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Energy + Magnetic Fields

- When talking about electric fields, there was the notion underlying our discussion that there was energy stored in the field (i.e. the electric potential)

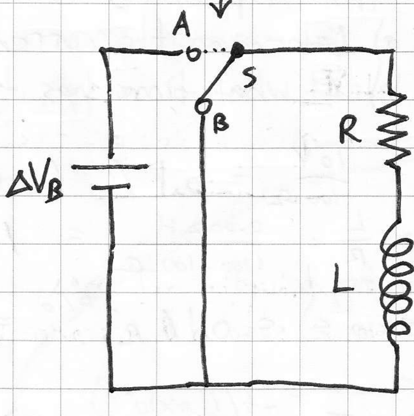
  ex. consider a charged capacitor in series w/ a resistor (but nothing else): the current set up discharges heat to the resistor, and thereby energy associated w/ $\bar{E}$ between the capacitor plates is dissipated into heat

  → Along similar lines, what can we say about the energy associated w/ a $\bar{B}$ field generated by a current passing through an inductor L?

- Let's return to our LR circuit (3/6/13 notes) but w/ a catch: we'll flip the switch back and forth periodically (w/ period T) in an on/off fashion:

  - this sort of 'oscillation' (we'll say more about that when we go back to ch. 11) is what's called a **square wave**
  - we end up w/ what we call an **AC** (alternating current) (more on this come ch. 22!)
  - assume $T \gg \tau = \frac{L}{R}$

switch flips back and forth between A and B w/ some period T



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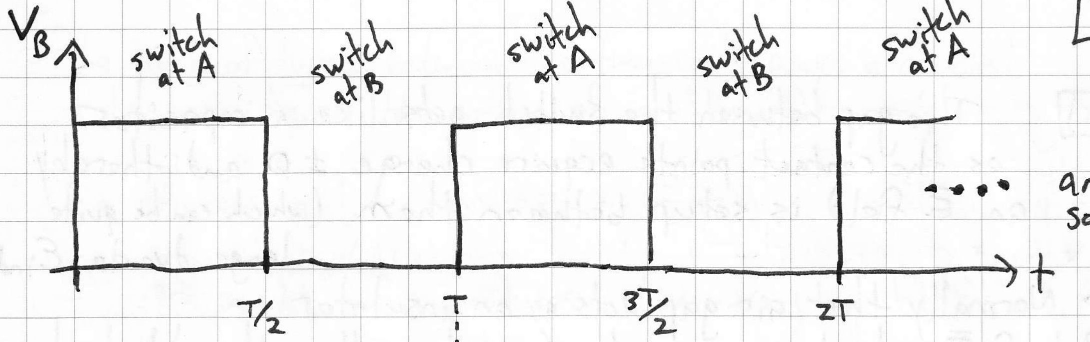
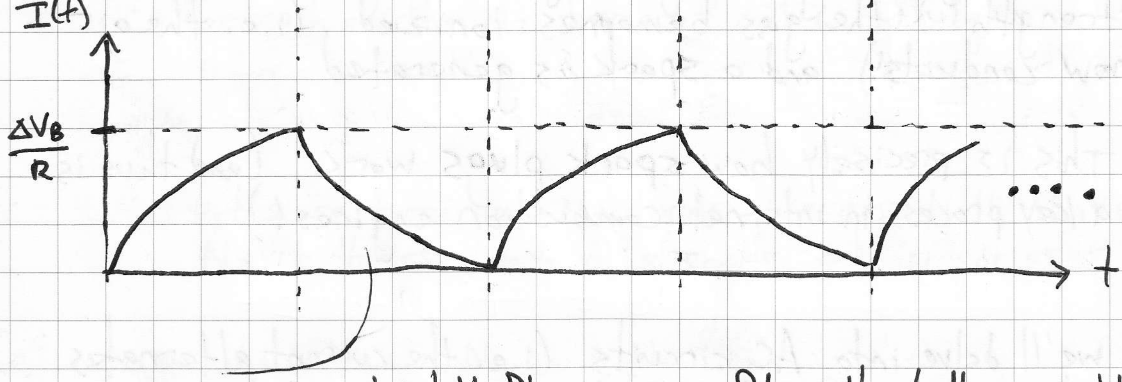


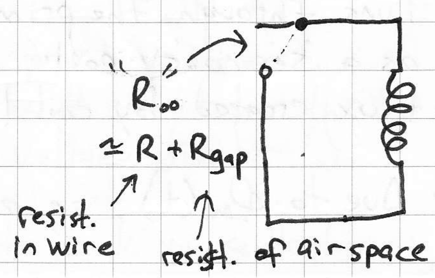


<u>NOTE</u> : current still flows even after the battery is 'turned off'!

$\Rightarrow$ when the battery is disconnected, the current continues to flow because of the $\vec{B}$ field's inertia. Put another way, the term $-L\,dI/dt$ provides a forward emf ($\mathcal{E}_{ind} = -\frac{d}{dt}\Phi_m$ where $\Phi_m = LI$) since $dI/dt < 0$ for a decreasing current

<u>ASIDE</u> : when the current stops suddenly, $\mathcal{E}_{ind}$ can be quite large and cause a spark to fly (discharge $\rightarrow$ electrical arc)

ex ~~FAB~~ Inductor w/ max. current through it is suddenly disconnected



Circuit is broken $\rightarrow$ current still flows
$\rightarrow$ why?

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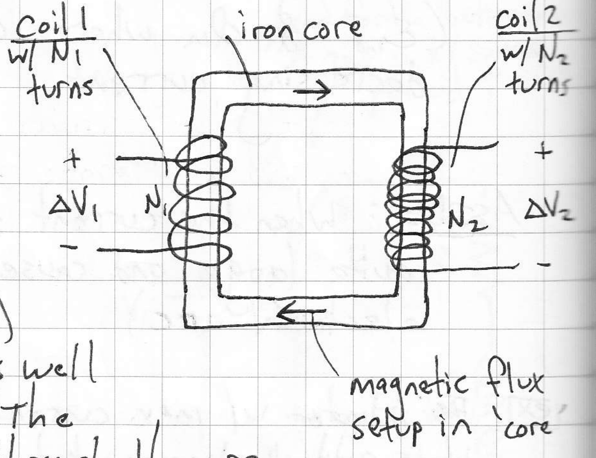
ex. (cont.) The gap between the switch acts like a capacitor as the contact points acquire charge $\pm Q$ and thereby an $\bar{E}$ field is setup between them (which can be quite large due to $\mathcal{E}_{ind}$)

→ Normally that air gap acts as an insulator. But if $\bar{E}$ gets large enough (greater than the 'dielectric strength') the gas becomes ionized (i.e. the air now conducts!) and a spark is generated

→ This is precisely how **spark plugs** work (and thus is a key process in internal combustion engines!)

⊡ Soon we'll delve into AC circuits (i.e. the current alternates polarity in some periodic fashion) in some detail. But from this point, we can see some interesting possibilities that might arise when dealing w/ inductors and non-constant electric currents. One key example is that of a **transformer** (see Giordano ch. 22.9)



- We drive a 'primary coil' (coil 1) w/ alternating current (AC). This generates a time-varying magnetic flux $\Phi_M(t)$
- A magnetizable 'core' (e.g. iron) runs through the primary coil as well as a 'secondary coil' (coil 2). The flux created by coil 1 "flows" through the core
- Due to $\Phi_M(t)$, an emf is induced in coil 2 (ΔV_2)

- The ratio of turns between the two coils plays a crucial role here, hence the notion of a 'transformer'

  - primary coil: $\Delta V_1 = -\frac{d\Phi_m}{dt} N_1$ $\quad \Phi_m$ = flux thru a single coil

    $\rightsquigarrow$ so each of the N_1 turns produces a time-varying flux $\Phi_m(t)$

  - For <u>each</u> of the N_2 turns of coil 2, changes in $\Phi_m(t)$ induce an emf:

$$\frac{d\Phi_m}{dt} = \mathcal{E}$$

    but since there are N_2 turns, we essentially have N_2 batteries (each of strength $\mathcal{E}$) in series

$$\Delta V_2 = N_2 \mathcal{E} = N_2 \frac{d\Phi_m}{dt}$$

  - But $d\Phi_m/dt$ has got to be the same for both coils, so:

$$\frac{|\Delta V_1|}{N_1} = \frac{|\Delta V_2|}{N_2} \longrightarrow \boxed{\frac{|\Delta V_2|}{|\Delta V_1|} = \frac{N_2}{N_1}}$$

    this is sometimes called the '<u>turns ratio</u>'

  - Since energy must be conserved here (assuming no losses), we can also consider the **power** to see how the currents are related:

$$P_1 = |I_1 \Delta V_1| = P_2 = |I_2 \Delta V_2| \longrightarrow \boxed{\frac{|\Delta V_2|}{|\Delta V_1|} = \frac{N_2}{N_1} = \frac{|I_1|}{|I_2|}}$$

    (Ideal Transformer Eqn.)

- Transformers are incredibly useful as they provide a means to step up/down voltages (by adjusting the # of turns in the two coils) w/o much loss of energy (unlike a voltage divider!)

- Very common in everyday electronics (e.g. the 'brick' you use for plugging in a laptop)

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Energy associated w/ $\bar{B}$

- For an electric field ($\bar{E}$) in a parallel-plate capacitor, we had derived that

potential diff. between plates

$$U = \frac{1}{2} C \Delta V^2 \quad \left(\text{where } |E| = \frac{\Delta V}{d}\right)$$

plate separation

- For $\bar{B}$, consider back to our RL circuit:

$$I(t) = \frac{\Delta V_B}{R}\left(1 - e^{-t/\tau}\right) \quad \text{where } \tau = \frac{L}{R},\ \frac{\Delta V_B}{R} = I_{max}$$

$$\Delta V_L = -L\frac{dI}{dt} = -L\frac{\Delta V_B}{R}\left(\frac{1}{\tau}e^{-t/\tau}\right) = -\Delta V_B e^{-t/\tau}$$

power in inductor: $P_L = |\Delta V_L| I = \frac{\Delta V_B^2}{R}\left(e^{-t/\tau} - e^{-2t/\tau}\right)$

To determine U_L, we need to know the change in power ~~over~~ which the current builds up (to $I_{max} = \Delta V_B/R$). When the smoke clears, we have:

$$\boxed{U_L = \frac{1}{2} L I^2} \quad \text{(where } I \text{ is the steady-state current)}$$

see Giordano Ch. 21.6

In more general terms: $U_L = L\int_0^I I\,dI = \frac{1}{2}LI^2$ (integrating over the current change)

$\Rightarrow$ Just like we determined the energy stored in a capacitor ($U_c = \frac{1}{2}C\Delta V^2$), we now have an expression for the energy stored in an inductor

NOTE: Since $B \propto I$, $U_L \propto B^2$ just like $U_c \propto E^2$ (since $\Delta V \propto E$ for a capacitor)