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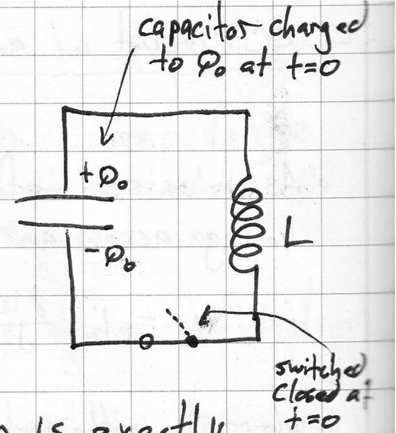
LC Circuit



- Let us consider an LC circuit (Giordano ch. 22.5) where there is no AC voltage source, but the capacitor is initially charged w/ $\pm Q_0$ across the plates. Assume no resistance at all.

<u>NOTE</u>: Something to keep in mind - this problem is exactly analogous to the simple harmonic oscillator (SHO, Giordano ch. 11.2 and our 3/13 notes). The capacitor acts like the spring, the inductor like the mass, and the initial charge $\pm Q_0$ is like us displacing the mass and then letting go at $t=0$.

- At $t=0$, there is no current. But there is an $\vec{E}$ field due to the capacitor. C starts to discharge by attempting to pass current through L. We can now apply Kirchoff's loop law:

$$\Delta V_C = \frac{Q}{C} \quad , \quad \Delta V_L = L\frac{dI}{dt}$$

$$\rightarrow \Delta V_C + \Delta V_L = 0 = \frac{Q(t)}{C} + L\frac{dI}{dt} = \frac{Q(t)}{C} + L\frac{d^2Q}{dt^2}$$

$$\rightarrow \frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0 \quad \longleftrightarrow \quad \ddot{Q} + \omega_0^2 Q = 0$$

where $\omega_0 = \sqrt{\frac{1}{LC}} = \frac{2\pi}{T}$ and $\ddot{Q} = \frac{d^2Q}{dt^2}$

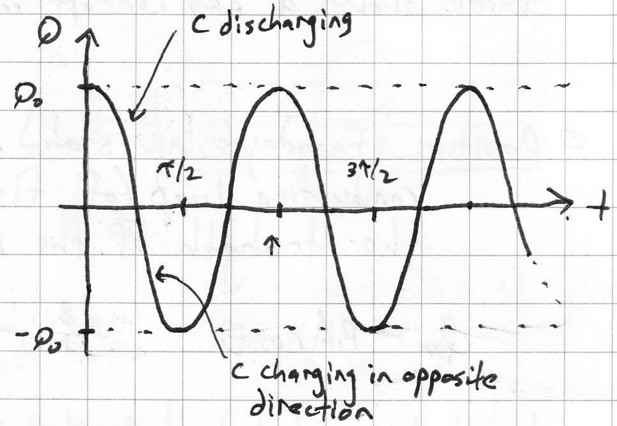
this is the same as our SHO eqn.!

- Note that $I = +\frac{dQ}{dt}$ because the current induced in the inductor opposes the change in current due to the charge on the capacitor. This is analogous to the velocity acting to oppose changes in inertia induced by the spring for the SHO

- Solution is $Q(t) = Q_0 \cos(\omega_0 t)$ $\boxed{3/22/13}$

So the charge across the capacitor plates transfers back and forth at a characteristic frequency ω_0 $\left(= \sqrt{\frac{1}{LC}}\right)$

- Let's consider what's going on at various stages:



$\boxed{0 \leq t \leq T/4}$

- current does not drop exponentially
- initially it remains const. due to counter-emf of L
- eventually that counter-emf is overcome and current flows more freely → energy is being transferred from $\bar{E}$ of capacitor to $\bar{B}$ of inductor

$\boxed{T/4 < t \leq T/2}$

- At $t = T/4$, C has a charge of 0
- However the current doesn't stop because $I = +\frac{dQ}{dt}$ is at a max. and the inductor (i.e. solenoid) has a max. $\bar{B}$ field
- Due to 'inertia' of $\bar{B}$, L produces a forward-emf to oppose a reduction in Φ_m
- As the current flows, it charges C back up, but in the opposite direction

$\boxed{T/2 < t \leq T}$

See Giordano Fig. 22.18

- Same basic process, but now in reverse

⟹ So ultimately, energy is getting transferred back and forth between an $\bar{E}$ field (via the capacitor) and the $\bar{B}$ field (via the inductor)

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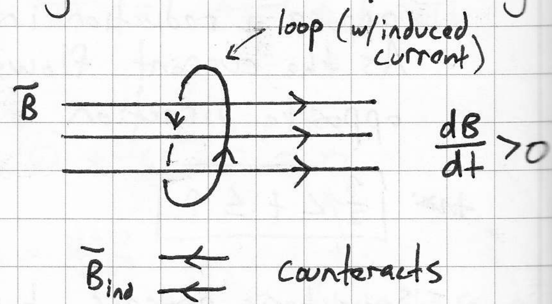
- This notion of $\bar{E}$ and $\bar{B}$ trading back and forth energy is a deep notion and provides the basis for allowing us to think about a key concept in physics: <u>**electromagnetic waves**</u>

- <u>Review</u>: Faraday's law stated that an emf is generated in a conducting loop (of fixed size and orientation) when the strength of the magnetic field varies w/ time:

$$\Phi_M = BA\cos\theta : \quad \frac{dB}{dt} \longrightarrow \frac{d\Phi_M}{dt} \longrightarrow \mathcal{E}$$

Lenz's law stated that the current induced in the loop generates an induced $\bar{B}$ field to counteract $d\Phi_M/dt$

- The induced emf ($\mathcal{E}$) responsible for the current is associated w/ an $\bar{E}$ field (otherwise there is no current!), but $\bar{E}$ here is a bit different than the electric fields we discussed earlier in semester
  - <u>Previously</u>: $\bar{E}$ starts at $q>0$ charge and ends at $q<0$ charge
  - <u>Here</u>: $\bar{E}$ goes around in a loop without beginning or end



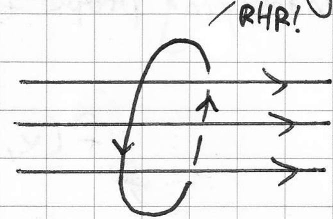
- Heinrich Hertz, around the late 1880s, used LC oscillators to show that when the oscillation freq. is relatively high ($100\text{ kHz} = 10^5\text{ Hz}$), this induction can happen **over distance**. Put another way, the induced $\bar{E}$ field can exist in air without a conducting metal loop!

$\rightarrow$ first expt. proof of **electromagnetic radiation**!!

☐ So what is going on here? what is electromagnetic radiation? $\boxed{3/22/13}$
How/why can an LC oscillator communicate w/ a reciever over some distance without a conducting pathway? To get some bearing here, we need to look at some <u>symmetries</u> that emerge stemming from our look at

- A time-varying $\bar{E}$ field surrounds itself w/ a circular $\bar{B}$ field

$\bar{E}(t)$



$$\frac{dE}{dt} \neq 0$$

→ this is just a generalized form of Ampère's Law

time-varying Φ_E through a virtual loop → $\bar{B}$ is generated

change in "electric flux"

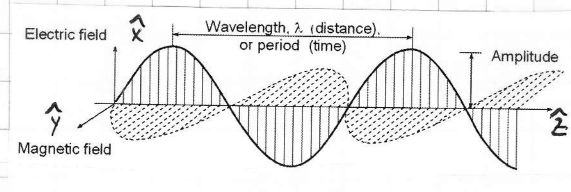
$$\oint_c \bar{B} \cdot d\bar{s} = \mu_0 I_{enc} \quad \longleftrightarrow \quad \frac{d}{dt}\Phi_E \longrightarrow \bar{B}$$

<u>NOTE</u>: $\begin{cases} \circ\ \bar{E} \text{ generated from } \Phi_m : \ \bar{E} \perp \bar{B} \\ \circ\ \bar{B} \text{ generated from } \Phi_E : \ \bar{B} \perp \bar{E} \end{cases}$ (Faraday's Law)

⇒ Symmetry here! $\left[\begin{array}{l} \frac{d}{dt}\Phi_m \rightarrow \bar{E} \\ \frac{d}{dt}\Phi_E \rightarrow \bar{B} \end{array} \right.$ the two can play off one another!!

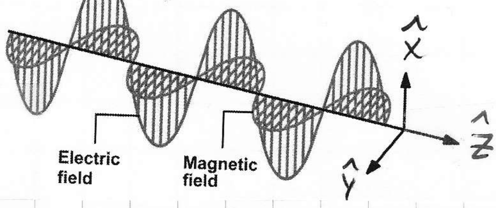
↓ snapshots @ fixed time

☐ This symmetry forms the basis for electromagnetic (EM) wave propagation



Electromagnetic wave

- $\bar{E}$ and $\bar{B}$ oscillate off one another to form a <u>transverse wave</u>



- EM waves are basis for light!

wave direction: $\hat{z}$

$\bar{E} = (E_x, 0, 0) \perp \hat{z}$

$\bar{B} = (0, B_y, 0) \perp \hat{z}$