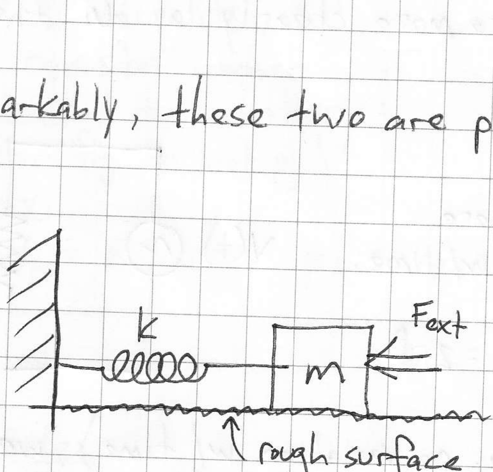
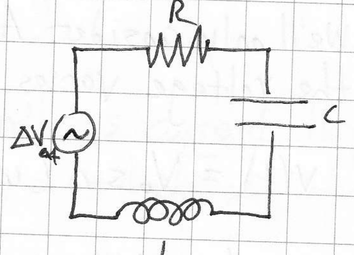
AC Circuits (Giordano ch. 22)

☐ Remarkably, these two are physically equivalent:



$\Longleftrightarrow$



$$m\ddot{x} + b\dot{x} + kx = F_{ext}$$

driving voltage

$$L\ddot{q} + R\dot{q} + \frac{q}{C} = V_{ext}$$

- mass $\leftrightarrow$ inductance
- spring $\leftrightarrow$ capacitor
- damping $\leftrightarrow$ resistor

mass, spring — storage of energy (reactance) $\boxed{X}$

damping — loss of energy (resistance) $\boxed{R}$

Note: Engineers commonly use the notion of impedance, which is a complex quantity and defined as

$$Z = R + iX \quad \longrightarrow \quad |Z| = \sqrt{R^2 \ast X^2}$$

$$= \left[R^2 + \left(2\pi f L - \frac{1}{2\pi f C} \right)^2 \right]^{\frac{1}{2}}$$

where

$$X = X_L - X_C$$
$$= 2\pi f L - \frac{1}{2\pi f C}$$

(this is eqn. 22.41 in Giordano's book)

and f is the driving freq.
(we assume Vext is sinusoidal at freq. f)

$\Rightarrow$ this leads to a more general form of **Ohm's Law**: $\boxed{V = IZ}$

(though Giordano states this in eqn. 22.39, he avoids the complex nature of things)

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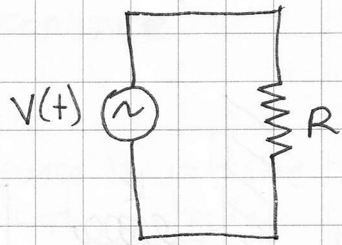
- Let's take a step back and try to more clearly lay out the pieces dealing w/ AC circuits

- We'll only consider AC circuits where the voltage varies sinusoidally w/ time:



$V(t) = V_0 \sin(\omega t)$ $\qquad \omega = 2\pi f$

<u>NOTE</u> when $V(t) = V_0 = \text{const.}$ (i.e. no variation w/ time), we call the voltage DC (direct current)

$\rightsquigarrow$ this is what we've dealt with primarily until now

- From Ohm's Law, we simply have: $\quad I(t) = \frac{V_0}{R}\sin(\omega t)$

- A useful quantity are <u>RMS</u> (root-mean-square) values, defined as:

$$V_{rms} = \sqrt{\tfrac{1}{2}V^2}$$

r - root
m - $\frac{1}{2}$
s - squared max.

<u>NOTE</u>: V is the max. amplitude of displacement

or more simiply: $\boxed{V_{rms} = \frac{V_{max}}{\sqrt{2}} \simeq 0.71\, V_{max}}$

$\longrightarrow$ Also applies to current as well: $\quad I_{rms} = 0.71\, I_{max}$

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- To see why rms quantities are useful in the context of AC circuits, let us consider power loss through a resistor when connected to a battery: (see fig. on last pg.)

Note: lowercase = instantaneous
uppercase = average

- Battery

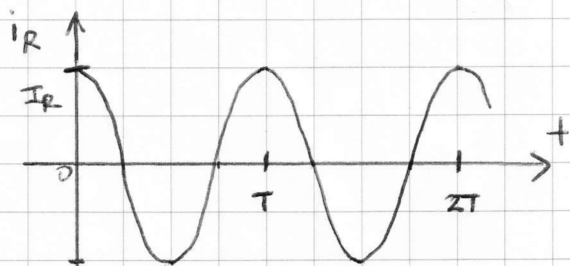
$$\Delta V_B = \mathcal{E} \longrightarrow p_b = i_B \mathcal{E}$$

$i_B \equiv$ instantaneous current through battery (not $\sqrt{-1}$ here!)

- Resistor

$$p_R = i_R V_R = i_R^2 R$$ (power dissipated by resistor)

- Assume circuit is AC: $i_R = I_R \cos \omega t$ $T = \frac{2\pi}{\omega}$



$$\longrightarrow p_R = I_R^2 R \cos^2 \omega t$$

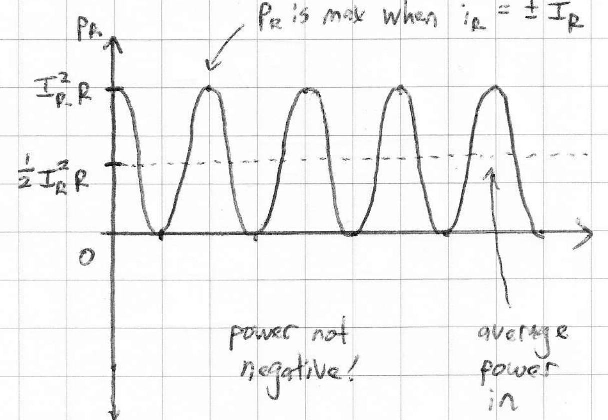
- Let $\mathcal{P}_R$ be the average power (i.e. total energy dissipated per second)

$$\mathcal{P}_R = I_R^2 R \cos^2 \omega t \Big|_{\text{one cycle}}$$

$$= I_R^2 R \left[\tfrac{1}{2} (1 + \cos 2\omega t) \right] \Big|_{\text{one cycle}}$$

$$= \tfrac{1}{2} I_R^2 R + \underbrace{\tfrac{1}{2} I_R^2 R \cos 2\omega t}_{=0 \text{ over one cycle}} \Big|_{\text{one cycle}}$$

$$= \tfrac{1}{2} I_R^2 R$$



$\mathcal{P}_R = \frac{1}{2} I_R^2 R$

but $\frac{1}{2} I_R^2 = \left(\frac{I_R}{\sqrt{2}} \right)^2 = I_{rms}^2$ (and $I_{rms} = \frac{V_{rms}}{R}$)

$$\longrightarrow \boxed{\mathcal{P}_R = I_{rms} \mathcal{E}_{rms} = I_{rms}^2 R}$$ ← average power dissipated over one cycle