t\right)$

- charge current : $I \equiv \frac{\Delta q}{\Delta t} = -e n_e \left(\frac{\pi}{4} d^2 v_d\right)$

$\Rightarrow$ So how many amps of current depends upon several things:
- drift velocity (faster → more current)
- cross-sectional area ($A = \frac{\pi}{4} d^2$; more = better)
- conduction electron volume density (n_e)

1/30/13

- So a given wire will be made from some material (i.e. n_e is fixed) and will have a certain diameter (the 'gauge' of the wire). But what factors dictate V_d?

$$V_d \propto E = \frac{\Delta V}{L}$$ (since the electric field is what drives the conduction electrons)

ASIDE $$V_d = \frac{e\tau}{m} E$$

this is a material property

τ - mean time between collisions of an electron and lattice atom

m - mass of electron

→ analogous to the kinetic theory of gases (we can deduce this by considering the 'average' path an electron takes)

→ since $I \propto V_d$ and $V_d \propto E$ and $E = \frac{\Delta V}{L}$

$$\Rightarrow I \propto \Delta V$$

- If current and voltage are proportional, we can specify a constant of proportionality:

$$\boxed{I = \frac{\Delta V}{R}}$$ Ohm's Law

Here, R is the **resistance**. For our wire, we can specify it as:

$$R \equiv \rho \frac{L}{A}$$

L - length

A - cross-sectional area

where ρ is a material property called the **resistivity** and is proportional to $\frac{1}{n_e \tau}$ (e.g. increasing n_e decrease ρ)

1/30/13

- Units for resistance are $[R] = \frac{V}{A} \equiv \Omega$ ← greek letter omega
  (ohm, named after Ohm!)

  → Note that resistivity has units of $\Omega \cdot m$

→ So n_e and $\uparrow$ are two key material properties that determine ρ and thereby the resistance of a given material

	Metal	$\rho\ [\Omega \cdot m]$
Conductors	Cu	1.7×10^{-8}
	Al	2.7×10^{-8}
	Au	2.2×10^{-8}
	Ag	1.6×10^{-8}
	Pb	22×10^{-8}

	other	$\rho\ [\Omega \cdot m]$
insulators	glass	$10^{9} - 10^{12}$
	rubber	$10^{12} - 10^{14}$
Semi-conductors	silicon	$10^{-1} - 10^{2}$
	Germanium	$10^{-3} - 10^{0}$

- Note that since $R \propto \rho$ and that $\rho \propto \frac{1}{n_e \uparrow}$, different materials can have different resistive characteristics depending upon their material properties; this gives rise to the notion of materials behaving like a **conductor** vs. **insulator** vs. **semi-conductor** [these latter materials being a key component of all modern electronics, as they are used in integrated circuits and relate directly to <u>Moore's Law</u> (i.e. transistor density doubles every 2 years)]

- Note that electron conduction can depend upon other parameters such as temperature and 'purity' of materials, factors we have ignored here for clarity

- Note that in an '**ideal wire**' (which we use extensively in our analysis of circuits), there is no voltage drop along its length and thus there is no resistance

1/30/13

ex what is the ~~[illegible]~~ current in a 2.0 mm copper wire if the electron drift speed in 1.0×10^{-4} m/s?

$$A = \pi r^2 = 3.14 \times 10^{-6}\ m^2$$

$$I = -e n_e A v_d = (-1.6 \times 10^{-19}\ C)(8.5 \times 10^{28}\ m^{-3})(3.14 \times 10^{-6}\ m^2) \cdot (1.0 \times 10^{-4}\ m/s)$$

$$= 4.27\ C/s = 4.27\ A \quad \text{(a large current!)}$$

Note that this means $\sim 2.7 \times 10^{19}$ electrons are moving across the wire's cross-section per second!!

- One important point relating the 'speed of electric current' (i.e. the relationship between drift velocity and how quickly a 'signal' can make it down a wire)
  - drift velocity is relatively small, but current moves quickly (nearly the speed of light!)
  - Analogy to water in a hose (Giordano ch.19.3): Not the velocity of an individual electron that matters, but how it affects the 'swarm'