3/25/13

Waves

- Though we are building up our knowledge in the context of electromagnetic waves ('EM radiation'), the notion of **waves** is a very general physical concept and waves manifest in a wide array of 'everyday' phenomena:

  - Sound
    - vibrating string (e.g. guitar, violin, piano)
    - speaker, drum head, eardrum (i.e. tympanic membrane)
    - compression of air
    - inner ear: cochlear traveling waves
  - Water (e.g. ocean waves, tsunami) → NOTE: Waves can move energy without transporting matter
  - Neurons - 'action potentials' convey information around the body (e.g. stepping on a tack)
  - EM radiation:
    - sunlight (and artificial light too!)
    - cell phone signal
    - wifi
    - microwave
    - 'black body radiation' (essentially heat)
    - X-ray imaging
    - MRI (in addition to strong magnets)

- The notion of waves is important in modern signal processing (e.g. wavelet transform, fourier transform)

- Most waves require a **medium** to travel in/along (e.g. sound requires air; no sound in outer space!). EM radiation is 'special' though in that it can propagate in vacuum (by means of induction; we'll come back to this)

- Waves can 'travel' but can also 'stand'
  → a **standing wave** is really a superposition of two opposite **traveling waves**

3/25/13

- Before venturing further ahead, it will help to take a step back of a moment to Giordano ch. 12 and examine underlying properties of **waves**

$$f(x,t) = f(\underbrace{x \pm ct})$$

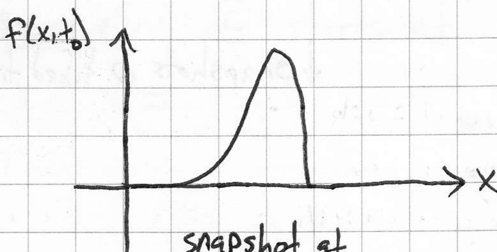
c – wave speed (see next pg.)

eqn. of a wave, depends upon both x and t

what is special about f (i.e. a wave) is that it actually depends upon x and t in a very ~~special~~ specific way

- $f(x-ct) \rightarrow$ 'right-traveling wave'
- $f(x+ct) \rightarrow$ 'left-traveling wave'

think about why given our conventions that x and t increase when 'going to the right'

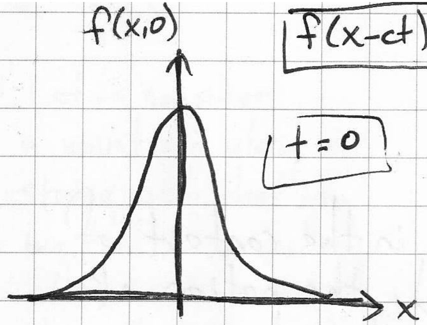


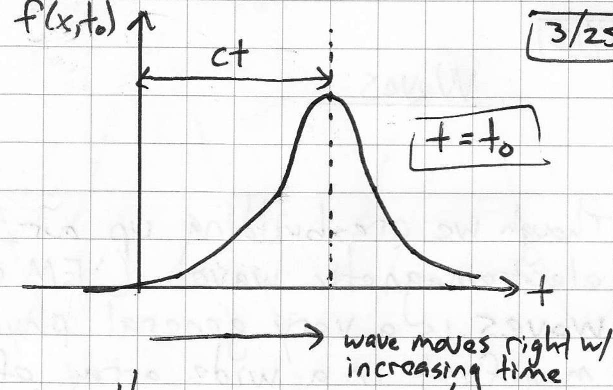
snapshot at $t = t_0$

snapshot at $x = x_0$

$\Rightarrow$ think that the 'shape' of f can stay constant if you tweak $x - ct$ appropriately . . .

$\boxed{f(x-ct)}$ $\boxed{3/25/13}$





- the 'wave' moves to the right with increasing t. Hence we say the wave 'travels to the right'
- Similar idea holds for $f(x+ct)$, except that we have a wave moving to the left
- Note that for the right-traveling wave, we require that

$$x - ct = \text{const.}$$

So what is we differentiate (w/ respect to t) both sides?

velocity of wave

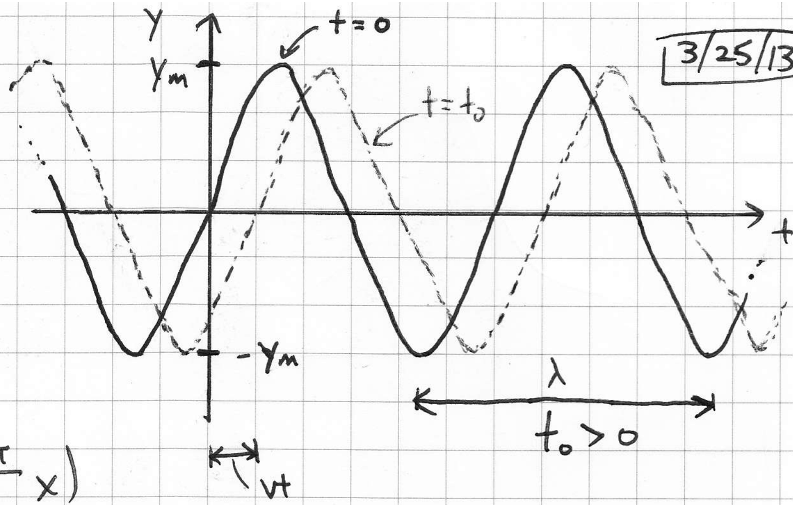
$$\frac{dx}{dt} - c = 0 \longrightarrow v = c$$

So c represents what we call the **phase velocity** of the wave, indicating the speed at which the wavefront moves to the right (for a left-traveling wave, $v = -c$)

□ Now we will consider a 'special' case where the wave repeats itself in a periodic fashion. This notion is useful in the context of a wide array of wave types, such as: EM waves, sound, (some types of) ocean waves

(distinction between a wave 'pulse' and a 'train')
$\longrightarrow$ the latter has a harmonic aspect to it!

3/25/13



- Let us consider a wavetrain along a string such that if we took a snapshot at $t=0$, we could describe the pattern by the equation:

$$y(x) = y_m \sin\left(\frac{2\pi}{\lambda} x\right)$$

such that y repeats itself at intervals of $x+\lambda,\ x+2\lambda, \cdots$ and λ is the **wavelength**

- Now consider a snapshot at a slightly later time (t_0) and that our wave is a right-traveling wave with phase velocity v. Then we have:

$$y(x,t) = y_m \sin\left[\frac{2\pi}{\lambda}(x - vt)\right] = y_m \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

  - T is the period of time required for the wave to travel a distance of one wavelength ($\lambda = vT$). So the wave will repeat itself at time intervals of $t+T,\ t+2T, \cdots$

- We make a slight change of variables to express things more compactly:

$$y(x,t) = y_m \sin(kx - \omega t)$$

k - **wave number** ($k \equiv \frac{2\pi}{\lambda}$)
ω - **angular freq.** ($\omega \equiv \frac{2\pi}{T} = 2\pi f$)

  - a left-traveling wave would instead be given by $y = y_m \sin(kx + \omega t)$
  - the phase velocity is simply $v = \frac{\lambda}{T} = \frac{\omega}{k}$
  - we can add in a phase offset if need be: $y(x,t) = y_m \sin(kx - \omega t - \phi_0)$
  - for a fixed location (e.g. $x = \frac{\pi}{k}$), this simply becomes $y(t) = y_m \sin(-\omega t + \phi)$ (this is just SHO!!)