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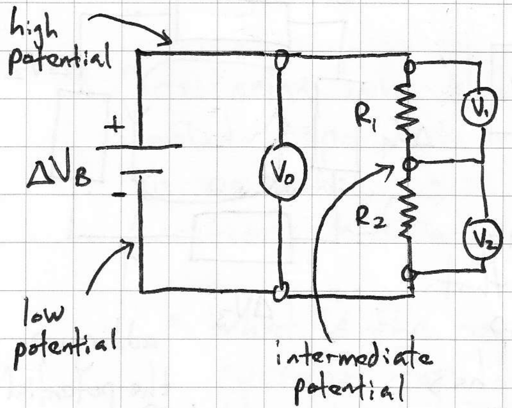
Putting the Pieces Together re Kirchoff's Circuit Laws

- A few key pieces we have learned so far:
  - voltage drop across a capacitor: $\Delta V_c = \frac{Q}{C}$ (definition of capacitance)
  - Voltage drop across a resistor: $\Delta V_R = IR$ (ohm's Law)
  - for a series combination of resistors or capacitors, the voltage drops add up to that provided by the battery



Ⓥ - indicates a 'voltmeter' (see Giordano ch. 19.6)

$V_0 = \Delta V_B$

$V_1 = -IR_1$

$V_2 = -IR_2$

$\longrightarrow$ $V_B = -V_1 - V_2$

(this was our 'voltage divider' example)

$\rightsquigarrow$ $V_1, V_2 < 0$ because the potential **drops** going across the resistor (whereas it increases across the battery)

$\Rightarrow$ So this last point is essentially firmed up via Kirchoff's loop rule: for any **closed path**, all the voltage changes **must add up to zero**

$$\Delta V_B + V_1 + V_2 = 0 \longrightarrow \Delta V_B - R_1 I - R_2 I = 0$$

$$\longrightarrow \Delta V_B = (R_1 + R_2) I$$

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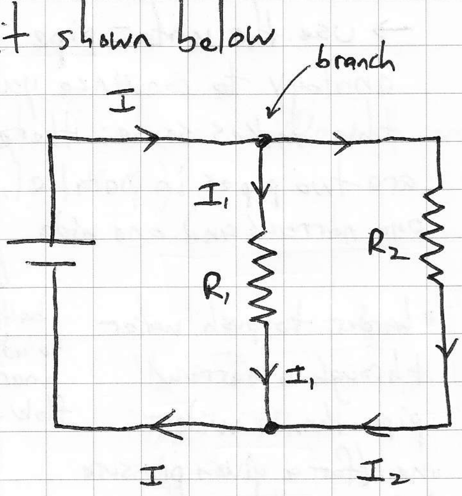
- So in the spirit of the voltage divider, we used Kirchoff's loop rule to deduce that for **resistors in series**:

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

- Let's now consider a branching circuit shown below
  - Now at each junction, Kirchoff's junction rule tells us that

$$\sum I_{in} = \sum I_{out}$$

⇝ our goal is to use this rule to determine the equivalent resistance of resistors in parallel



we can measure the current using an ammeter (see next pg.)

1) $I = I_1 + I_2$ (junction rule)

2) $\Delta V_1 = \Delta V_2 = \Delta V_B$ (loop rule)

3) $\left.\begin{array}{l} \Delta V_B = R_{eq} I \\ \Delta V_1 = I_1 R_1 \\ \Delta V_2 = I_2 R_2 \end{array}\right]$ (Ohm's Law x3)

$$\rightarrow \frac{\Delta V_B}{R_{eq}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} \rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\text{so } R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

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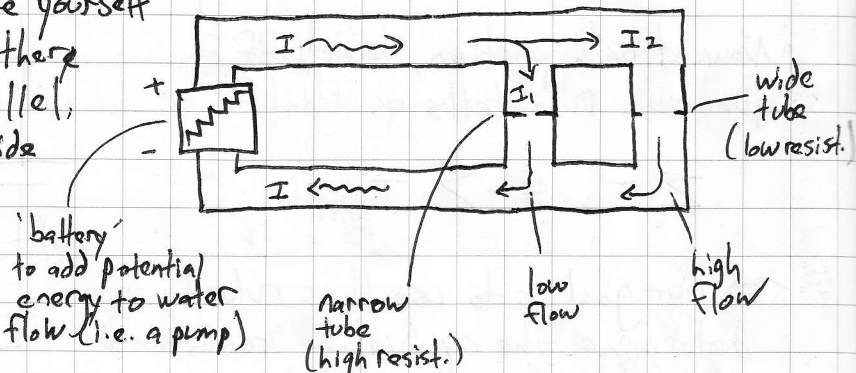
- So for combinations of two resistors:

  <u>Series</u>: $R_{eq} = R_1 + R_2$ (same as capacitors in parallel)

  <u>parallel</u>: $R_{eq} = \dfrac{R_1 R_2}{R_1 + R_2}$ (same as capacitors in series)

→ use the water pipe analogy to convince yourself this makes sense: there are two pipes in parallel, one narrow and one wide

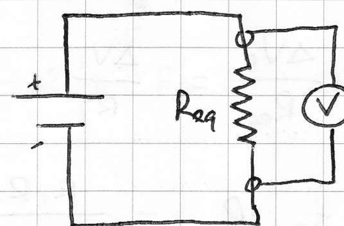


- harder to push water through a narrow pipe than a wider one (for a given pressure difference) → more water will flow through the wider tube (i.e. higher current there, since resistance is lower)

(this basic idea is used in microfluidic devices to help isolate/diagnose disease pathogens!)

- There are actually devices that allow us to measure either voltage or current within a circuit (as alluded to earlier)

  - **<u>voltmeter</u>**
    → allows for the measurement of voltage; must be connected in <u>parallel</u> w/ the circuit element of interest and ideally has a **infinite** resistance (so not to take energy away)



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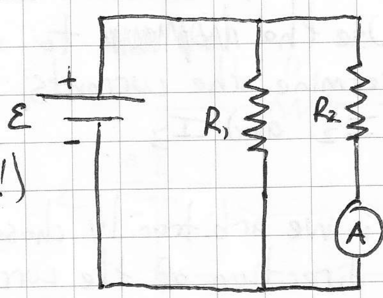
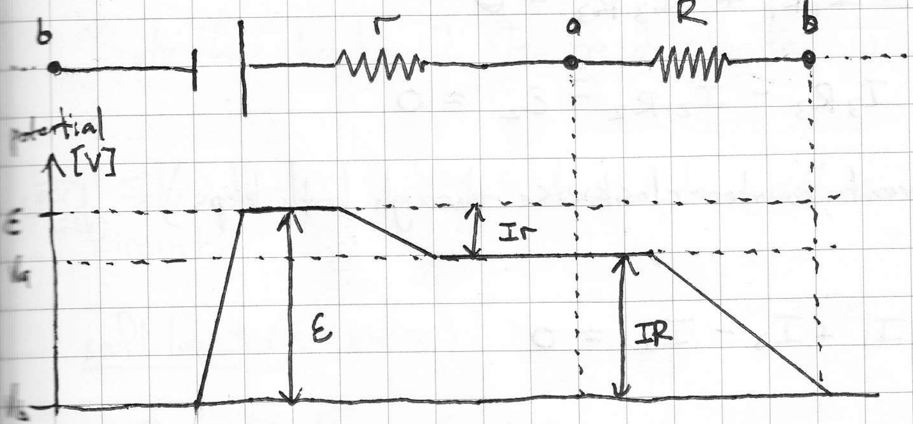
- ammeter
  → allows for the measurement of current; must be connected in series w/ the branch of interest (remember the junction rule!) and ideally should ~~not~~ not affect the value of the current (i.e. it has zero resistance



☐ For clarity, let's visualize a simple circuit in three different ways that contains a 'real battery' (Giordano ch. 19.4)



$V_a = Ir$
$V_{ab} = IR$
$V_b = 0$

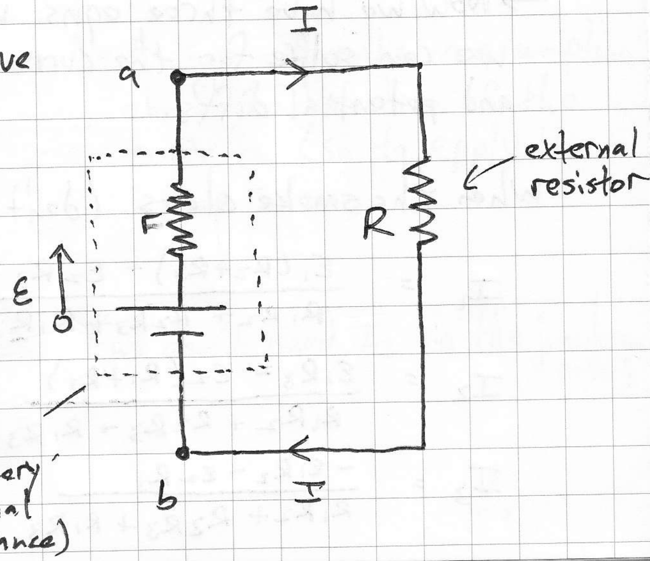
→ applying the loop rule above, we have

$$V_b = \varepsilon - V_a - V_{ab} = V_b$$

$$\rightarrow \varepsilon = I(r+R)$$

(resistors in series again!)

⇒ helps to visualize how potential changes around circuit



[to think about for 2/5/13 tutorial]

Determine the currents I_1, I_2 and I_3 through each of the resistors: