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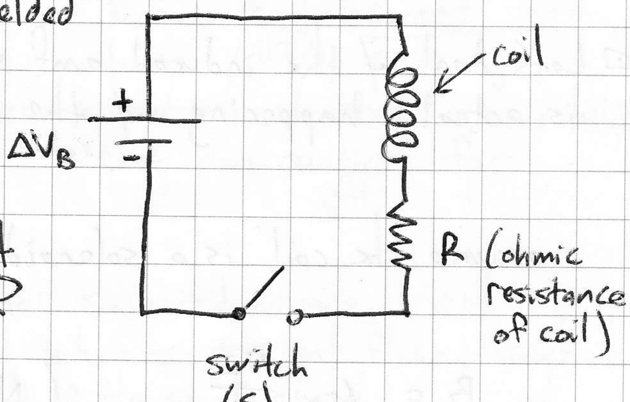
Inductance and RL Circuits

- Faraday's Law (combined w/ Lenz's Law) basically states that an electromotive force is generated so to resist change. or put another way that there is some sort of 'inertia' associated with magnetic fields

(i.e. an N-turn solenoid)

- Consider a coil placed into a circuit containing a battery. Assume the coil has some resistance (R) associated with it (we can represent this as a resistor in series w/ the coil). Also assume there is no magnetic flux through the coil (e.g. it is shielded from earth's field)



- When the switch S is closed (at $t=0$), what is the current flowing through the circuit?

Guess: $I = \frac{\Delta V_B}{R}$

⇝ If this is true, then there'd be a magnetic flux Φ_B instantly set up in the coil. From Faraday's Law:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$ → instant turn on of current mean $\frac{d\Phi_B}{dt} = \infty$! (not possible!!)

⇒ An emf will be induced in the coil to prevent an instant turn-on of the current

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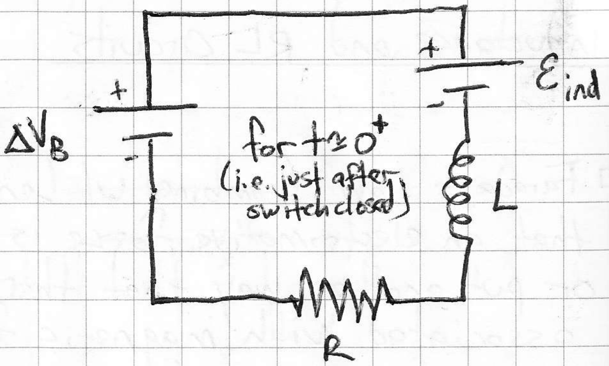
- the coil acts as an inductor and tries to prevent an increase in Φ_m (from zero) by countering the battery emf (ΔV_B) with an induced emf ($\mathcal{E}_{ind}$)



- The induced emf is **transient**: as time goes on the current grows and a magnetic field is built up. As this happens, the rate of change of Φ_m decreases and $\mathcal{E}_{ind}$ decreases

□ Let's deal w/ the induced emf in detail by considering what is actually happening w/ the coil:

- assume the 'coil' is a solenoid ($\underbrace{\text{length}}_{}^{=d} \gg$ radius)

$$B = \mu_0 \frac{NI}{d} \qquad (N = \text{\# of turns})$$

A – cross-section area of solenoid

- For a single turn: $\Phi_m^{(1)} = BA = \mu_0 \frac{NI}{d} A$
- For N turns (i.e. total flux): $\Phi_m = N\Phi_m^{(1)} = \mu_0 \frac{N^2 I}{d} A$
- Faraday's Law: $\mathcal{E}_{ind} = -\frac{d\Phi_m}{dt} = -\mu_0 \frac{N^2 A}{d} \frac{dI}{dt} \equiv -L\frac{dI}{dt}$

$$\rightsquigarrow L \equiv \frac{\mu_0 N^2 A}{d} \quad \text{(physical property of the solenoid)}$$

$$\Rightarrow \mathcal{E}_{ind} = -L\frac{dI}{dt} \quad \text{(self-induced emf)}$$
$$= \Delta V_L \quad \text{(potential 'drop' across inductor)}$$

☐ We call L the inductance

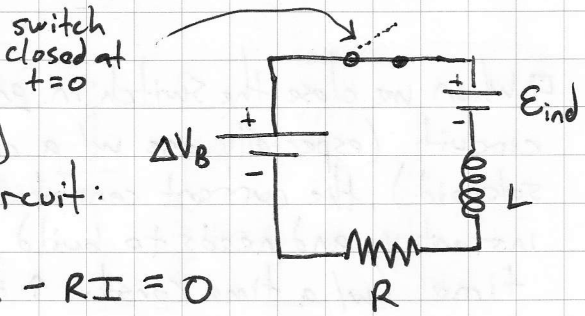
→ some general comments on inductance:

- a physical property of a solenoid (or coil) that one can measure
- has units of Henrys (H), which is equivalent to

$$1\ H = \frac{m^2 \cdot kg}{C^2} = \frac{V \cdot s}{A} = \Omega \cdot s$$

- Turning on a current ($dI/dt > 0$) means $\mathcal{E}_{ind}$ opposes the battery. Turning off the battery ($dI/dt < 0$, e.g. disconnecting it) means the current can not stop immediately and $\mathcal{E}_{ind}$ exists such that the current can continue for some time
- A simple straight wire also has some inductance, as it surrounds itself w/ a $\bar{B}$ field when a current flows through it (their inductance L is relatively quite small though)
- The induced emf has something to do w/ **energy storage**: building up $\bar{B}$ via a current implies energy storage analogous to that in an electric field ($\bar{E}$) of a capacitor

LR Circuit



☐ Can apply Kirchoff's Law (loop rule) to determine the dynamics of the circuit:

$$\Delta V_B + \Delta V_L + \Delta V_R = \Delta V_B - L\frac{dI}{dt} - RI = 0$$

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⇝ So now we have a (differential) equation that tells us about the current through the circuit and how it varies w/ time:

- we likely expect an asymptotic response (i.e. exponential)
- at $t=0$, the current through the circuit should be zero (since the induced $\mathcal{E}_{ind}$ opposes the flow of current so to prevent the flux Φ_m from changing)
- we can introduce a new time constant (τ) as follows:

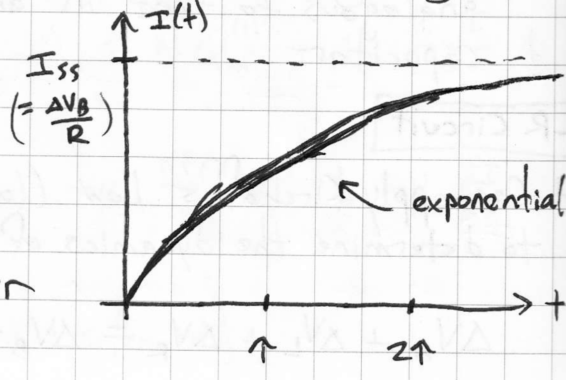
$$\frac{\Delta V_B}{R} - \frac{L}{R}\frac{dI}{dt} - I = 0 \longrightarrow I_{ss} - \tau \frac{dI}{dt} - I = 0$$

- $I_{ss} \equiv \frac{\Delta V_B}{R}$ (steady-state current, happens when $t \to \infty$)
- $\tau \equiv L/R$ (inductive time const.), const.

[Math Aside: this is an example of a linear, homogeneous, autonomous first order ordinary differential equation] $\equiv$ ODE

$$\Longrightarrow \text{Solution: } \boxed{I(t) = I_{ss}\left(1 - e^{-t/\tau}\right)}$$

[verify that this solution satisfies the ODE stemming from Kirchoff's Loop Law]



☐ When we close the switch in any circuit (especially one w/ a coil or solenoid), the current cannot turn on instantly and needs to build up over time (w/ a time const. $\tau = L/R$)

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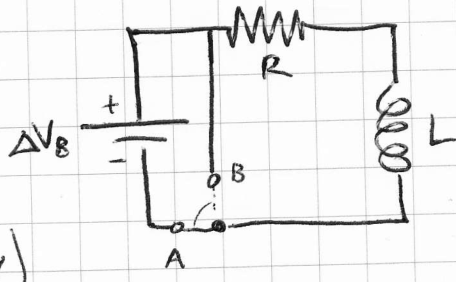
☐ Note that the door swings both ways here in that disconnecting the battery (but keeping a closed circuit) will cause an induced current to resist the change



- assume switch is at A for a long time. Then the current through the battery is $I_0 = \Delta V_B / R$

- Now flip the switch to B (i.e. essentially disconnect the battery)

Kirchoff's Loop Rule: $\Delta V_R + \Delta V_L = 0$

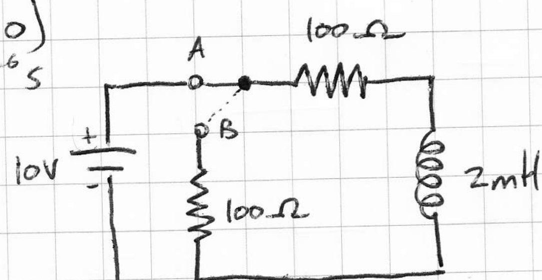
$$-RI - L\frac{dI}{dt} = 0$$

⇝ similar to before, but one less term!

Solution: $I(t) = I_0 e^{-t/\tau}$ $\quad (\tau = L/R)$

⇒ exponential decay from $\Delta V_B / R$ down to zero

ex See circuit below. Switch initially at A for a long time, then flipped to B at $t=0$.
a) Determine the current at $t = 5.0\,\mu s$
b) At what time has the current decayed to 1% its initial value?



$I_0 = \frac{10\,V}{100\,\Omega} = 0.1\,A$ (current before $t=0$)

$\tau = \frac{L}{R} = \frac{0.002\,H}{(100+100\,\Omega)} = 10\,\mu s = 10\times10^{-6}\,s$

$I(5\times10^{-6}\,s) = 0.1\,A\, e^{-5/10} = 0.061\,A$

$0.01 I_0 = I_0 e^{-t/(1.0\times10^{-5})} \rightarrow t = 46\,\mu s$

(use natural log!)