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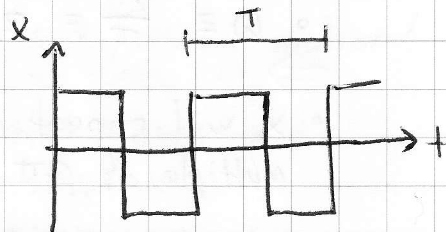
Oscillations + Harmonic Motion

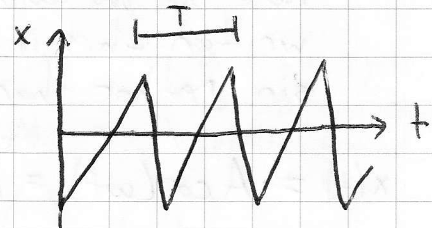
- Before delving into AC circuits in ch. 22, it is first helpful to get some mathematical tools under our belt as well as examine some basic physical concepts associated w/ sinusoidal motion (e.g. a mass-on-a-spring)

  → heading back to Giordano ch. 11



- Basics about oscillations we'll consider:

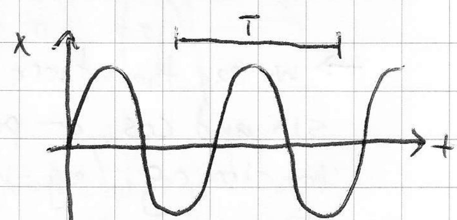
  - occur about an **equilibrium** position



  - motion is **periodic**, meaning that it repeats itself after some period T

  - we can also talk about the **frequency** of an oscillation, which is just

$$f = \frac{1}{T} \qquad \left[\text{NOTE that } [f] = \frac{1}{s} = Hz\right]$$



ex Sound can be described in terms of different frequency components. Suppose a speaker is creating a 5 kHz tone. What is its oscillation period?

$$T = \frac{1}{f} = \frac{1}{5\,kHz} = \frac{1}{5000\,Hz} = 2.0 \times 10^{-4}\,s = 200\,\mu s$$

→ this means that the speaker cone vibrates back and forth over the course of 200 μs! (the air displaced near its surface leads to the sound our ears pick up)

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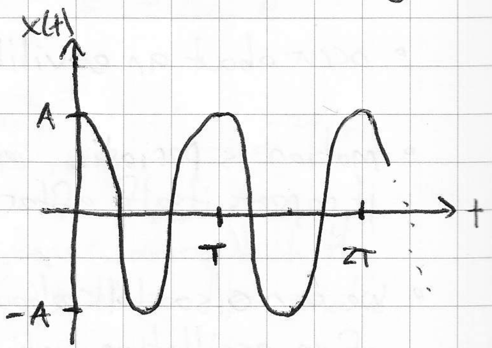
e.g. sin, cos

- One effective way to describe harmonic motion mathematically is via sinusoidal functions. Consider:

$$x(t) = A\cos\left(\frac{2\pi t}{T}\right) = A\cos(\omega t)$$

- A - **amplitude**, T - period
- $\omega \equiv \frac{2\pi}{T} = 2\pi f$ (we call this the **angular frequency**)
- x will repeat itself everytime $2\pi f t = \omega t$ is an integer multiple of 2π
- Note that we could have also written this in terms of sin rather than cos



$$x(t) = A\cos(\omega t) = A\sin\left(\omega t + \frac{\pi}{2}\right)$$

→ we say that there is a difference in **phase** between sin and cos, or put another way there is just a difference in timing (e.g. where does the peak occur?)

- In a slightly more general description, we can consider a **phase constant** ϕ_0

$$x(t) = A\cos(\omega t + \phi_0)$$

→ by varying ϕ_0, we can shift (horizontally along the t-axis) the structure of the sinusoid

⇝ we'll come back to this in a moment

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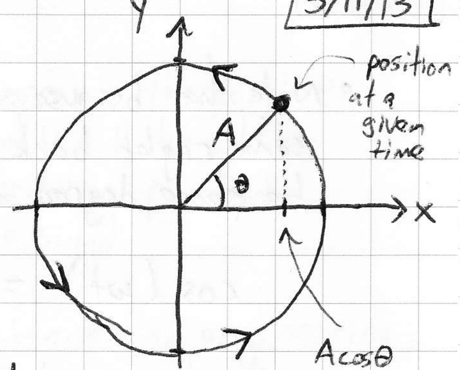


- A very useful way to think about harmonic motion is visually via a **circle** (and via a little trigonometry)

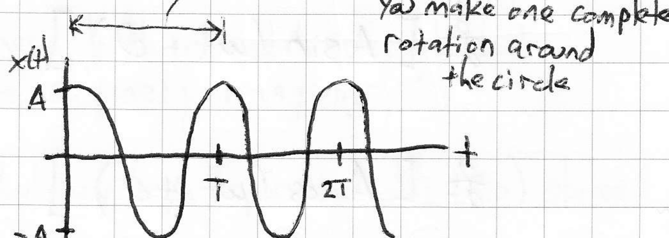
  - consider a circle of radius A centered at the origin

  - As t goes forward, you move around the circle in a counter-clockwise fashion. After some interval T, you end up where you started $\rightarrow$ periodic motion!

  - consider $x(t)$ as your projection onto the x-axis and θ is the angle made relative to the positive x-axis [note that $\theta = \theta(t)$!]. Then we have:

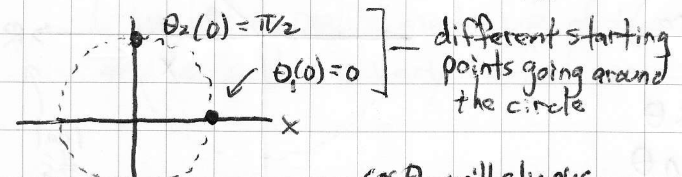
$$x(t) = A\cos(\theta(t)) = A\cos(\omega t)$$

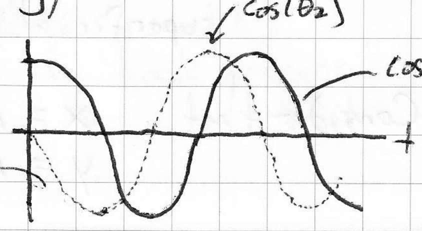
  - Assuming $\theta(0) = 0$, then drawing this out as a function of time shows:



$\rightarrow$ Thus, a sinusoid can readily be visualized in terms of going around on a circle (we'll come back in a bit as to how to think about that vertical axis y)

  - what if $\theta(0) \neq 0$, but instead is something like $\frac{\pi}{3}$ or $\frac{5\pi}{6}$?
    $\rightarrow$ think back to the circle analogy!





$\cos\theta_2$ will always 'lead' $\cos\theta_1$

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- Note that because once you go around a circle fully (i.e. you come right back to where you started) there will be some degree of ambiguity. Put another way

$$\cos(\omega t) = \cos(\omega t + 2\pi) = \cos(\omega t + 2000\pi)$$

⇝ you are not sure if you've gone around the circle once, 2000 times, or not at all

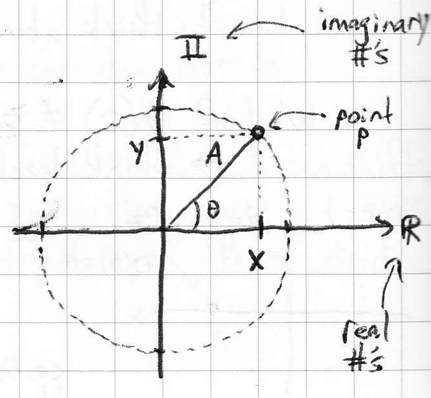
⇒ this issue is known as **phase ambiguity** and manifests is a wide range of applications/problems (such as our ability to localize sound!)

- Some useful derivatives to know here are:

$$\frac{d}{dt}\left[A\sin(\omega t + \phi)\right] = \omega A\cos(\omega t + \phi)$$

$$\frac{d}{dt}\left[A\cos(\omega t + \phi)\right] = -\omega A\sin(\omega t + \phi)$$

Note that $\frac{d^2}{dt^2}\sin(\omega t) = -\omega^2\sin(\omega t)$ (and similarly for cos)
→ we'll come back to this shortly



Math Aside

Complex Numbers
(we'll only touch upon this very superficially here)

Consider that $x = A\cos\theta$
$y = A\sin\theta$

Math Aside (cont.)

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- Notice that there are two numbers that describe the point P. These are A and θ. or conversely x and y. Put another way, there are different ways of describing P, but ultimately we need two numbers to do such.

- As very common in many branches of science and engineering, (but not necessarily intuitive), imaginary numbers are put forth to create **complex numbers** and thereby create a means to more efficiently describe things.

- imaginary number (by definition): $i \equiv \sqrt{-1}$

ex. $i^2 = i \cdot i = \sqrt{-1} \cdot \sqrt{-1} = -1$, similarly $i^3 = -i$ (weird!)

- a complex number z is the sum of a real and imaginary #

$z = a + ib$ (where both a and b are real, but ib is 'purely imaginary')

- Euler's formula (very cool, but way beyond purview of 1410) states:

$$e^{ix} = \cos x + i \sin x$$

→ or in terms of our figure from the last page:

$$Ae^{i\theta} = A\cos\theta + i\sin\theta$$

A - tells you how 'big' things are
θ - tells you something about timing

⇒ $Ae^{i\theta}$ has the virtue that it is a single (complex) number (though it is really made up of two numbers, a and b) and as such is very useful in terms of 'book keeping'

Summary: Complex #s are very physical (i.e. real world) in the sense that they provide a compact way of expressing two values (A and θ)