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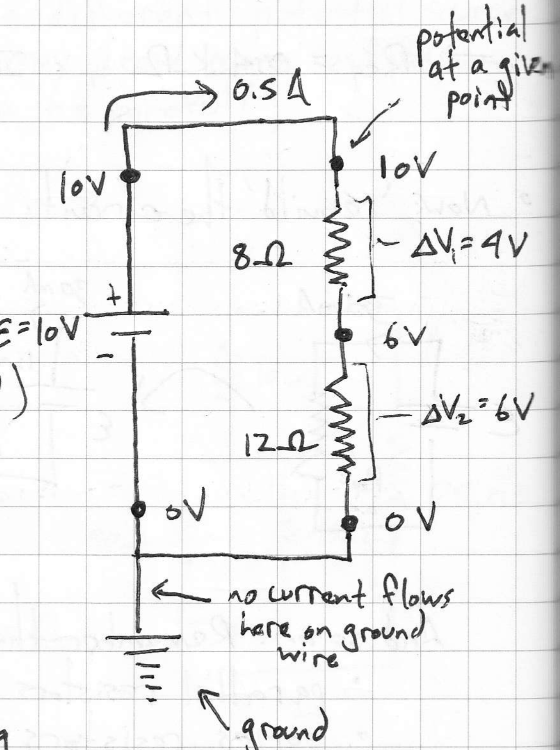
Ground

- We have been a bit loose in defining an appropriate reference (i.e. where is the potential zero?). When dealing with potential diffs. this is not an issue.

  → However when connecting different circuits together, you need a common reference

- Consider the earth as a conductor upon whose potential we define as 0 V. Suppose we connect a wire from our circuit to the earth (such that there is **no closed loop** to ground!)



  - Ground wire is not part of complete circuit, so no current flows along it

  - Since wires are equipotential, it gives connecting points along the wire the same potential (i.e. 0 V)

  - Though the ground wire can change the potential, it <u>**does not affect**</u> the way the circuit functions (i.e. total resistance stays unchanged, voltage drop across resistor is unchanged, current through resistors is unchanged)

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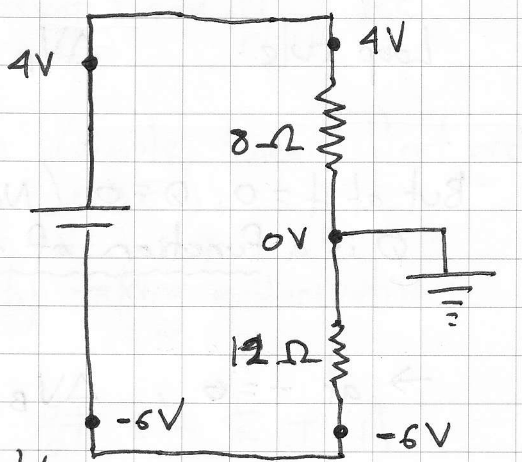
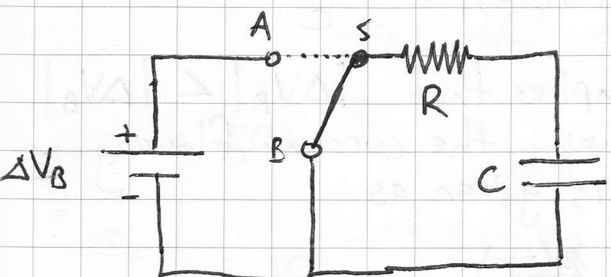
(*) → what the ground allows us to do is specify the potential at a given point along the circuit
(not just a potential **difference** across a given element)

ex What if we re-wired the the circuit such that the ground was inbetween the two resistors?



→ the only thing that has changed is the potential (not the potential difference!) at a given point along the circuit, simply just because we changed our reference point!

<u>RC Circuits</u> → what happens when a capacitor is added to the mix?



Switch connects either:

- S to A (battery changes C via R)
- S to B (battery disconnected → C discharged via R)

<u>Basic Idea</u>: When connected to battery, current flows through R and charges C. Once battery is disconnected, that energy stored in C gets 'discharged' via R.

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1) Start w/ S-B connection. No charge on C (i.e. $Q=0$)

2) Connect S-A. Current starts to flow due to battery

Loop rule: $\Delta V_B - \underbrace{IR}_{\Delta V_R} - \underbrace{\frac{Q}{C}}_{\Delta V_C} = 0$

potential drop due to capacitor

But at $t=0$, $Q=0$ (Note that Q is a <u>function of time</u>, as is I)

$\rightarrow$ at $t=0$, $\Delta V_B = IR \longrightarrow I(t=0) = \frac{\Delta V_B}{R}$

$= \Delta V_R$

- since we have a closed loop, that means that at $t=0$ we have a current flowing through C (limited by R)

3) For $t>0$, charge $Q(t)$ starts to build up on the capacitor plates

$$|V_c(t)| = \frac{Q(t)}{C}$$

from the loop rule, this implies that $|\Delta V_R| < |\Delta V_B|$ for $t>0$. Put another way, the current flowing (through both R and C) is given as

$$I(t) = \frac{1}{R}\left(\Delta V_B - \frac{Q(t)}{C}\right)$$

$\rightarrow$ Can we obtain a more specific expression for $I(t)$ that doesn't depend upon $Q(t)$?
[we will need a bit of calculus]

4) Remember that $I(t) \equiv \frac{dQ}{dt}$ (by definition). So re-writing things:

$$\frac{dQ}{dt} = \frac{\Delta V_B}{R} - \frac{1}{RC} Q(t) = I_0 - \frac{Q(t)}{\tau}$$

where $I_0 = \frac{\Delta V_B}{R}$ (initial current) and $\tau \equiv RC$ (this is called the RC time const.)

<u>NOTE</u>: This is an example of a (simple, linear first order ordinary differential eqn., or **ODE**)

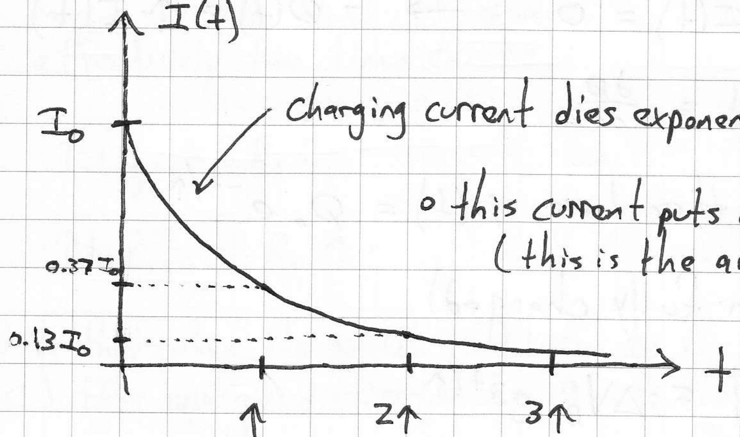
⇝ we can get back to $I(t)$ by taking a derivative:

$$\frac{d^2Q}{dt^2} = -\frac{1}{\tau}\frac{dQ}{dt} \quad \longleftrightarrow \quad \frac{dI}{dt} = -\frac{1}{\tau} I(t)$$

(another ODE!)

6) This is a fairly simple/common ODE that you can more or less guess the answer based upon a result from calculus (i.e. $\frac{d}{dx}e^x = e^x$) combined w/ the chain rule

$$\longrightarrow \quad I(t) = I_0 e^{-t/\tau}$$



Charging current dies exponentially w/ time const. τ

- this current puts change $Q(t)$ on the plate (this is the area under the $I(t)$ curve)
- this charge builds up the potential $\Delta V_c = \frac{Q(t)}{C}$

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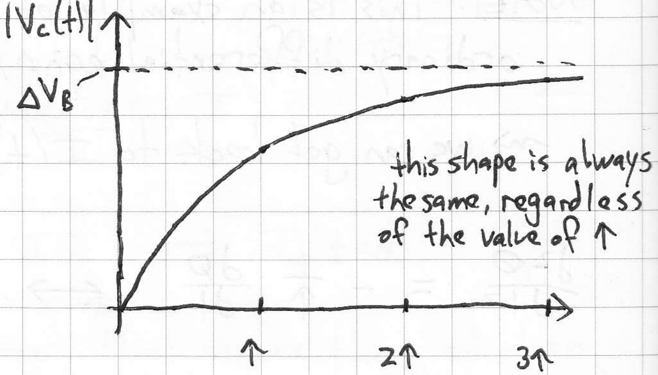
7) Can we determine what $\Delta V_c(t)$ does? $\rightarrow$ use loop law!

$$\Delta V_B + \Delta V_R + \Delta V_C = 0 \quad \longrightarrow \quad \Delta V_c = -\Delta V_B + RI(t)$$

$$\text{so} \quad |V_c(t)| = \left| \Delta V_B \left(-1 + \frac{RI_0}{\Delta V_B} e^{-t/\tau}\right) \right| = \Delta V_B \left(1 - e^{-t/\tau}\right)$$

$\rightarrow$ so potential diff. across capacitor is also an exponential function asymptoting to ΔV_B



8) What if we flipped the switch from S-A to S-B?

$\rightarrow$ same basic picture, but now rather than 'charge' the capacitor will 'discharge'

Loop Law: (no battery) $\quad \Delta V_c(t) + \Delta V_R(t) = 0$

$$-\frac{Q(t)}{C} - RI(t) = 0 \quad \longrightarrow \quad -Q(t) = \tau I(t)$$

$$\text{so} \quad -\frac{Q(t)}{\tau} = I(t) = \frac{dQ}{dt}$$

similar to before, we then have $Q(t) = Q_0 e^{-t/\tau}$

- $Q_0 = C\Delta V_B$ (when fully charged)

$$\Longrightarrow \quad |V_c(t)| = \Delta V_B e^{-t/\tau}$$

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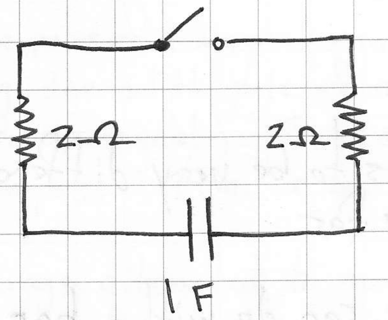
ex



Once the switch is closed, what is the time const. for the discharge of the capacitor?

→ two resistors in series ($R_{eq} = 4\Omega$)

$$\text{so } \tau = RC = (4\Omega)(1\,F) = 4\,s$$

Note that the resistors do not 'cancel out' such that the capacitor doesn't discharge!

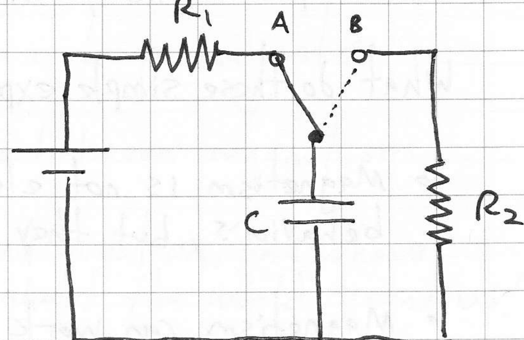
NOTE $\tau = RC$ and $[\tau] = s$ ⇝ does this make sense?

$$[R] = \Omega = \frac{V}{A} = \frac{kg\cdot m^2/A\cdot s^3}{C\cdot s} = \frac{J\cdot s}{C^2}$$

$$[C] = F = \frac{A\cdot s}{V} = \frac{C^2}{J}$$

$$\text{so } [R][C] = \frac{J\cdot s}{C^2}\cdot\frac{C^2}{J} = s \checkmark$$

▫ For the circuit to the right, the picture doesn't change much except that there are effectively two time consts.



$$\tau_{charge} = R_1 C$$

$$\tau_{discharge} = R_2 C$$

Note that when the switch is flipped to B (from A, after fully charging C) the energy dissipated by R_2 is the same as the energy stored in C