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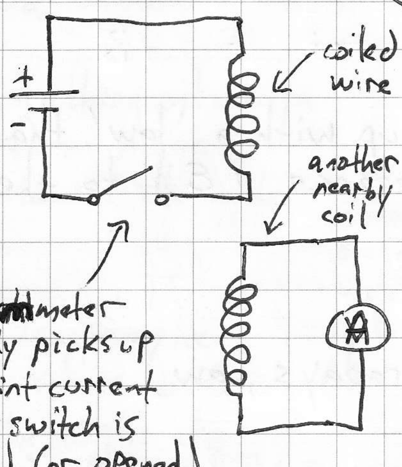
Magnetic Induction

- Oersted's 1820 discovery generated a lot of excitement about magnetism

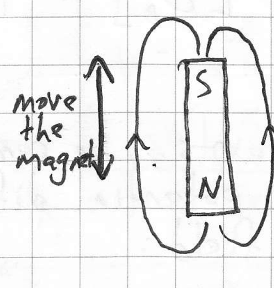
  → Current creates a magnetic field. But can a magnet be used to create a current?

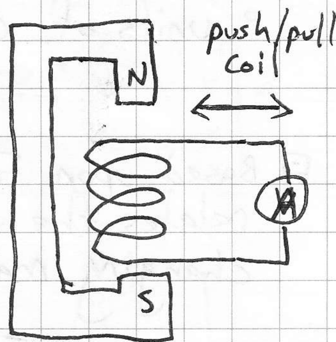
- A lot of fiddling along these lines proved inconclusive until 1831, when Faraday and Henry indepently discovered the process of **electromagnetic induction**

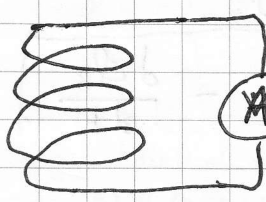
  → the key to getting a magnet to create a current was that the magnet (and the associated $\vec{B}$) had to be **changing**



~~Ohm~~ Ammeter only picks up a faint current when switch is closed (or opened)







moving magnet or coil continuously induces a current

- stationary magnet → nothing happens
- approaching magnet → voltage
- moving-away magnet → voltage (or opposite sign)
- the more turns, the bigger the effect
- reverse magnet orientation → voltage sign flips

<u>NOTE</u>: can use either an ammeter or voltmeter to observe that the changing magnetic field is <u>inducing a current</u>

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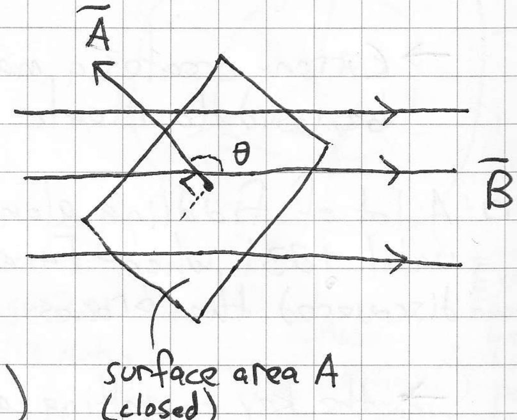
- To understand what's going on, we introduce the notion of **magnetic flux** (which is analogous to the idea of $\bar{E}$ when we dealt w/ Gauss' Law)

$$\Phi_B = \bar{B} \cdot \bar{A}$$

$$= BA\cos\theta$$



A is a surface that has a 'direction' perpendicular to its surface (see Giordano fig. 21.3)

- units of Φ_B : $[\Phi_B] = T\,m^2 \equiv Wb$ (weber)

- Based upon Faraday's expts., he came up with a 'law' that relates the electric potential difference ($\mathcal{E}$) to the changing magnetic field:

$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}} \quad \text{(Faraday's Law)}$$

→ so a small change in the magnetic field ($d\Phi_B$) divided by the time over which that occurs (dt) yield the induced electromotive force ($\mathcal{E}$)

- So $|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right|$, but why the negative sign?

  → Lenz's Law (we'll get there shortly)

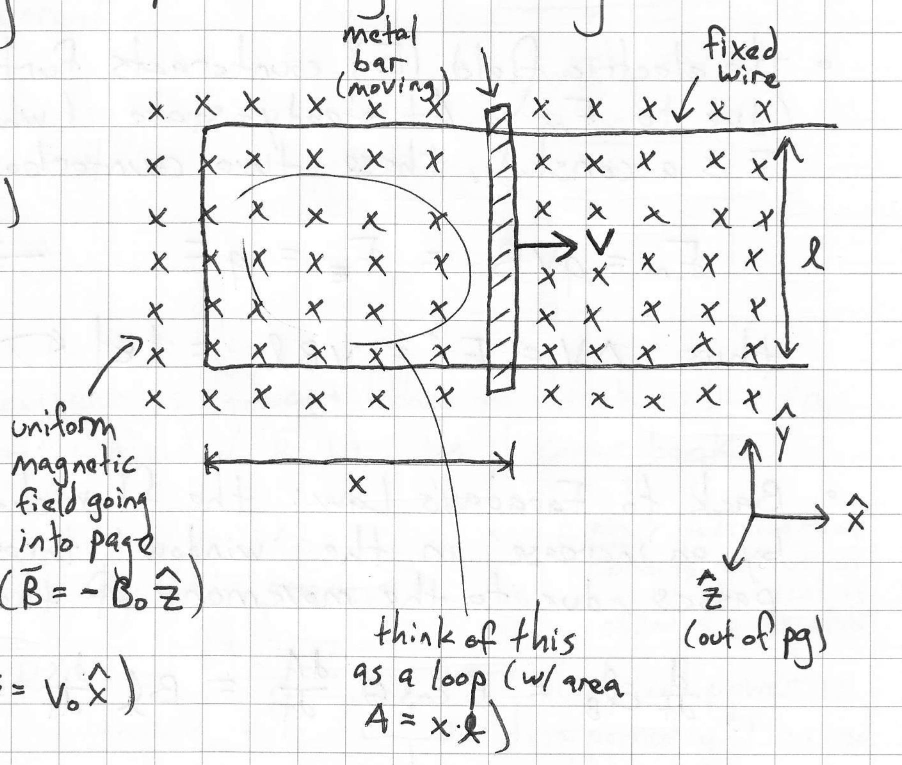
$\Longrightarrow$ So, can we gain some physical insight into why a time-varying flux leads to an induced electromotive force? Let's consider what $\frac{d\Phi_B}{dt}$ implies: 3/1/13

$$\frac{d}{dt}\Phi_B = \frac{d}{dt}\left(BA\cos\theta\right) = \underbrace{A\cos\theta\,\frac{dB}{dt}}_{\bar{B}\text{ field varies}} + \underbrace{B\cos\theta\,\frac{dA}{dt}}_{\text{area of loop changes}} + \underbrace{BA\,\frac{d}{dt}\cos\theta}_{\text{orientation of loop changes}}$$

(chain rule!)

▫ let's first understand the piece dealing w/ a change in area (i.e. $B\cos\theta\,\frac{dA}{dt}$)

- Consider a metal bar resting atop a wire (i.e. a conducting rail)
- Everything sits in a uniform magnetic field w/ strength B_0
- Now suppose the the bar is moved to the right at a constant velocity ($\bar{v} = v_0\hat{x}$)



Q: What happens to the conduction electrons in the bar?

$\rightarrow$ Since they are charged objects ($q = -e$) and moving through a $\bar{B}$ field, they'll experience a force

$$F_m = q\left(\bar{v}\times\bar{B}\right) \qquad \text{(points along } -\hat{y}\text{ direction!)}$$

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→ put another way, electrons get pushed towards the bottom of the bar, leaving the top of the bar positively charged and the bottom negatively charged

- So F_m separates charge, thereby creating an electric field ($\bar{E}$) along the bar and thus a potential difference

$$\Delta V = E\ell \qquad \text{(like a parallel-plate capacitor!)}$$

- the electric field (E) counteracts further charge separation (due to F_m). At steady-state (which occurs when $\bar{v}$ is a const.), these two counterbalance one another:

$$F_m = qvB = F_E = qE \quad \rightarrow \quad E = vB$$

thus $\Delta V = E\ell = vB\ell = |\varepsilon|$ ← we'll deal w/ the sign a bit later

- Back to Faraday's Law: the flux change $\frac{d\Phi_B}{dt}$ is caused by an 'increase' in the 'window' through which $\bar{B}$ passes, due to the movement of the bar:

$$\frac{d}{dt}\Phi_B = B\underbrace{\cos\theta}_{\substack{\theta=0 \text{ here,}\\ \text{so this is just one}}}\frac{dA}{dt} = B\ell\frac{dx}{dt} = B\ell v = |\varepsilon|$$

⇒ So Faraday's Law can be thought of as a balance between electric force and the force due to a charged particle moving through a magnetic field!

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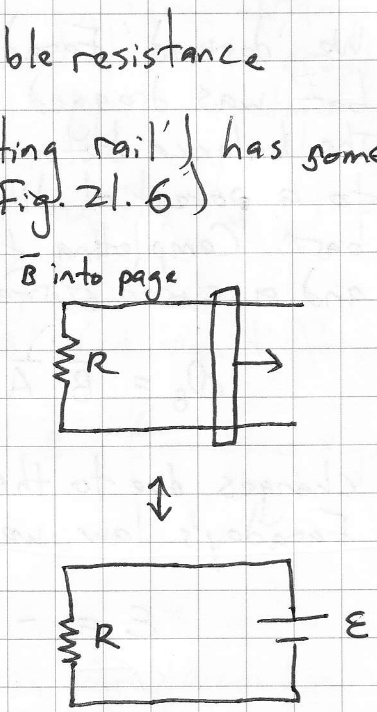
- So $\frac{d\Phi_B}{dt}$ generates (i.e. induced) an electromotive potential difference ($\Delta V = \mathcal{E}$), but what about a current?

  - assume the moving bar has negligible resistance
  - assume the fixed wire (i.e. 'conducting rail') has some net resistance R (see Giordano Fig. 21.6)

→ now we have a circuit w/ the moving bar acting as the generator and we can use Ohm's Law to determine the current



$$|RI| = \Delta V = \mathcal{E} = B\ell v$$

$$\rightarrow \quad I = \frac{B\ell v}{R}$$

- Note that since a current is induced, that in itself will also generate a magnetic field ($\bar{B}_{ind}$); we'll come back to this in a bit

- Is power dissipated by this process?

NOTE: A device that converts mechanical energy into electrical is called a **generator**

$$P = \Delta V \cdot I = RI^2 = \frac{B^2 \ell^2 v^2}{R}$$

⇝ electrical power is dissipated and turned into heat

⟹ the source of this power comes from the work done to move the bar at constant velocity: when a current is induced in the bar (i.e. electrons move down along $-\hat{y}$), $\bar{F}_m = q(\bar{v} \times \bar{B})$ ~~create~~ is a force opposing the bar's movement (i.e. $\bar{F}_{ext}$ to the right is required for $\bar{v} = $ const.)