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Loops vs. Coils vs. Solenoids

- At center of current-carrying loop:

$$B_{\text{loop}} = \frac{\mu_0 I}{2R} \quad (\text{at center of loop!})$$



- For an N-turn coil (i.e. length $\ll R$)

$$B_{\text{coil}} = N \frac{\mu_0 I}{2R} \quad (\text{at center})$$

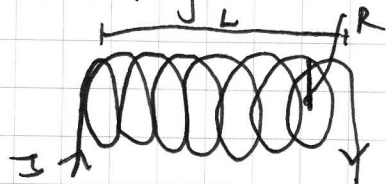
NOTE: the field inside the coil can be strengthened by adding in a 'core' of magnetizable material such as iron (similar somewhat in spirit to a dielectric for a parallel-plate capacitor)

ex coil of radius $R = 0.1 \text{ m}$, current $I = 1 \text{ A}$ (single turn)

$$B(\text{at center}) = \frac{\mu_0 I}{2R} = \frac{1.26 \times 10^{-6}}{2} \frac{1.0}{0.1} \frac{\text{T} \cdot \text{m}}{\text{A}} \cdot \frac{\text{A}}{\text{m}}$$
$$= 6.5 \times 10^{-6} \text{ T}$$

→ this is significantly weaker than earth's field ($\approx 50 \mu\text{T}$); can get around this by using more turns, but eventually if you stack too many coils, there is some length to your stack of coils that needs to be accounted for (→ solenoid)

- When we put the coils together such that the length exceeds the radius ($L \gg R$), we call it a solenoid



Using Amperé's Law, the B field in the solenoid can be deduced as:

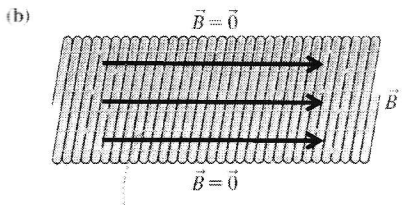
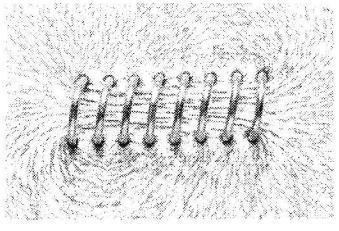
$$B_{\text{solenoid}} = \frac{\mu_0 N I}{L}$$

(note that the radius is not even specified here!)

[see Figs. below as well as slides on website for a clearer picture]

FIGURE 32.31 The magnetic field of a solenoid.

(a) A short solenoid

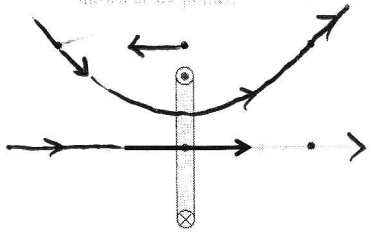


The magnetic field is uniform inside this section of an ideal, infinitely long solenoid. The magnetic field outside the solenoid is zero.

FIGURE 32.30 Using superposition to find the magnetic field of a stack of current loops.

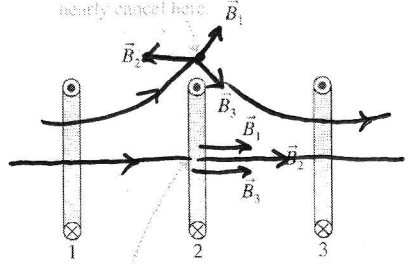
(a) A single loop

The magnetic field vector is shown at six points.



(b) A stack of three loops

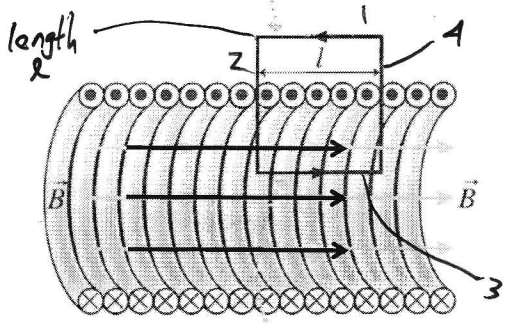
The fields of the three loops nearly cancel here



The fields reinforce each other here.

FIGURE 32.32 A closed path inside and outside an ideal solenoid.

This is the integration path for Ampère's law. There are N turns inside.



B is tangent to the integration path along the bottom edge.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}} = \mu_0 N I$$

$$= (\text{side 1}) + (\text{side 2}) + (\text{side 3}) + (\text{side 4})$$

$$= 0 + 0 + B\ell + 0$$

$\underbrace{\quad}_{\text{no } \vec{B} \text{ outside}}$
 $\underbrace{\quad}_{\vec{B} \perp d\vec{s}}$
 $\underbrace{\quad}_{\vec{B} \perp \text{to } d\vec{s}}$

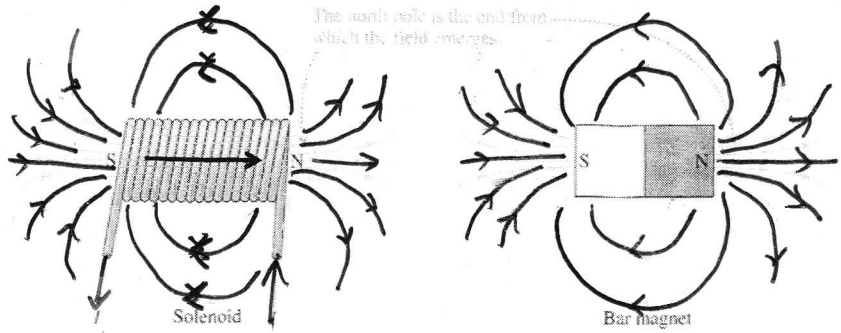
$$\rightarrow B = \frac{\mu_0 N I}{\ell}$$

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FIGURE 32.33 The magnetic fields of a finite-length solenoid and of a bar magnet.

□ Note that a solenoid acts in a similar way to a bar magnet

→ elektromagnet



□ Magnetic Forces

- Given a uniform (homogeneous) magnetic field \vec{B} , a moving charge (q, \vec{v}) experiences a deflection at right angles to both \vec{v} and \vec{B}

[Historical Aside: following Oersted's discovery, Ampère did some expts. w/ current-carrying wires to deduce some of these properties experimentally]

$$\vec{F}_m = q (\vec{v} \times \vec{B})$$

Cross-product

• mag. given as

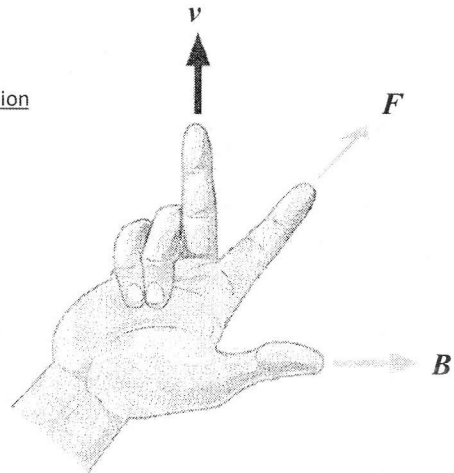
$$|F_m| = q |\vec{v}| |\vec{B}| \sin \theta$$

θ is angle between \vec{v} and \vec{B}

• direction given by right-hand rule (#2; see right)

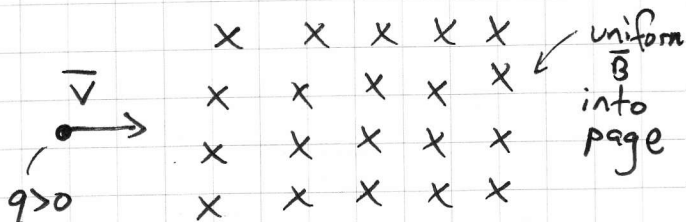
FIGURE 32.36 The right-hand rule for magnetic forces.

alternate version



Note that the sign of q and angle between the velocity and magnetic field are crucial in terms of determining the corresponding force

ex) Positive charge moves into a region of uniform \vec{B} . Which way is the resulting force?



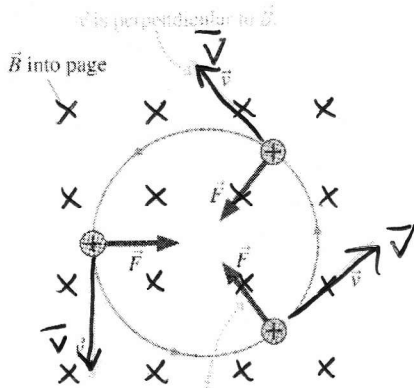
Use right-hand rule \rightarrow force will be upwards

but that will change the velocity (i.e. the particle's path will bend upwards) and thereby the direction of the force changes too (since it is \perp to both \vec{v} and \vec{B})

\rightarrow leads to circular motion

as shown in Giordano Fig. 20.16 we also have cases where one can get helical motion as well (Giordano fig. 20.17 shows a neat expt. demonstration of this)

FIGURE 32.39 Cyclotron motion of a charged particle moving in a uniform magnetic field.



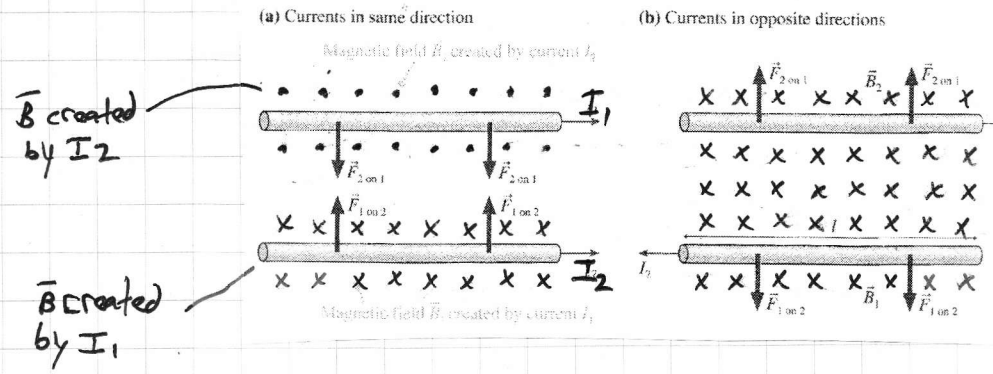
The magnetic force is always perpendicular to \vec{v} , causing the particle to move in a circle.

Note that when \vec{B} and \vec{v} are parallel, there is no force and that flipping the sign of the charge flips the direction of the force

$\otimes \rightarrow$ Hall effect (see Giordano Fig. 20.29)

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FIGURE 32.47 Magnetic forces between parallel current-carrying wires.



Now we can understand Ampère's expts. w/ two wires:

- one wire creates a magnetic field
 - the other wire is subjected to that field and because it has charge moving through it, a force is created
- so one needs to apply the RHR rule twice (using both versions of Giordano's rule)

NOTE: we don't consider self-interactions here (i.e. the force a wire feels from its own magnetic field)