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Combinations of Waves

- For a spherically propagating wave, the intensity at a location r from the source is $I = \frac{P_s}{4\pi r^2}$ (P_s - Power source) (i.e. energy is spread over the surface of sphere)

- **Principle of superposition**: when two or more waves are simultaneously present at a single point in space, the displacement of the medium at that point is the **sum** of the displacements due to each individual wave

$$D_{total} = D_1 + D_2 + \cdots \sum_n D_n$$

D_n - displacement due to wave n alone

→ unlike particles, two waves can pass directly through one another and emerge unchanged

1) Standing Waves

Q: What happens when a left- and right-traveling sinusoidal wave pass one another?

- Assume $v = v_0 > 0$ and $v_0 = \frac{\lambda}{T}$

Note sign change re 3/25 notes (not a problem!)

L→R wave: $D_1(x,t) = A\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right) = A\sin(\omega t - kx)$

R→L wave: $D_2(x,t) = A\sin\left(\frac{2\pi}{T}t + \frac{2\pi}{\lambda}x\right) = A\sin(\omega t + kx)$

$$D_{total}\ (= D_t) = A\left[\sin(\omega t - kx) + \sin(\omega t + kx)\right]$$

$$= A\left[\sin(\omega t)\cos(kx) - \cos(\omega t)\sin(kx) + \sin(\omega t)\cos(kx) + \cos(\omega t)\sin(kx)\right]$$

NOTE: $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

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$$= \underbrace{2A\sin(\omega t)}_{\text{time-dependent amplitude}} \underbrace{\cos(kx)}_{\text{spatial structure (doesn't change w/ time)}} = D_+(x,t)$$

→ spatial structure that changes w/ time, but does not move

- ~~These~~ This notion of a 'standing wave' describes the behavior of a string fixed at both ends (e.g. guitar or piano string) but also other types of waves such as sound or EM radiation (e.g. flute)

2) Interference

- In a more general way, we can examine how two waves 'interfere' (a fancy way of saying 'interact' via superposition). The analysis can get tricky, but we'll make some simplifying assumptions here:
  - consider two sinusoidal waves, each created at some source
  - both travel to the right ($+x$ direction)
  - both have the same amplitude and freq.
  - but one could be shifted relative to the other
  - for some point x, x_1 is the distance between x and source 1 (ditto for x_2)

$$D_1(x_1,t) = A\sin(kx_1 - \omega t + \phi_{10}) = A\sin\phi_1$$

$$D_2(x_2,t) = A\sin(kx_2 - \omega t + \phi_{20}) = A\sin\phi_2$$

where $\phi_1(x_1,t)$ and $\phi_2(x_2,t)$ are the **phases** of the waves. ϕ_{10} and ϕ_{20} are consts. determined by whatever source created the waves.

- We now can quantify the **phase difference** $(\equiv \Delta\phi)$

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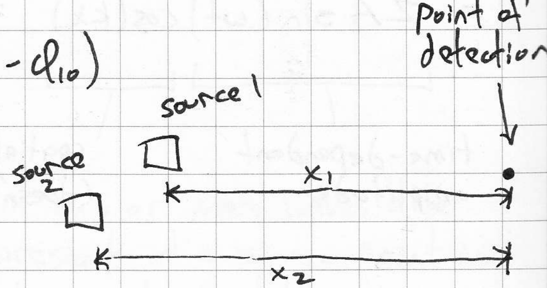
$$\Delta\phi = \phi_2 - \phi_1 = k(x_2 - x_1) + (\phi_{20} - \phi_{10})$$

$$= 2\pi \frac{\Delta x}{\lambda} + \Delta\phi_0$$

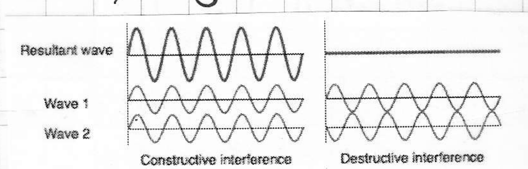


where $\Delta x \equiv x_2 - x_1$ (called the **path-length difference**) and
$\Delta\phi_0 \equiv \phi_{20} - \phi_{10}$ (inherent phase diff. between sources)

- When the phases line up correctly (crests aligned w/ crests, troughs aligned w/ troughs), ~~the~~ get a big response. This is called **constructive interference** and occurs when $\Delta\phi = 0, 2\pi, 4\pi, \ldots$

condition for max. constructive interference $\quad \Delta\phi = 2\pi \frac{\Delta x}{\lambda} + \Delta\phi_0 = m \cdot 2\pi$ radians
where $m = 0, 1, 2, 3, \ldots$

- Similarly, the converse holds such that when $\Delta\phi = \pi, 3\pi, 5\pi, \ldots$ the two are perfectly misaligned such that the crest of one aligns w/ the trough of the other ~~and~~ you get **destructive interference**



cond. for max. destructive interference

$$\Delta\phi = 2\pi \frac{\Delta x}{\lambda} + \Delta\phi_0 = \left(m + \tfrac{1}{2}\right) \cdot 2\pi$$

☐ other important properties/behavior of waves include **reflection**, **refraction**, and **diffraction** (this latter one being a complicated case of interference). These ideas form the foundation of **optics** (e.g. Giordano ch. 24, 25)

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☐ Let us return back to the idea of EM waves. First, let us compile some of the pieces we've gained along the way so far:

Maxwell's Eqns.

$$\oint \bar{E}\cdot d\bar{A} = \frac{Q_{in}}{\epsilon_0}$$

<u>Gauss' Law</u> - charged particles create electric fields

$$\oint \bar{B}\cdot d\bar{A} = 0$$

<u>Gauss' Law of magnetism</u> - no magnetic monopoles

$$\oint \bar{E}\cdot d\bar{s} = -\frac{d\Phi_m}{dt}$$

<u>Faraday's Law</u> - an electric field can be created by a changing magnetic field

$$\oint \bar{B}\cdot d\bar{s} = \mu_0 I_{through} + \underbrace{\epsilon_0 \mu_0 \frac{d\Phi_E}{dt}}_{\text{we have seen this yet}}$$

<u>Ampere-Maxwell Law</u> - currents create magnetic fields <u>and</u> a magnetic field can be created by a changing electric field

$$\bar{F} = q(\bar{E} + \bar{v}\times\bar{B})$$

<u>Lorentz force Law</u> - An electric ~~field~~ force is exerted on a charged particle in an electric field <u>and</u> a magnetic force is exerted on a charge moving in a magnetic field

→ These five eqns. are the backbone/foundation of electromagnetism. Everything else (e.g. Ohm's Law, Kirchoff's Law, Lenz's Law) can be derived from them.

→ Though beyond the scope of 1410, it follows from the equations that an electromagnetic wave arises and has the properties:

- $\bar{E}$ and $\bar{B}$ are perpendicular to one another and propagation direction → transverse wave
- $\bar{E}\times\bar{B}$ is in direction of propagation
- wave travels in vacuum at speed $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$
- $E = cB$ at any point on the wave