PHYS 1410 (Winter 2013) - Exam II

Name:
Student number:

Instructions:

- Show all work clearly in order to get full credit. Points can be taken off if it is not clear to see how you arrived at your answer (even if the final answer is correct).
- Please keep your written answers brief and to the point. Circle your final answers.
- This test has 4 problems (plus two extra credit problems) and is worth 100 points (with 6 additional extra credit points possible). It is your responsibility to make sure that you have done all the problems!

1. (25 points)
The figure below shows the current as a function of time for an electric circuit.

![Figure 1: Problem 1.](image)

Note: Let each box represent a unit measure for both $t$ and $y$. For example, the $x$ is at $t=3$, $y=0$. The $0$ is at $t=6$, $y=2$.

a. Sketch both a simple circuit that gives rise to this sort of response, as well as a corresponding mechanical analog (i.e., a physical system that exhibits similar behavior).

Electrical

Mechanical

-3 Not sketching both

-2 Circuit doesn't have $L$ and $C$

-1 Some degree of explanation

Other combination of an RLC ($L$ and $C$ needed for oscillation, $R$ needed for decreasing amplitude)
b. Estimate the fraction of the energy that is "lost" to resistance over the course of one cycle.

- We need to estimate the amplitude as a function of time, specifically how it decreases over the course of one full cycle.

\[
\text{Consider } t = 0, \ y_0 = 3 \quad \Rightarrow \quad \frac{y_2}{y_0} = \frac{1}{2}
\]

- So the amplitude decreases by a factor of \( \frac{1}{2} \)

\[
\frac{E_0}{E_2} = \frac{\frac{1}{2}ky_0^2}{\frac{1}{2}ky_2^2} = \left( \frac{y_2}{y_0} \right)^2 = \frac{1}{4} \Rightarrow 75\% \text{ of the energy is lost over one cycle}
\]

- c. How does your answer to the last part change with time?

- There is no change with time: the fraction of energy lost per cycle for a harmonic oscillator stays constant (even though the overall amplitude decreases). For example, consider that \( y_4 \approx 0.75 \) such that \( \frac{y_4}{y_2} \approx \frac{1}{2} \) (some fractional decrease)

- d. Is the system critically damped? Explain.

- No. The system is under-damped (otherwise there would not be any oscillations)
2. (25 points)
A bar magnet is dropped through a loop of wire as shown in the figure below.

![Diagram of bar magnet and loop](image)

**Figure 2:** Current-carrying wire (and reference coordinates) for Problem 2.

**a.** When the bar magnet is above the plane of the loop, the current induced in the loop is counterclockwise as viewed from above. Is the orientation of the bar magnet north pole up or south pole up? Explain.

- Direction of magnet will matter since the field has direction and thereby so does the flux through the hoop.
- The induced current in the loop creates a flux pointing upwards. This flux is such that it is resisting the change due to the moving magnet.
- This means that as the magnet moves downwards, the change in flux (due to the magnet's field) must be such that it is increasing in the downward direction.
- This can only be the case if the south end of the magnet is pointing up.

-3 wrong answer
-3 No clear rationale made clear (e.g., Lenz's Law)
→ 1 if no mention of flux
b. What is the direction of the resulting magnetic force on the bar magnet? Explain.

Since the induced field points upwards, that will create a **repulsive** force (or upwards). Energy-wise this makes sense, since we will have to do work to push the bar towards the hoop (and thereby generate that induced current).

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c. If the orientation of the bar magnet is inverted, does the direction of the current change? Explain.

Yes. Since the flux will be upward-pointing, the induced field will point down and thus the induced current will go in the opposite direction.

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d. If the orientation of the bar magnet is inverted, does the direction of the resulting magnetic force on the bar magnet change? Explain.

No. Both the flux and induced flux change direction, thus the force will still be **repulsive** (i.e. upwards). Energy-wise, this makes sense for the same reasons outlined before.
3. (25 points)
Consider an RL circuit shown in the figure below. Assume that $V = 9.1$ V, $L = 7.0$ mH, and $R = 1.75$ kΩ. After the switch has been left open for a long time, the switch is closed at $t = 0$.

![RL Circuit Diagram](image)

Figure 3: Problem 3.

a. What is the current the instant after the switch closed? Explain.

At $t=0$, the inductor will create an emf to resist the change in flux (i.e. zero). That induced emf will be $-9.1$ V such that no current flows through the circuit (i.e. $I(0) = 0$).

b. What is the current a very long time after the switch is closed?

After a long time, the induced emf of the inductor is zero and thereby provides no opposition. Thus the current through the circuit is as if $L$ was not even there (i.e. it's just what passes through the resistor).

$$V = \frac{I(\infty)R}{R} \rightarrow I(\infty) = \frac{V}{R} = \frac{9.1}{1.75 \times 10^3} \Omega = 5.2 \text{ mA}$$

c. How long does it take the current in the circuit to reach 30% of its final (long time) value?

For an RL circuit, we have: $I(t) = I(\infty) \left( 1 - e^{-\frac{t}{\tau}} \right)$ where $\tau = \frac{L}{R}$.

To reach 30% of $I(\infty)$, we have: $0.3I(\infty) = I(\infty) \left( 1 - e^{-\frac{t_30}{\tau}} \right)$

$0.3 = 1 - e^{-\frac{t_30}{\tau}} \rightarrow t_{30} = -\ln(0.7)

= - \frac{7.0 \times 10^{-3}}{1.75 \times 10^3} \frac{H}{\Omega} \ln(0.7) = 1.43 \text{ ms} = t_{30}$
4. (25 points)
Consider a cell phone that emits electromagnetic radiation with a frequency of 1.0 GHz \( (1.0 \times 10^9 \text{ Hz}) \) with a power of 1.0 W.

a. What is the wavelength of the emitted radiation?

For an EM wave, \( C = \lambda f \) where \( C = 3.0 \times 10^8 \text{ m/s} \)

So \( \lambda = \frac{C}{f} = \frac{3.0 \times 10^8 \text{ m/s}}{1.0 \times 10^9 \text{ Hz}} = 0.3 \text{ m} \)

b. What are the approximate amplitudes of both the electric field and magnetic field at a distance of 1.2 km from the phone? (Assume it radiates with spherical wavefronts)

\[
I = \frac{P_{\text{source}}}{4\pi r^2} = \frac{E_0 C}{2} \Rightarrow E_0 = \sqrt{\frac{2P_{\text{source}}}{4\pi r^2 E_0 C}} = \sqrt{\frac{1.0\text{ J/s}}{2\pi (1200\text{ m})^2 (3.0 \times 10^8 \text{ m/s}) (8.85 \times 10^{-12} \text{ F/m})}} = 6.5 \times 10^{-3} \text{ V/m} = E_0
\]

Since \( |E| = c|B| \), \( B_0 = \frac{E_0}{c} = 2.2 \times 10^{-11} \text{ T} = B_0 \)

Since the electric field amplitude at your ear as you use the phone? (Again assume it radiates with spherical wavefronts and that the distance between your ear and the phone is 1 cm)

\[
E_0 = \sqrt{\frac{1.0\text{ W}}{2\pi (0.01\text{ m})^2 (3.0 \times 10^8 \text{ m/s}) (8.85 \times 10^{-12} \text{ F/m})}} = 774 \text{ V/m} = E_0
\]

\[
B_0 = \frac{E_0}{c} = 2.6 \times 10^{-6} \text{ T} = 2.6 \mu\text{T} = B_0
\]

d. If the phone can be turned on and transmitting for 8 hours before discharging the battery, how much energy can be stored in the battery?

Since power = \( \frac{\text{Joules}}{s} = 1.0 \text{ W} \) and 8 hours = 28,800 s

then \( \text{(power)} \times \text{(duration)} = \text{(energy stored)} \Rightarrow E \)

\[
E = 1.0 \frac{\text{J}}{s} \times 2.88 \times 10^4 \text{ s} = 2.9 \times 10^4 \text{ J}
\]
**Extra Credit # 1** (3 Points):

A published abstract started off as follows:

*Preliminary findings reported at the 145th meeting of the Society suggested that confrontational tiger roars contain energy in the infrasonic portion of the electromagnetic spectrum.*

Can you identify anything fishy about this? Explain.

Tiger vocalizations are acoustic in nature (i.e. longitudinal compression waves), not electromagnetic waves. Thus, there is no such thing as the "infrasonic portion of the electromagnetic spectrum".

\[ +1 \text{ something} \]
\[ +1 \text{ realizing sound not part of EM spectrum} \]
\[ +1 \text{ explanation} \]

**Extra Credit # 2** (3 Points):

Express \( z = 6e^{i\pi/2} \) in cartesian form.

\[ cis(\theta) \]

Consider that \( e^{i\theta} \) can be thought of as a point on the unit circle in the complex plane at angle \( \theta \) relative to the positive real axis. As such,

\[ e^{i\pi/2} = i \]

Thus \( 6e^{i\pi/2} = 6i = 0 + 6i \) (purely imaginary)

\[ +1 \text{ something} \]
\[ +1 \text{ reasonable rationale} \]
\[ +1 \text{ correct answer} \]