Analysis of the Middle-Ear Function. Part I: Input Impedance

J. Zwislocki

Bioacoustics Laboratory, Syracuse University, Syracuse, New York
(Received March 12, 1962)

A quantitative theory of the middle-ear acoustics is developed and expressed in terms of an electric analog. The analog network is based on the functional anatomy of the middle ear. The numerical values of its elements are derived from impedance measurements on normal and pathological ears and from anatomical data. It is shown that the input impedance of the analog agrees within the experimental error with the acoustic impedance at the eardrum, and that changes in analog parameters corresponding to known anatomical changes produce the same effect on its impedance characteristics as measured at the eardrum.

INTRODUCTION

After four centuries of empirical research on the middle ear,[1] our knowledge of its acoustic properties seems to be approaching an asymptote. We went through periods of pure speculation, pseudotheories, working hypotheses, rugged empiricism, and it is possible that we reached a stage where an integrative theory is within our reach. By theory is meant here a formal system based on empirical evidence and concerned with interpolation among experimental data rather than with extrapolation—in other words, a system that shows how various experimental results fit together.

This paper describes an attempt to produce such a theory. The procedure used is similar to that of a jigsaw puzzle where, at the end of many trials, a coherent picture emerges. The pieces of the puzzle are bits of information produced by various kinds of research ranging from anatomy to physiology and acoustics. Experiments on living subjects as well as on post-mortem preparations are included.

There is one important difference between a jigsaw puzzle and piecing together a scientific theory. Whereas, in the first instance, all pieces are made to fit exactly for all practical purposes, in the second, the fit is usually much less perfect and some pieces do not seem to fit anywhere. The effect of a certain noise in the system is compensated to some extent by redundancy. Nevertheless, some blurred spots remain in the final picture. They should vanish gradually, as the experimental redundancy accumulates.

The knowledge of the acoustic function of the middle ear may be synthesized in various ways. The most primitive way would be to build an artificial middle ear that would replicate the natural middle ear in every detail. This is impractical, however, and the analytic information that could be derived from such an undertaking would be low. Any more sophisticated model must be based on mathematical theory, explicitly or implicitly. As a consequence, the theory developed in further portions of this paper is essentially mathematical, although it is presented in the disguise of an electric analog. Equations describing the action of various parts of the middle ear, as well as of the whole middle ear, are complex and their solution is tedious. It is more economical to express them by means of electrical networks and to study the input–output relationships. In this form, the theory appears reasonably simple and straightforward.

ANATOMICAL EVIDENCE

The acoustically important parts of the middle ear are shown schematically in Fig. 1. Proceeding from the outer-ear canal, the first part is the eardrum. To it is connected the chain of ossicles that consists of the malleus, incus, and stapes (hammer, anvil, and stirrup). The foot plate of the stapes is imbedded in the oval window of the inner ear and its motions are transmitted to the inner-ear fluids. The volume displacements produced by the stapes are compensated by nearly

[1] G. von Békésy and W. A. Rosenblith, "The Early History of Hearing—Observations and Theories," J. Acoust. Soc. Am. 20, 727 (1948).

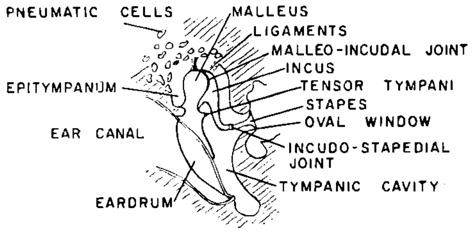

FIG. 1. Schematic of the middle-ear mechanism.

equally large displacements of the membrane of the round window.

Békésy[2] has shown that at low and medium frequencies the major central portion of the eardrum moves nearly as a rigid body and with the same amplitude as the part of the malleus attached to it. The eardrum appears to rotate around its upper edge—a mode of motion facilitated by a flexible fold near its lower edge. Nevertheless, not the entire membrane moves with the same amplitude. Toward the edges, the motion decreases to zero. Also, due to their flexibility, parts of the eardrum can move even after a complete fixation of the ossicles. At high frequencies, the mode of motion changes and different sections vibrate in different phases.[2] All this means that, although the coupling between the eardrum and the malleus is close, it is not perfect, and some of the acoustic energy activating the eardrum is not transmitted to the ossicles.

Further energy losses may be expected in the ossicular joints which are not completely rigid. This is particularly true for the frail incudo–stapedial joint.[3] Its pronounced flexibility becomes apparent from the following observations. After a complete fixation of the stapes through an otosclerotic ossification, it is possible to move the incus without much effort. In normal conditions, the incus and the stapes rotate around two different axes that are almost perpendicular to each other. A contraction of the stapedius muscle may change the plane of stapedial rotation by ninety degrees.[2] Finally, the extremely small cross-sectional area of the joint must lead to high stresses during sound transmission. Such stresses would produce noticeable deformations even in a material with a high constant of elasticity.

By contrast to the incudo–stapedial joint, the malleo–incudal joint appears robust, and it may be assumed that the two ossicles move as one unit.[3] They appear to rotate around their point of gravity so that the effective mass of the system is reduced to a minimum.[2,4]

The ossicles are held in place by ligaments and two muscles, the tensor tympani and the stapedius. The tensor tympani is attached to the hammer; when contracted it pulls the eardrum toward the middle ear. The stapedius is attached to the stapes and pulls this ossicle sideways. While the acoustic effect of the tympanic muscle is uncertain, that of the stapedial muscle is noticeable. It produces a change in the impedance measured at the eardrum and in the transmission characteristic of the ear.[5]

The eardrum, the ligaments and the muscles of the middle ear, the ligaments holding the stapedial foot plate in the oval window, and the elastic properties of the round-window membrane provide the middle-ear system with stiffness and introduce some frictional resistance. The main resistive component is contributed by the input impedance of the cochlea.[6]

Motions of the eardrum are transmitted to the ossicular chain as well as to the air-filled cavities of the middle ear. As a consequence, these cavities affect the acoustic characteristics of the middle ear and should not be ignored. From the acoustic point of view, they may be divided into three parts: the tympanic cavity immediately behind the eardrum, the epitympanum which extends upward from the tympanic cavity, and the more or less extensive system of pneumatic cells of the temporal bone which communicate with the other cavities through the antrum. Most of the epitympanum is filled by the body of the ossicles, so that only a narrow passage remains between the tympanic cavity and the antrum with the adjoining pneumatic cells. Seen from the ear-

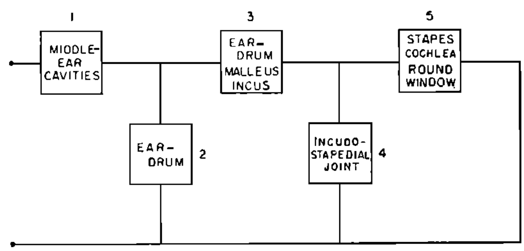

FIG. 2. Block diagram of the middle-ear mechanism.

drum, such a system may be expected to produce a resonance and an antiresonance effect, as has been recently demonstrated by Onchi.[7]

On the basis of the functional anatomy, it is possible to draw a block diagram of the acoustic units of the middle ear. This has been done in Fig. 2, which contains five functional units. Some justification for these units and their placement appears necessary. First of all, it may not be obvious why the block representing the middle-ear cavities precedes the blocks associated with the eardrum. This order is dictated by the fact that a displacement of any part of the eardrum is reflected in a compression of air enclosed in the middle-ear cavities while it is not necessarily transmitted to the ossicular

[2] G. von Békésy, *Experiments in Hearing* (McGraw-Hill Book Company, New York, 1960).
[3] J. Zwislocki, "Some Impedance Measurements on Normal and Pathological Ears," J. Acoust. Soc. Am. **29**, 1312 (1957).
[4] E. Bárány, "A Contribution to the Physiology of Bone Conduction," Acta Oto-Laryngol. Suppl. **26** (1938).
[5] O. Metz, "The Acoustic Impedance Measured on Normal and Pathological Ears," Acta Oto-Laryngol. Suppl. **63** (1946).
[6] J. Zwislocki, "Theorie der Schneckenmechanik" (Theory of Cochlear Mechanics), Acta Oto-Laryngol. Suppl. **72** (1948).
[7] Y. Onchi, "Mechanism of the Middle Ear," J. Acoust. Soc. Am. **33**, 794 (1961).

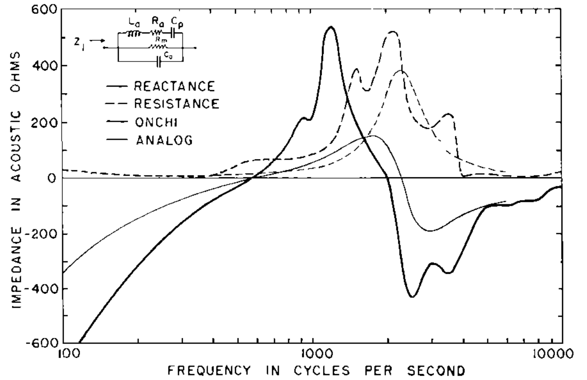

FIG. 3. Acoustic input impedance of the middle-ear cavities and their electric analog. Heavy lines show the impedance components obtained by Onchi on one temporal bone; thin lines indicate analog results based on average data.

chain. The second block represents the parts of the eardrum whose motion differs from that of the ossicles. Since a portion of the eardrum, the malleus and the incus, may be considered as one unit vibrating with the same amplitude and phase, they may be included in one block. The next block, number four, accounts for the fact that not all acoustic energy is transmitted across the incudo–stapedial joint. Finally, block number five stands for the stapes with its attachment to the oval window, the input impedance to the cochlea, and the round-window membrane. One block is sufficient for this complex structure, since the stapes and the round-window membrane vibrate with approximately the same amplitude and phase.

The next step is an analysis of the network blocks of Fig. 2 and a determination of the numerical values of their elements.

ACTION OF THE MIDDLE-EAR CAVITIES

The acoustic action of the middle-ear cavities has been ascertained on the basis of static volume determinations and impedance measurements.

The volume of the middle-ear cavities was determined on four preparations of temporal bones after the soft tissue had been removed. The bones were sealed hermetically with wax and connected to a vacuum pump by means of a perforated earplug secured in the ear canal. A valve arrangement made it possible to switch from the vacuum pump to a container filled with colored water. The amount of water that flowed into the bone could be determined from the level difference in the container. It was observed that, unless the air was evacuated from the bone, the cavities were not filled completely. The volumes of the middle-ear cavities obtained on the four bones amounted to 2.43, 4.1, 8.6, and 10.5 cc, respectively. These values are larger than the average values generally accepted in the literature.[8] Consequently, further experiments were undertaken. They included acoustic measurements on the four temporal bones and on patients with a reflected eardrum. Most of these measurements were performed by means of an acoustic bridge of the Schuster type.[5] On one temporal bone, the sound pressure produced in the ear canal by means of a high-impedance source was measured directly and compared to that produced in a standard cavity.[9] In this method, the volume to be determined is equal to the product of the standard volume and of the sound-pressure ratio. From bridge measurements on the temporal bones, the following volumes of the middle-ear cavities were obtained: 2.1, 3.4, 11.7, and 17.44 cc. With the exception of the 17.44-cc figure, these values are reasonably close to those obtained by means of water filling. The method of direct sound-pressure measurement confirmed the value of the largest volume obtained with the help of the acoustic bridge. It is possible, therefore, that in the first method the water did not fill all the cavities. The mean volume resulting from bridge measurements on the four temporal bones amounts to 8.6 cc.

Since it could be suspected that the large volumes obtained on the temporal bones were due to the absence of soft tissue, bridge measurements were repeated on otosclerotic patients during the mobilization operation. During one phase of the operation, a portion of the eardrum is reflected in order to make the stapes visible. It then becomes possible to measure acoustically the effective volume of the middle-ear cavities in the low-frequency range. Frequencies of 150, 200, and 300 cps were used. They produced comparable results. The volume of the middle-ear cavities measured on six patients ranged from 5.5 to 14.0 cc with a median of 9.3 cc and a mean of 8.7 cc. These values agree well with those obtained on the bone preparations.

The acoustic action of the middle-ear cavities at medium and high frequencies was recently determined by Onchi[7] who performed impedance measurements on various parts of fresh ear preparations. Unfortunately, the impedance of the middle-ear cavities is given for one specimen only. Onchi estimated its total middle-ear volume to be 3.45 cc, which is below our average. Nevertheless, the general shape of the impedance curves of the specimen should not depart in principle from the average. For this reason, Onchi's impedance curves are reproduced in Fig. 3 by means of heavy lines. The thin lines show the real and the imaginary components of the impedance of the electrical analog shown in the inset. No effort was made to match Onchi's individual curves exactly.

The electrical analog has been designed mainly on the basis of the anatomical evidence. The electrical elements shown in Fig. 3 have the following significance. The inductance L_a is an analog of the acoustic inertance due

[8] G. von Békésy and W. A. Rosenblith, "The Mechanical Properties of the Ear," in *Handbook of Experimental Psychology*, edited by S. S. Stevens (John Wiley & Sons, Inc., New York, 1951).

[9] J. Zwislocki, "Some Measurements of the Impedance at the Eardrum," J. Acoust. Soc. Am. **29**, 349 (1957).

to the narrow passage between the tympanic cavity and the remainder of the cavities. The resistance R_a stands for the acoustic resistance arising from the same narrow passage. The capacitance C_p represents the compliance due to the air volume of the antrum and of the pneumatic cells. The pneumatic cells have a complex structure and the capacitance C_p can be considered an adequate analog only at low frequencies. Some sort of a transmission line, as suggested by Onchi, would have been a more likely structure. However, there are no sufficient data for the design of such a line, and a study of Onchi's impedance curves has shown that a simple capacitance can satisfy approximately the input conditions. Beyond the resonance point, which is situated around 600 cps, the importance of the pneumatic cells decreases. The resistance R_m represents the sound absorption in the walls of the tympanic cavity and the Eustachian tube. Its value is also affected by the sound absorption in the pneumatic cells. Finally, the capacitance C_t stands for the acoustic compliance of the middle-ear cavities. The sum of the capacitances C_p and C_t corresponds to the total volume of the middle-ear cavities. If we accept the acoustic compliance as defined by the equation

$$C=V/\rho c^2,$$

with V meaning the volume of air, ρ the average density of air, and c the speed of sound, then

$$C_p+C_t=V_m/\rho c^2.$$

Accepting for the total volume V_m an approximate value of 8 cc, we obtain $C_p+C_t=5.45\times10^{-6}$ cgs units or, in electrical terms, 5.45 μF. The average volume of the tympanic cavity may be estimated to be of the order of 0.5 cc. This leads to $C_t=0.35\mu$F and $C_p=5.1\mu$F. The value of L_a was so chosen that the zeros of the analog reactance agree approximately with the zeros of Onchi's reactance curve. This value amounts to 14 mH. Other values were tried and rejected on the basis of subsequent experiments. The resistance values of R_a and R_m were determined from the phase relationships that result from Onchi's impedance measurements on the middle-ear cavities and on the basis of impedance measurements at the eardrum of normal and pathological ears.

It must be emphasized that Onchi's impedance curves could have been duplicated considerably better than in Fig. 3. A smaller capacitance C_p, a larger inductance L_a, and resistance R_m would bring the analog and real ear-impedance curves to close coincidence. Such an undertaking would be in conflict with other empirical evidence, however, particularly with the determination of the average volume and with the impedance measurements at the eardrum of normal and pathological ears. Since the impedance of the middle-ear cavities is low by comparison to other parts of the middle ear, its effect on the impedance at the eardrum and on the sound transmission to the inner ear is not critical. As a consequence, the analog of Fig. 3 may be regarded as a sufficient approximation.

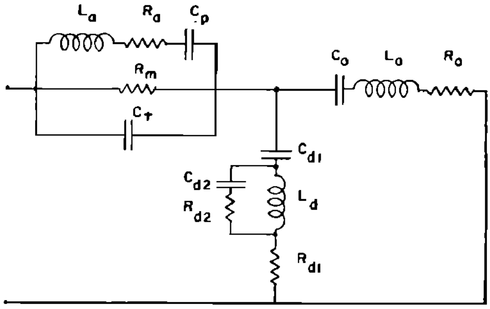

FIG. 4. Electric analog of the middle ear without incus. Elements denoted by subscripts a, p, m, and t belong to the middle-ear cavities, those with the subscript d to a portion of the eardrum, and those with the subscript o to the malleal complex.

EAR WITHOUT INCUS

In certain surgical procedures, the incus is removed without otherwise affecting the middle-ear structures. Ears without the incus are acoustically simpler than normal ears.[3] The effects of the incudo–stapedial joint, of the stapes, of the cochlea, and of the round-window membrane are eliminated. The analog circuit contains only three units of Fig. 2, i.e., middle-ear cavities, eardrum, and malleus. The acoustic properties of the middle-ear cavities are known already, so that only the characteristics of the malleus with its attachments and of the eardrum remain to be considered.

The malleal structures consist of the malleus, the ligaments holding it in place, the tensor tympani, and the portion of the eardrum that vibrates with the same amplitude and phase as the malleus. The whole structure can be approximated in the electric analog by a series connection of a capacitance, a resistance, and an inductance.

At low frequencies, the portion of the eardrum that is not coupled directly to the malleus can be represented by a capacitance in series with a resistance.[3] At higher frequencies, the mass of the eardrum becomes effective, and an inductance must be added to the circuit. At still higher frequencies, the eardrum vibrates in sections, and a complex electrical network is required. A transmission line would probably constitute the best analog. However, it could be demonstrated that the simplified circuit of Fig. 4 is sufficient for low and medium frequencies.

Figure 4 shows the complete analog network of the ear without incus. The elements corresponding to the middle-ear cavities have been described already. The elements with the subscript o refer to the malleal complex, those with the subscript d to the remaining portions of the eardrum.

The numerical values of the elements in the two latter groups, with the exception of L_d, C_{d2}, and R_{d2}, were determined from impedance measurements at the

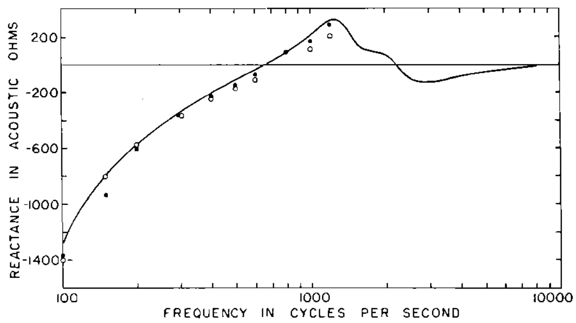

FIG. 5. Reactance at the eardrum of an ear without incus. Points show data obtained on two patients; curves indicate analog results.

eardrum. The experimental method was described in two previous papers,[3,9] and only small technical improvements were introduced for the purposes of the present investigation.[10] These improvements concern the coupling between the probe tubes and the ear canal, and the measurement of the volume of the ear canal. First, an otological speculum was secured in the ear canal by means of a plastic earplug and so oriented that the eardrum could be seen. Second, the probe tubes were connected to the speculum through a tightly fitting adapter. In this way, an obstruction of the tubes by the walls of the ear canal could be avoided. After the termination of the acoustic measurements, the tubes were withdrawn from the speculum and the ear canal was filled with alcohol up to the tip of the speculum, using a calibrated syringe and a blunt hypodermic needle. The amount of injected alcohol served as a measure of the volume of air between the tips of the probe tubes and the eardrum. Since the volume of the ear canal is contained in the computational formulas for the impedance at the eardrum, its determination was undertaken with great care.

Typical experimental results obtained on two patients with removed incus are shown in Figs. 5 and 6. The curves indicate the input impedance of the electrical analog with the following numerical values:

$$C_0=1.4\,\mu\text{F},\quad C_{d1}=0.23\,\mu\text{F},$$
$$L_0=40\text{ mH},\quad C_{d2}=0.40\,\mu\text{F},$$

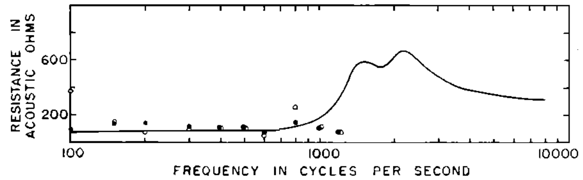

FIG. 6. Resistance at the eardrum of an ear without incus. Points show data obtained on two patients; curves indicate analog results.

[10] J. Zwislocki, "Electrical Model of the Middle Ear," J. Acoust. Soc. Am. **31**, 841(A) (1959).

$$R_0=70\,\Omega,\qquad L_d=15\text{ mH},$$
$$R_{d1}=40\,\Omega,$$
$$R_{d2}=220\,\Omega.$$

The agreement between the experimental data and the analog results may be considered satisfactory. This is particularly true for the reactance. In the resistance data, a discrepancy appears in the vicinity of 1000 cps. While the resistance measured in the ear seems to decrease with frequency, the resistance of the analog increases. The latter effect is in agreement with anatomical considerations, and it is possible that the decrement in the experimental resistance data is due to an artifact. Onchi[7] points out that sound absorption in the ear canal may be the cause. In his experiments, the effect of the ear canal could be eliminated, and the resistance shows an upward trend similar to that of the analog.

A method of determining the network elements was described in a previous paper in which a highly simplified electric analog of the middle ear was discussed. Although, with increasing amount of information it became possible to devise more refined models, the basic method remained the same. As a consequence, there is no need for its renewed discussion at this place. It may be of interest, however, to compare the magnitude of the analog elements with the available anatomical data. Such a comparison is possible for the inductances L_0 and L_d which correspond to the effective mass of the malleus and of approximately one-half of the eardrum, respectively. The elements of the analog are numerically equal to acoustic impedance elements. In order to obtain the actual mechanical values, they have to be multiplied by the square of the effective area of the eardrum. According to Békésy,[2] this area amounts to 0.55 cm^2 on the average. Consequently, the inductance L_0 is equivalent to a mass of 12 mg and the inductance L_d to one of 4.5 mg. According to Békésy and Rosenblith,[8] the total mass of the malleus is of the order of 23 mg and is, therefore, approximately twice as great as the effective mass determined by means of the analog. This agrees with the observation that the ossicles tend to rotate around their center of gravity, so that the effective mass is reduced. Although extraction of the incus may be expected to throw the system out of balance,[3] the effective mass should remain smaller than the total mass of the malleus. The total mass of the eardrum may be calculated from the geometrical dimensions and the specific density. Békésy[2] estimates the total area of the eardrum to be 0.85 cm^2 and the thickness to be 0.01 cm. Assuming a specific density of 1.1 g/cm^3, which is slightly larger than that of water, we obtain 9.35 mg for the total mass. If an area of 0.55 cm^2 is rigidly coupled to the malleus, the remaining area, which accounts for L_d, amounts to 0.3 cm^2 and its mass to 3.3 mg. In view of the difficulties in estimating the dimensions of the eardrum, this value must be considered in agreement with that obtained from the analog.

It may be mentioned that Wever and Lawrence[11] give for the mass of the tympanic membrane a value of 14 mg which would lead to an equivalent mass of L_d of 4.6 mg.

OTOSCLEROTIC EAR

In the otosclerotic ear, the stapes is immobilized in the oval window and, as a consequence, the cochlea disconnected from the middle-ear system.[3] In terms of the analog network, this means an interrupted circuit between the blocks 3 and 5. Block 4, which corresponds to the incudo-stapedial joint, remains in the network, and its elements have to be determined from anatomical considerations and from impedance measurements at the eardrum.

It is rather obvious that the elastic and fricative forces in a joint between two bones should play a preponderant role. The simplest electrical analog of such a system is a capacitance in series with a resistance.[3] It could be demonstrated that these two elements suffice to account for the effect of the incudo-stapedial joint on the impedance at the eardrum. Their numerical values

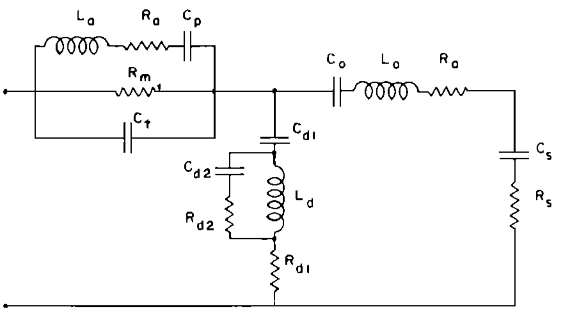

FIG. 7. Electric analog of the otosclerotic ear. Elements denoted by subscript *s* belong to the incudo-stapedial joint. The other elements are the same as in Fig. 4.

can be easily determined from impedance measurements at the eardrum, provided it is possible to assume that the parts of the network determined on ears without incus do not require any modifications. Such an assumption is open to criticism, as has been pointed out in a preceding paper[3] and elaborated on by Møller.[12] First of all, the incus has been added to the malleus. Second, the motion of the two ossicles is probably affected by the fixation of the stapes, and, according to Møller, their effective mass should be increased. The change in the ossicular mode of motion may, in turn, affect the acoustic properties of the eardrum.

Studies on the electric analog have demonstrated that, although the effective mass of the ossicles may well be increased by a fixation of the stapes, the effect on the impedance at the eardrum is negligible. This is so because of the high impedance of the incudo-stapedial

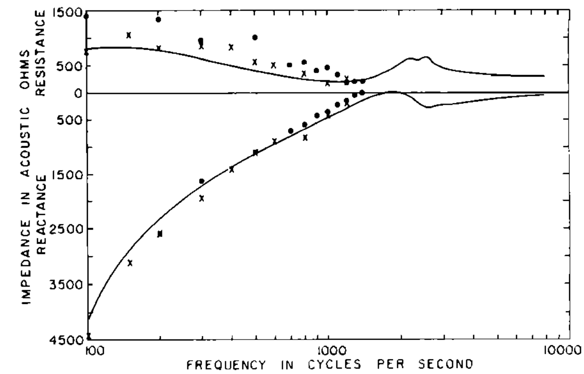

FIG. 8. Average reactance and resistance at the eardrum of otosclerotic ears. Closed circles indicate experimental results of an earlier series; crosses those of a more recent series where an improved technique was used; curves show analog results.

joint at all frequencies, and because of the low coupling impedance of the eardrum at high frequencies. It was also possible to duplicate the otosclerotic impedance at the eardrum without any changes in the analog circuit of the eardrum. For these reasons, the analog network of the middle ear without the incus was accepted as the analog of the otosclerotic ear after the analog circuit for the incudo-stapedial joint had been added. The analog is shown in Fig. 7. Its numerical values are $C_s = 0.25\ \mu F$ and $R_s = 3000\ \Omega$, in addition to those determined previously. The value of L_0 can be doubled or even tripled without producing any appreciable effect.

The experimental and the analog impedance data are shown in Figs. 8 and 9. Figure 8 contains medians of two series of experiments performed on 14 ears (14 patients)[3] and 17 ears (9 patients),[10] respectively. Closed circles denote the results of the former, crosses of the latter series in which the more recent, improved technique was used. The curves correspond to the analog data. The agreement between the experimental and the analog results should be considered satisfactory, although some discrepancies are apparent. The absolute value of the analog reactance is somewhat too small at low and somewhat too large at medium frequencies. The analog

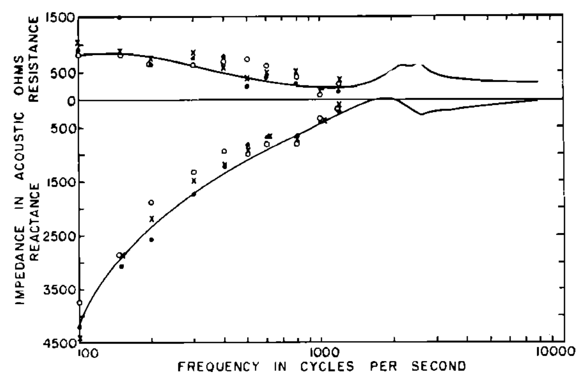

FIG. 9. Reactance and resistance at the eardrum of otosclerotic ears. Circles and crosses indicate data obtained on 3 patients; curves show analog results (same as in Fig. 8).

[11] E. G. Wever and M. Lawrence, *Physiological Acoustics* (Princeton University Press, Princeton, New Jersey, 1954).

[12] A. R. Møller, "Network Model of the Middle Ear," J. Acoust. Soc. Am. **33**, 168 (1961).

resistance is somewhat too small over the entire range. In general, the agreement is better with the second experimental series. The discrepancies could be easily corrected by a slight modification of the eardrum circuit. Although such a modification could be justified in view of probable changes in the mode of ossicular motion, its physical meaning could not be ascertained at present. Consequently, it was considered superfluous for the purposes of this article. The effect of various parameters on the impedance at the eardrum will be discussed in a subsequent paper.

Figure 9 compares empirical data obtained on three typical patients to the analog characteristics. It can be seen that the agreement is at least as good as with the median data, and that little information was lost in the preceding figure as a result of the averaging procedure.

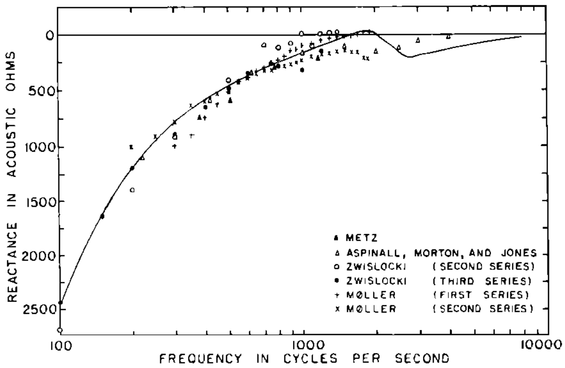

Fig. 11. Average reactance at the eardrum of normal ears. Symbols indicate data obtained by several investigators; curve shows analog results.

NORMAL EAR

Functionally, the otosclerotic ear contains all the elements of the normal ear, except the input impedance of the cochlea and the impedances of the stapes and of the round-window membrane. A simple circuit for the

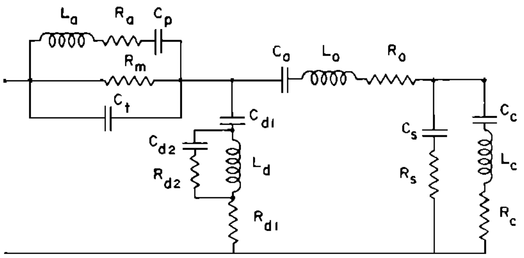

Fig. 10. Electric analog of the normal ear. Elements with subscript *c* are added to those of Figs. 4 and 7; they belong to the cochlear complex.

missing complex may be derived from the theory of electroacoustic analogies.[3] It consists of a capacitance and a resistance for each of the two windows, and of a resistance and an inductance for the cochlea. All these elements are connected in series, so that the capacitances and the resistances that stand for the two windows may be lumped together. Furthermore, the practically negligible mass of the stapes may be added to the mass resulting from the inner-ear impedance to constitute one inductance. All the resistances may be lumped together also. As a result, the network of Fig. 10 is obtained. It is an analog of the normal middle ear, in which all the elements except C_c, L_c, and R_c have already been determined.

The elements of the cochlear complex have been so chosen that the input impedance of the analog matches the average impedance at the eardrum.

During the matching experiments, it was found that the input impedance is so insensitive to the inductance L_c that this element may be completely eliminated. Nevertheless, L_c has been left in because of anatomical considerations, and because Békésy's experiments indicate the presence of a mass component in the input impedance of the inner ear. This mass component is not due to the entire column of perilymph in the cochlea, as has been assumed erroneously by several authors, but is produced by a short column of fluid in the vestibule, between the cochlea proper and the oval window. The input impedance of the cochlea itself is resistive, due to the interaction between the mass of the perilymph and the compliance of the basilar membrane. This is clearly evident from the general hydrodynamical theory of wave propagation,[6] and has been confirmed experimentally by Mundie[13] on the guinea-pig ear.

The mass component of the inner ear cannot be determined directly from Békésy's experiments, since the conditions prevailing at the oval window may have been altered during the experimental procedure. However, it may be estimated from anatomical sections that the column of perilymph between the oval window and the cochlea proper does not exceed 2 mm. Since the average area of the stapes foot plate has been estimated to be 3.2 mm², the mass of the effective perilymph column should amount to approximately 6.4 mg. This value is increased by the mass of the stapes (2.5 mg)[8] to 8.9 mg. In order to obtain the acoustic mass, the latter figure must be divided by the area of the stapes squared. The calculation yields 8.7 g/cm⁴. The equivalent acoustic mass referred to the impedance at the eardrum is obtained by division by the square of the acoustic

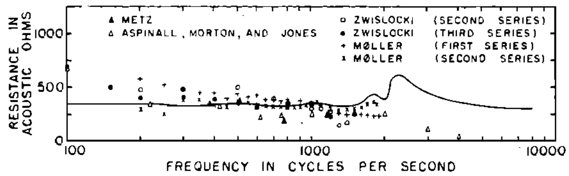

Fig. 12. Average resistance at the eardrum of normal ears. Symbols indicate data obtained by several investigators; curve shows analog results.

[13] J. R. Mundie and D. F. Henges, "Some Factors Influencing the Acoustical Impedance of Guinea Pig Ear," J. Acoust. Soc. Am. **32**, 1495(A) (1960).

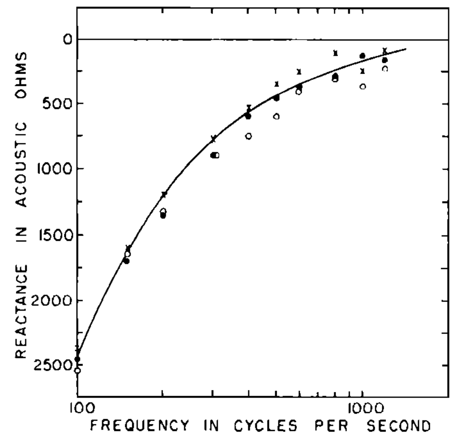

FIG. 13. Reactance at the eardrum of normal ears. Circles and crosses indicate data obtained on three subjects; curve shows analog results (same as in Fig. 11).

transformer ratio between the oval window and the eardrum. Békésy arrives at a theoretical transformer ratio of 22. Consequently, the equivalent acoustic mass at the eardrum amounts to approximately 20 mg. This corresponds to an inductance of 20 mH.

The other values of the cochlear complex that fit the measured impedance at the eardrum are

$$C_c = 0.6\,\mu\text{F},\ R_c = 600\,\Omega.$$

The resistance agrees very well with the value predicted in earlier papers.[3,6]

The experimental data obtained by several investigators as well as the analog results are shown in Figs. 11 and 12. The data of Møller[12,14] and Zwislocki[3,10] were taken directly from original papers. The values given by Metz[5] had to be converted from specific impedance expressed in ρc units to acoustic impedance expressed in cgs units. The conversion was based on a characteristic acoustic impedance of air of 130 cgs units which could be derived from the description of Metz's experimental setup. The results of Morton and Jones[15] were adjusted because of their somewhat crude estimates of the volume

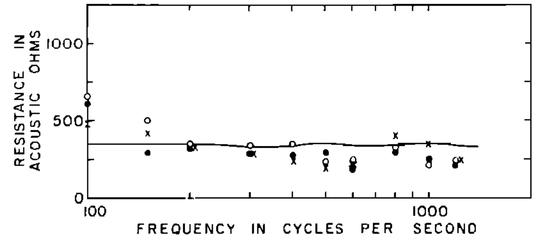

FIG. 14. Resistance at the eardrum of normal ears. Circles and crosses indicate data obtained on three subjects; curve shows analog results (same as in Fig. 12).

[14] A. R. Møller, "Improved Technique for Detailed Measurements of the Middle Ear Impedance," J. Acoust. Soc. Am. **32**, 250(A) (1960).

[15] J. Y. Morton and R. A. Jones, "The Acoustical Impedance Presented by Some Human Ears to Hearing Aid Earphones of the Insert Type," Acustica **6**, 339 (1956).

of the ear canal. Morton and Jones measured the distance from the probe to the eardrum and multiplied it by the average cross-sectional area of the ear canal. In this way, they obtained a volume of 0.48 cc which is smaller by 0.07 cc than the average volume usually obtained under similar conditions. Since, on the one hand, their impedance values at the entrance to the ear canal agreed rather well with data discussed in a previous paper,[10] and, on the other, the calculated impedance at the eardrum appeared somewhat too small, it was decided to adjust their data to a volume of 0.55 cc.

Figures 11 and 12 show that the experimental results are in a fairly good agreement with each other. The largest discrepancies appear above 1000 cps for the reactance and below 300 cps for the resistance. The probable reason for this effect can be found in the magnitude ratio of the two components. At low frequencies, the reactance is large and the resistance small; above 1000 cps, the reverse is true. The smaller component is very sensitive to the phase measurement. Consequently, an error in phase determination produces a magnified error in the smaller impedance component.

The curves of Figs. 11 and 12 indicate the results obtained on the electrical analog. In the frequency range between 100 and 1500 cps for the reactance and between 200 and 1500 cps for the resistance, they agree with the experimental data within the error of measurement.

Figures 13 and 14 compare the analog impedance characteristics to typical data obtained on three subjects with normal hearing. The agreement is as good as with the average data, which leads to the conclusion that the analog is a valid representation of a typical middle ear.

COMPARISON WITH PREVIOUS MODELS

The electrical analog discussed in this paper evolved as a result of successive approximations. The first crude model was published in 1957, a somewhat improved one was discussed before the Acoustical Society of America in 1959. Several other models remained unpublished. It may be of interest to compare the numerical values of the principal elements in the three published analogs in order to see whether there is some consistency. Using the nomenclature of this paper, we can make the following comparisons. The capacitance C_0 representing the compliance of the malleo-incudal system had in the first model a value somewhat higher than 0.56 μF, in the second it grew to 1.64 μF, and in the third it decreased a little to 1.4 μF. The inductance standing for the acoustic mass of the ossicular chain changed from 60 to 50 to 40 mH. The capacitance in the complex of the incudo-stapedial joint varied irregularly from 0.35 and 0.52 to 0.25 μF. The analog capacitance of the compliance of the cochlear windows had an unreasonably high value of 2.6 μF in the first model, but decreased to 0.7 μF in the second, and further to 0.6 μF in the third. The input resistance of the cochlea remained reasonably stable changing only from 540 to 400 to 600 Ω. It is evident that, despite the growing complexity of the analog, the principal elements kept their order of magnitude. It is probable, therefore, that they are representative of the actual conditions in the middle ear.

In this context, it may not be superfluous to mention that the inductance L_0 obtained for the malleo–incudal complex in the latest analog is exactly equivalent to the effective mass of these ossicles as determined by Frank.[16] By letting the ossicles oscillate around their axis of gravity, Frank obtained an effective mass of 12 mg. When the value of 40 mg/cm^4, equivalent to the inductance $L_0=40$ mH, is multiplied by the square of the effective area of the eardrum, an effective mass of 12 mg results.

A considerable number of electrical analogs have been suggested by various investigators. The older ones became obsolete in the light of accumulating experimental evidence. The more recent ones, designed on the basis of quantitative investigations, need to be discussed.

Møller[12] has suggested an analog based on impedance measurements at the eardrum of clinically normal ears. In order to determine the circuit elements, he varied the response of the ear by coating the eardrum with collodion and other substances and by letting the stapedius muscle contract. This method is somewhat analogous to the one discussed in this and a preceeding paper. However, changes in the middle-ear characteristics were produced in a different way.

There are three aspects of Møller's approach that appear open to criticism. First of all, the complex effect of the middle-ear cavities is ignored; second, it is assumed that the stapedius muscle effectively immobilizes the stapes; and third, it is assumed that the ossicles rotate around the incudo–stapedial joint when the stapes becomes immobile. The effect of the middle-ear cavities is small up to about 1000 cps, and, if the validity of the model is restricted to the low frequencies, the omission of this effect is not critical. The assumption of stapedial immobility due to stapedius contraction may be of greater consequences. A comparison of Møller's impedance curves with analogous curves obtained on otosclerotic patients indicates that the stapedius action is considerably less effective than otosclerotic fixation. While, in the first situation, the impedance at low frequencies increases by a few percent, in the second, the change amounts to 100% and often more. The mode of ossicular motion is of primary importance in the design of an electrical analog, and Møller's assumption that the malleus and the incus can rotate around the incudo–stapedial joint makes his analog substantially different from the analog described in this paper. At first glance, Møller's assumption appears quite attractive. Closer scrutiny leads to some doubts. Rotation of the two ossicles around the incudo–stapedial joint would necessitate a motion of the portion of the malleus imbedded in the eardrum in a plane tangent to the surface of the eardrum. Such a motion would be opposed by an extremely high mechanical impedance which is not borne out by experiments. Rotation of the two ossicles around the incudo–stapedial joint would also produce a high mass component in the impedance at the eardrum. This mass would be greater than the total mass of the malleo–incudal complex and, as a consequence, would exceed 50 mg. In the electrical analog, this mass would be reflected by an inductance of 167 mH instead of about 50 mH, as calculated by Møller for his analog.

From the analysis of Møller's work, there emerges an interesting conclusion that the middle-ear structure effectively prevents any motion of the ossicles other than rotation around an axis passing through, or at least near, the center of gravity of the malleo–incudal complex.[2]

Another model of the middle ear has been suggested by Onchi.[7] It was presented for the first time in 1949 as a mechanical analog.[17] In a recent paper, Onchi shows a conversion of the mechanical analog into an electrical one. The electrical version is based on essentially the same assumptions that underly the analog of this paper, and, as a consequence, the general structures of the two analogs are similar. Nevertheless, there are some differences that should be mentioned. They concern mainly the portion of the eardrum that is not rigidly coupled to the malleus, and the incudo–stapedial joint. Both systems are represented by a capacitance in Onchi's paper. Such a simplification should be permissible for the eardrum at low frequencies. In the frequency range above 1000 cps, a more complex network appears necessary. In the incudo–stapedial joint, where the lenticular process of the incus moves relative to the head of the stapes, friction must arise. It should be represented in the electrical analog by a resistance. Impedance measurements on otosclerotic ears indicate that this resistance is substantial.

It is not possible to make a comparison with Onchi's analog on a quantitative basis, since Onchi omitted giving the numerical values of his elements. However, judging from his impedance data, a considerable disagreement would result. These data, obtained on ear preparations, show much higher impedance values than those encountered in the ears of living subjects with normal hearing. Onchi's explanation that the low impedance measured on living subjects is due to the acoustic conditions in the ear canal does not seem acceptable, since a high impedance is found in otosclerotic patients. It appears far more probable that post-mortem changes account for the high impedance of Onchi's specimens.

[16] O. Frank, "Die Leitung des Schalles im Ohr (Sound Transmission in the Ear)," Sitzber. math-physik. Kl. bayer. Akad. Wiss. München **53**, 11 (1923).

[17] Y. Onchi, "A Study of the Mechanism of the Middle Ear," J. Acoust. Soc. Am. **21**, 404 (1949).

This point will be further elaborated on in a subsequent paper.

CONCLUSIONS

In an effort to synthesize the available empirical information on the acoustic function of the middle ear into a quantitative theory, an electric analog of the middle ear has been devised. The analog may be considered an expression of the underlying mathematical theory. Although it would have been possible to obtain the same result by means of a set of algebraic equations, these equations would have been so complex that their meaning would not become immediately apparent. An electric analog provides a simple visual image of the system, and makes it possible to recognize functional relationships almost on sight. Another advantage is the time economy in obtaining data for various kinds of input signals and for various changes in the parameters of the system.

It is true that an electrical analog is an imperfect model and that its validity is restricted to a limited range of the variables involved. The same is true for all possible models or theories, however, and cannot be considered a specific disadvantage. All the simplifications deemed necessary in the electric analog described in this paper have been checked carefully against empirical evidence, and they were not accepted unless the response of the analog matched that of the real ear within the experimental uncertainty. It is expected that future research will narrow down the tolerances and make certain modifications necessary. This should be viewed as scientific inevitability, however, rather than as an inherent weakness of the method.

For the present, it is probably safe to state that the devised electric analog answers the following questions. It specifies the function of various parts of the middle ear in a quantitative way. It correlates changes in the acoustic impedance at the eardrum to anatomical changes in the middle-ear structures. It also shows that the numerical values of its parameters, which were derived from average data, are a true representation of conditions in typical ears. In anticipation of a subsequent paper, it is possible to state further that the analog provides information on the transmission properties of the middle ear, on the correlation of these transmission properties to the impedance at the eardrum for various anatomical conditions, and finally, on the transient responses.

The frequency range of validity for the analog of the normal ear extends from at least 100 to 2000 cps. Its response characteristics were recorded up to 8000 cps. However, the empirical data beyond 2000 cps are meager and in poor agreement with each other. For pathological ears, no data seem to be available beyond 1400 cps. It is hoped that these gaps will be filled in the near future, although experimental difficulties increase rapidly at frequencies higher than 1000 cps.

ACKNOWLEDGMENTS

I should like to express my gratitude to Dr. G. D. Hoople, to Dr. W. H. Bradley, and to Dr. A. S. Feldman for their help in obtaining data on patients and on temporal bones. My thanks also go to Mr. V. H. Sandoval, to Mr. S. Madonna, and to Mr. R. Gardinier for their help in computation of the impedance data and in the construction of the electrical analogs.

This work was supported by a research grant from the National Institute of Neurological Diseases and Blindness, National Institutes of Health, Public Health Service, U. S. Department of Health, Education, and Welfare.