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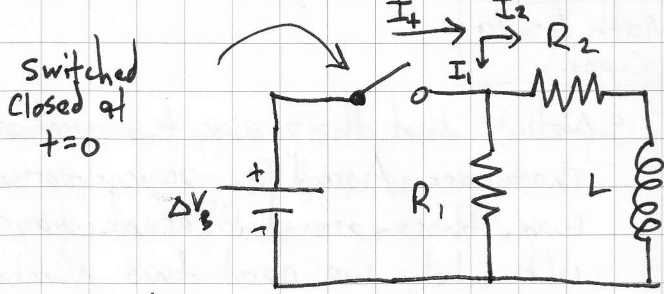
ex. (Giordano P.21.42)



a) At $t=0$, inductor prevents current flow through R_2 branch (i.e. $\Delta V_{R2} \neq 0$)

$R_1 = 0.8\,k\Omega$
$R_2 = 0.5\,k\Omega$
$L = 4.5\,mH$
$\Delta V_B = 3.0\,V$

$$\Delta V_{R1} = I_1 R_1 = \Delta V_B$$

$$\rightarrow I_1 = \frac{\Delta V_B}{R_1} = \frac{3.0\,V}{800\,\Omega} = 3.8 \times 10^{-3}\,A = 3.8\,mA$$

b) At $t=0$, there is no potential drop across R_2. Put another way, L is essentially in parallel w/ R_1 (i.e. they have the same potential across them)

$$|\Delta V_L (t=0)| = 3\,V$$

$\rightsquigarrow$ put yet another way, this is the $\mathcal{E}_{ind}$ that opposes current flow through the R_2 branch

c) After the switch has been closed for a long time, $\mathcal{E}_{ind}$ ($= \Delta V_L$) $= 0$, since the current (and thereby the mag. flux) has reached steady-state. So we just have two resistors in parallel, both w/ the same potential diff. ($\Delta V_B = 3\,V$) across them:

total current

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \simeq 307.7\,\Omega \quad \rightarrow \quad I_t = \frac{3\,V}{0.31\,k\Omega} = 9.7\,mA$$

$$I_1 + I_2 = I_t, \quad I_2 = \frac{R_2}{R_1+R_2} I_t \simeq 6.0\,mA \text{ (to right)}, \quad I_1 = 3.7\,mA \text{ (downward)}$$

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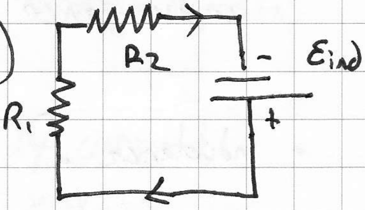
d) Switched opened at $t = t_{open}$ (i.e. battery essentially disconnected)

→ inductor creates $\mathcal{E}_{ind}$ to oppose change and induces clockwise current.

at $t = t_{open}$



$|\mathcal{E}_{ind}| = 3\,V$ + contrib. due to R_1, R_2 (see e))

so current through R_1 is $\frac{\mathcal{E}_{ind}}{R_1} = 6.0\,mA$ and points in the opposite direction of I_1 from when the switch was closed (!)

e) Use Kirchoff's Loop Rule:

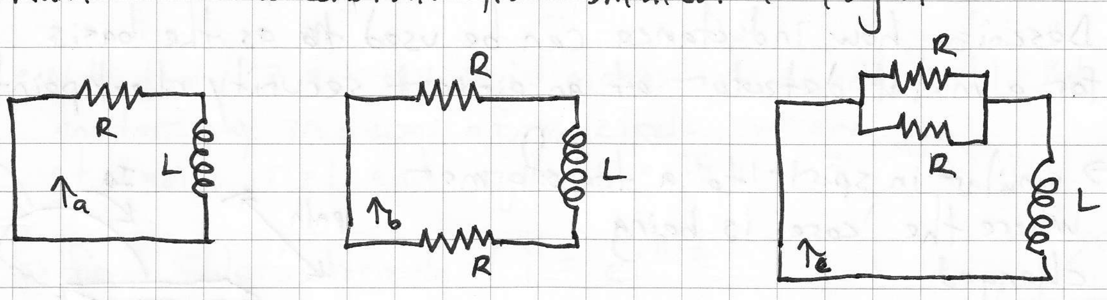
$$\Delta V_L + \Delta V_{R1} + \Delta V_{R2} = 0$$

$$\Delta V_L = -\Delta V_{R1} - \Delta V_{R2} = I_2 (R_1 + R_2) = (6.0\,mA)(800\,\Omega + 500\,\Omega)$$

$$= 7.8\,V$$

∴

$\boxed{ex}$ Rank the time constants from smallest to largest:



$\tau = \frac{L}{R}$ → $R_b > R_a > R_c$ → $\tau_c > \tau_a > \tau_b$

∴

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ex An inductor is made by tightly wrapping 0.30 mm diameter wire around a 4.0 mm diameter cylinder. What must the length of the cylinder be in order to get an inductance of 10 μH?

- Inductance of a solenoid is given by $L = \frac{\mu_0 N^2 A}{l}$

  → we are looking for l

- N (the total # of turns) must be given by $N = \frac{l}{d}$ (think about it; put another way, $l = Nd$ since all turns span the length of the cylinder)

r = radius of cylinder (= 2.0 mm)

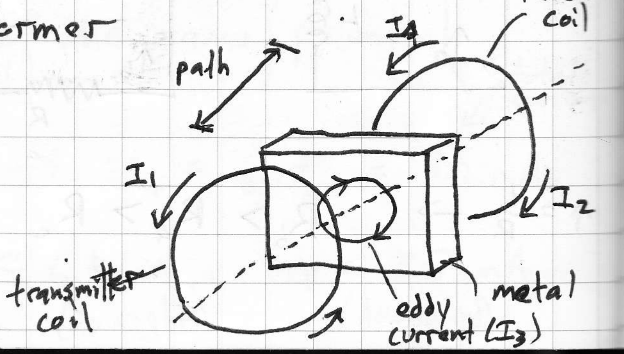
so $$L = \frac{\mu_0 A}{l}\left(\frac{l}{d}\right)^2 = \frac{\mu_0 \pi r^2 l}{d^2} \rightarrow l = \frac{L d^2}{\mu_0 \pi r^2}$$

$$= \frac{(1.0 \times 10^{-5}\ H)(3.0\times 10^{-4}\ m)^2}{(4\pi \times 10^{-7}\ Tm/A)\,\pi\,(2.0\times10^{-3}\ m)^2} = 0.057\ m = 5.7\ cm$$

$\therefore$

ex Describe how inductance can be used ~~the~~ as the basis for a metal detector at an airport security checkpoint.

→ similar in spirit to a transformer where the 'core' is being changed



- two coils are set up, one a 'transmitter', the other a 'receiver'

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- Drive the transmitter w/ an AC high frequency 'control' signal (I_1). This will induce an oppositely-oriented current in the receiver (I_2)

- The scanned object (e.g. a person walking through) passes between the two coils.

- If the object is metal, it will produce an **eddy current** in a plane parallel to the coils. Put another way, a current will be induced in the metal (I_3) due to the field created by the transmitting coil

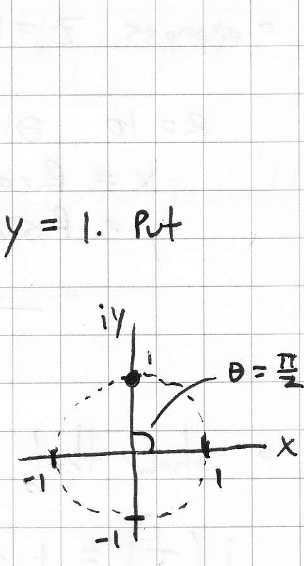
- The receivers picks up the superposition of what comes from the transmitter and metal. Specifically, the eddy current (I_3) induces a current in the receiver opposite that produces by the transmitter. This ultimately leads to a reduction in $|I_2|$.

- Circuitry detects changes in $|I_2|$, thereby raising the alarm

$\therefore$

ex Determine i^i.

- Note that $z = x + iy = i$ means $x = 0$ and $y = 1$. Put another way in terms of a circle, we are at radius $r = 1$ and angle $\theta = \frac{\pi}{2}$



- so from Euler's formula: $i = e^{i\pi/2}$

- Exponentiating exponentials: $(e^a)^b = e^{ab}$

$\rightarrow$ $i^i = (e^{i\pi/2})^i = e^{-\pi/2} \simeq 0.208$ (since $i^2 = -1$) $\therefore$

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$\boxed{\text{ex}}$ A solenoid has inductance $L = 50\,H$ and a resistance $R = 30\,\Omega$. If connected to a 100 V battery, how long will it take to reach one half its final equilibrium value?

$$I(t) = I_{max}\left(1 - e^{-t/\tau}\right) \qquad I_{max} = \frac{\Delta V_B}{R}$$

$$\tau = \frac{L}{R} \simeq 1.67\,s$$

so $\frac{1}{2} I_{max} = I_{max}\left(1 - e^{-t_0/\tau}\right)$

$\rightarrow 1 - e^{-t_0/\tau} = \frac{1}{2}, \quad e^{-t_0/\tau} = \frac{1}{2}, \quad -t_0/\tau = \ln\left(\frac{1}{2}\right)$

$\Rightarrow t_0 = -\tau \ln\left(\frac{1}{2}\right) = -(1.67)\ln\left(\frac{1}{2}\right) \simeq 1.2\,s$

$\boxed{\text{ex}}$ Complex # arithmetic

$$\underbrace{Re^{i\theta}}_{\text{polar form}} = \underbrace{x + iy}_{\text{cartesian form}}$$

$$\begin{cases} R = \sqrt{x^2+y^2} \\ \theta = \tan^{-1}\left(\frac{y}{x}\right) \end{cases}$$

$$\begin{cases} x = R\cos\theta \\ y = R\sin\theta \end{cases}$$

- express $z = 10^{i\pi/2}$ in cartesian form

$R = 10, \quad \theta = \frac{\pi}{2}$

$x = R\cos\theta = 10\cos\frac{\pi}{2} = 0$

$y = R\sin\theta = 10\sin\frac{\pi}{2} = 10$

$\longrightarrow z = 10e^{i\pi/2} = 10i$

- Show that $\frac{1}{i} = -i$

$i\left(\frac{1}{i}\right) = 1 \overset{?}{=} i \cdot -i = -1 \cdot i^2 = -1 \cdot -1 = 1$ ✓

(multiply both sides by i)

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- Show that $i = e^{i\pi/2}$

$i = 0 + 1 \cdot i \rightarrow x = 0, y = 1 \rightarrow R = 1$

$\theta = \arctan\left(\frac{1}{0}\right) = \arctan(\infty) = \frac{\pi}{2}$

$\rightarrow i = Re^{i\theta} = e^{i\pi/2}$ ✓

Basics of complex arithmetic:

Addition: $(a+ib) + (c+id) = (a+c) + i(b+d)$

subtraction: $(a+ib) - (c+id) = (a-b) + i(b-d)$

multiplication: $(a+ib)(c+id) = (ac-bd) + (bc+ad)i$

division: $\frac{(a+ib)}{(c+id)} = \frac{(a+ib)(c-id)}{(c+id)(c-id)} = \left(\frac{ac+bd}{c^2+d^2}\right) + \left(\frac{bc-ad}{c^2+d^2}\right)i$

NOTE: if $z = c+id$, then $z' = c-id$ is called the complex conjugate of z. It has the nice property that:

$$z \cdot z' = (c+id)(c-id) = c^2 - d^2 \quad \text{(which is real!)}$$