PHYS 1410 (Winter 2013) - Exam II

SOLUTIONS

Name:
Student number:

Instructions:

- Show all work clearly in order to get full credit. Points can be taken off if it is not clear to see how you arrived at your answer (even if the final answer is correct).
- Please keep your written answers brief and to the point. Circle your final answers.
- This test has 4 problems (plus two extra credit problems) and is worth 100 points (with 6 additional extra credit points possible). It is your responsibility to make sure that you have done all the problems!

1. (*25 points*)
The figure below shows the current as a function of time for an electric circuit.

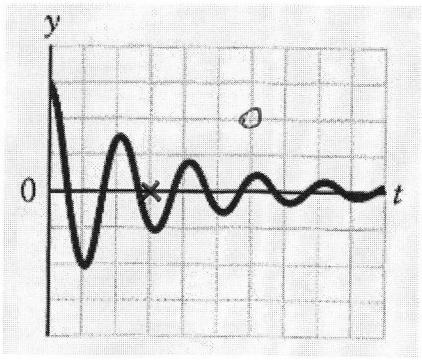


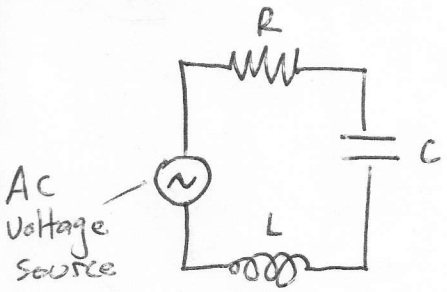
Figure 1: Problem 1.

Note: Let each box represent a unit measure for both t and y. For example, the x is at t=3, y=0. The O is at t=6, y=2.

7

a. Sketch **both** a simple *circuit* that gives rise to this sort of response, **as well as** a corresponding *mechanical analog* (i.e., a physical system that exhibits similar behavior).

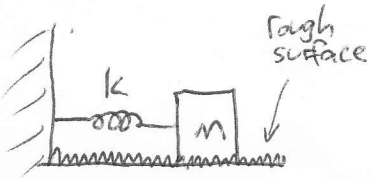
Electrical



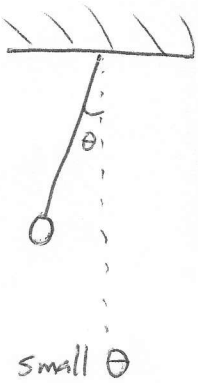
(not necessary here)

→ other combination of an RLC (L and C needed for oscillation, R needed for decreasing amplitude)

Mechanical



→ mass-on-a-spring or a simple pendulum



small θ

-3 Not sketching both

-2 Circuit doesn't have L and C

-1 some degree of explanation

8 **b.** Estimate the fraction of the energy that is "lost" to resistance over the course of one cycle.

We need to estimate the amplitude as a function of time, specifically how it decreases over the course of one full cycle.

Consider $t=0,\ y_0 = 3$ → $\frac{y_2}{y_0} = \frac{1}{2}$

one full cycle $t \simeq 2,\ y_2 = 1.5$

So the amplitude decreases by a factor of $\frac{1}{2}$

$E_0 = \frac{1}{2}ky_0^2$, $E_2 = \frac{1}{2}ky_2^2$

$\frac{E_2}{E_0} = \frac{\frac{1}{2}ky_2^2}{\frac{1}{2}ky_0^2} = \left(\frac{y_2}{y_0}\right)^2 = \frac{1}{4}$ ⇒ $\boxed{\text{75\% of the energy is lost over one cycle}}$

-1 No rationale

-4 wrong answer

say $\frac{1}{2}$ only amounts to $\frac{1}{2}$ credit (if there is explanation)

5 **c.** How does your answer to the last part change with time?

There is no change with time: the fraction of energy lost per cycle for a harmonic oscillator stays constant (even though the overall amplitude decreases).
For example, consider that $y_4 \simeq 0.75$ such that $\frac{y_4}{y_2} \simeq \frac{1}{2}$
(same fractional decrease)

-3 wrong answer

-1 no clear rationale

5 **d.** Is the system critically damped? Explain.

No. The system is underdamped (otherwise there would not be any oscillations)

-2 wrong answer

-2 No rationale spelled out

2. (*25 points*)
A bar magnet is dropped through a loop of wire as shown in the figure below.

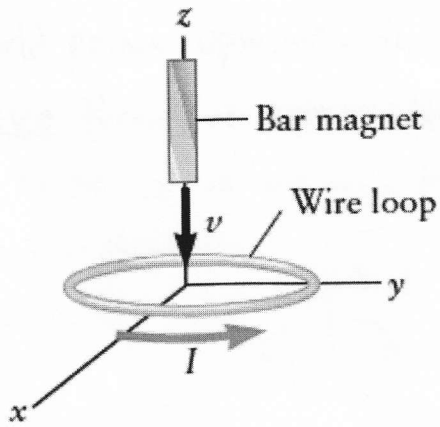


Figure 2: Current-carrying wire (and reference coordinates) for Problem 2.

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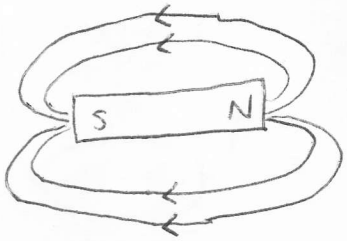
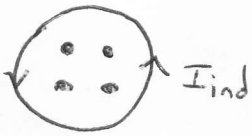
a. When the bar magnet is above the plane of the loop, the current induced in the loop is counterclockwise as viewed from above. Is the orientation of the bar magnet north pole up or south pole up? Explain.

- Direction of magnet will matter since the field has direction and thereby so does the flux through the hoop



direction of magnet field for the bar magnet

- The induced current in the loop creates a flux pointing upwards. This flux is such that it is resisting the change due to the moving magnet.

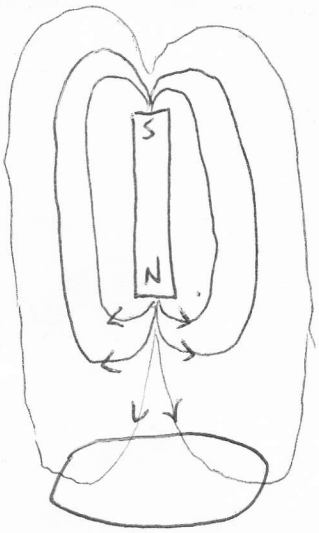


loop (looking down from above)

→ Use right-hand rule!!

- This means that as the magnet moves downwards, the change in flux (due to the magnet's field) must be such that ~~the flux~~ it is increasing in the downward direction

→ this can only be the case if the <u>south end of the magnet is pointing up</u>



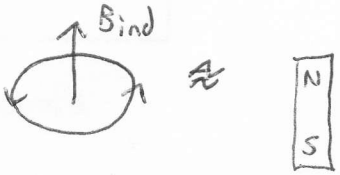
-3 wrong answer

-3 No clear rationale made clear (e.g. Lenz's Law)

→ -1 if no mention of flux

7 **b.** What is the direction of the resulting magnetic force on the bar magnet? Explain.

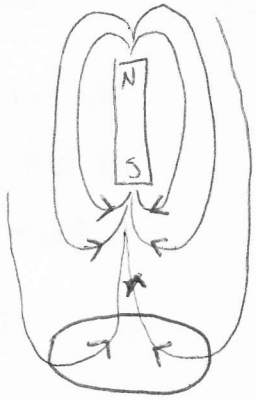
Since the induced field points upwards, that will create a <u>repulsive</u> force (or <u>upwards</u>). Energy-wise this makes sense, since we will have to do work to push the bar towards the hoop (and thereby generate that induced current)



- 3 wrong answer
- 1 No connection back to energy
- 2 directionality doesnt make sense

5 **c.** If the orientation of the bar magnet is inverted, does the direction of the current change? Explain.

<u>Yes</u>. Since the increasing flux will be upward-pointing, the induced field will point down and thus the induced current will go in the opposite direction



↓ magnet moving down mean increasing upward-point flux

5 **d.** If the orientation of the bar magnet is inverted, does the direction of the resulting magnetic force on the bar magnet change? Explain.

<u>No</u>. Both the flux and induced flux change direction, thus the force will still be repulsive (i.e. upwards). Energy-wise, this makes sense for the same reasons outlined before.

c). -2 wrong answer

-2 clear rationale

d) -2 wrong answer

-2 rationale

3. (*25 points*)
Consider an RL circuit shown in the figure below. Assume that $V = 9.1$ V, $L = 7.0$ mH, and $R = 1.75$ kΩ. After the switch has been left open for a long time, the switch is closed at $t = 0$.

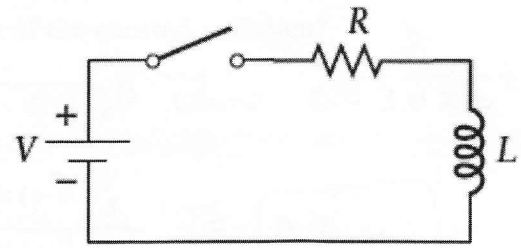


Figure 3: Problem 3.

7 **a.** What is the current the instant after the switch closed? Explain.

At $t=0$, the inductor will create an emf to resist the change in flux (i.e. zero). That induced emf will be -9.1 V such that <u>no current</u> flow through the circuit (i.e. $I(0)=0$)

a) −4 wrong answer
−2 explanation
−1 no mention that L opposes change

7 **b.** What is the current a very long time after the switch is closed?

b) similar to above

After a long time, the induced emf of the inductor is zero and thereby provides no opposition. Thus the current through the circuit is as if L was not even there (i.e. it's just what passes through the resistor)

$$V = \frac{I(\infty)R}{\cancel{R}} \rightarrow I(\infty) = \frac{V}{R} = \frac{9.1\text{ V}}{1.75\times10^{3}\ \Omega} = \boxed{5.2\text{ mA}}$$

11 **c.** How long does it take the current in the circuit to reach 30% of its final (long time) value?

- For an RL circuit, we have: $I(t) = I(\infty)\left(1-e^{-t/\tau}\right)$ where $\tau = \frac{L}{R}$
- To reach 30% of $I(\infty)$, we have: $0.3I(\infty) = I(\infty)\left(1-e^{-t_{30}/\tau}\right)$

$$0.3 = 1-e^{-t_{30}/\tau} \rightarrow t_{30} = -\tau\ln(0.7)$$

$$= -\frac{7.0\times10^{-3}\text{ H}}{1.75\times10^{3}\ \Omega}\ln(0.7) = \boxed{1.43\ \mu\text{s} = t_{30}}$$

−2 wrong answer
−5 not identifying $\tau = \frac{L}{R}$
−4 putting τ in the right context

4. (*25 points*)
Consider a cell phone that emits electromagnetic radiation with a frequency of 1.0 GHz (1.0×10^9 Hz) with a power of 1.0 W.

5 **a.** What is the wavelength of the emitted radiation?

For an EM wave, $c = \lambda f$ where $c = 3.0 \times 10^8$ m/s

so $\lambda = \frac{c}{f} = \frac{3.0 \times 10^8 \text{ m/s}}{1.0 \times 10^9 \text{ Hz}} = \boxed{0.3 \text{ m}}$

a) −3 note $c = \lambda f$
−2 carry thru calculation

b) −5 not realizing $I = \frac{P}{4\pi r^2}$
−3 connecting E and I
−2 calculation

10 **b.** What are the approximate amplitudes of **both** the electric field and magnetic field at a distance of 1.2 km from the phone? (Assume it radiates with spherical wavefronts)

- $I = \frac{P_{source}}{4\pi r^2} = \frac{\epsilon_0 c}{2} E_0^2 \rightarrow E_0 = \sqrt{\frac{2 P_{source}}{4\pi r^2 \epsilon_0 c}} = \sqrt{\frac{1.0 \text{ J/s}}{2\pi (1200 \text{ m})^2 (3.0 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \frac{F}{m})}}$

$\boxed{= 6.5 \times 10^{-3} \text{ V/m} = E_0}$

- Since $|E| = c|B|$, $B_0 = \frac{E_0}{c} = \boxed{2.2 \times 10^{-11} \text{ T} = B_0}$

5 **c.** What is the approximate electric field amplitude at your ear as you use the phone? (Again assume it radiates with spherical wavefronts and that the distance between your ear and the phone is 1 cm)

Same basic idea as the last part, but now $r = 0.01$ m

$E_0 = \sqrt{\frac{1.0 \text{ W}}{2\pi (0.01 \text{ m})^2 (3.0 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ F/m})}} = \boxed{774 \text{ V/m} = E_0}$

$B_0 = \frac{E_0}{c} = 2.6 \times 10^{-6} \text{ T} = \boxed{2.6 \ \mu\text{T} = B_0}$

c) −3 not realizing need to apply same idea
−2 calculation

5 **d.** If the phone can be turned on and transmitting for 8 hours before discharging the battery, how much energy can be stored in the battery?

- Since power $= \frac{\text{Joules}}{\text{s}} = 1.0$ W and 8 hours $= 28800$ s

then (power)(duration) = (energy stored) $\equiv E$

$E = 1.0 \frac{J}{s} \cdot 2.88 \times 10^4 \text{ s} = \boxed{2.9 \times 10^4 \text{ J}}$

d)
−3 not connecting power + energy
−2 calculation

Extra Credit # 1 (3 Points):

A published abstract started off as follows:

Preliminary findings reported at the 145th meeting of the Society suggested that confrontational tiger roars contain energy in the infrasonic portion of the electromagnetic spectrum.

Can you identify anything fishy about this? Explain.

Tiger vocalizations are acoustic in nature (i.e. longitudinal compression waves), not electromagnetic waves. Thus, there is no such thing as the 'infrasonic portion of the electromagnetic spectrum'

+1 something

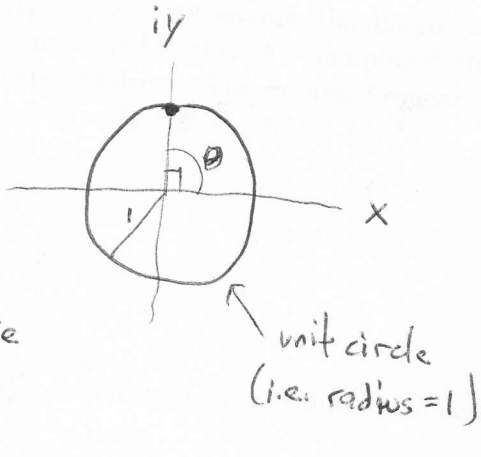
+1 realizing sound not part of EM spectrum

+1 explanation

Extra Credit # 2 (3 Points):

Express $z = 6e^{i\pi/2}$ in cartesian form.

Consider that $e^{i\theta}$ can be thought of as a point on the unit circle in the complex plane at angle θ relative to the positive real axis. As such,



$$e^{i\pi/2} = i$$

Thus $6e^{i\pi/2} = 6i = 0 + 6i$ (purely imaginary)

real part is zero

+1 something

+1 reasonable rationale

+1 correct answer