Linear momentum conservation and collisions

☆ 2 particles w/o external forces (e.g. 2 billiard balls on table if friction is negligible)

Newton-3 : $\vec{F}_{1\,on\,2} = -\vec{F}_{2\,on\,1}$

calculate momentum changes of both particles:

$$\left\{\begin{aligned} \Delta\vec{p}_2 &= \vec{p}_{2f} - \vec{p}_{2i} = \int_{t_i}^{t_f} \vec{F}_{1\,on\,2}(t)\,dt \\ \Delta\vec{p}_1 &= \vec{p}_{1f} - \vec{p}_{1i} = \int_{t_i}^{t_f} \vec{F}_{2\,on\,1}(t)\,dt = -\int_{t_i}^{t_f} \vec{F}_{1\,on\,2}(t)\,dt \end{aligned}\right.$$

$$\Rightarrow \boxed{\begin{aligned} &\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0 \\ \Leftrightarrow\ &\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \end{aligned}}$$

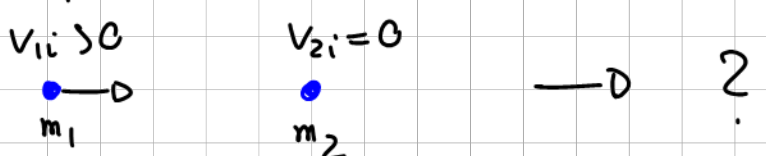
total momentum is conserved!

this can be generalized:

for a system of N particles w/o external forces (i.e. for a <u>closed</u> system), the total momentum never changes (i.e. is conserved):

$$\boxed{\vec{P}_{tot,i} = \sum_{k=1}^{N} \vec{p}_{k,i} = \sum_{k=1}^{N} \vec{p}_{k,f} = \vec{P}_{tot,f}}$$

example: 1D billiard (mentioned before)



- momentum conservation (for $\hat{i}$ components)

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$

we need another equation to calculate the velocities of both balls after the collision. This second equation is kinetic energy conservation of the 2-particle system:

$$KE_i = \frac{m_1}{2} v_{1i}^2 = \frac{m_1}{2} v_{1f}^2 + \frac{m_2}{2} v_{2f}^2 = KE_f$$

(note that there is no PE present before and after the collision, i.e. $TE_i = TE_f \Rightarrow KE_i = KE_f$).

These two equations can be combined to obtain

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \, , \quad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

For the special case $m_1 = m_2$ this results in $v_{1f} = 0, v_{2f} = v_{1i}$

(more details: see challenge problem # 5)

- Collisions with $KE_i = KE_f$ are called elastic collisions.
- Collisions with $KE_i \neq KE_f$ are called inelastic collisions.

Special case for the latter: sticky collision

characterized by $\vec{v}_{1f} = \vec{v}_{2f} = \vec{v}_f$

example: car accident (book, ex 7.6)

① $P_{tot,i} = m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f = P_{tot,f}$

$$\Leftrightarrow \quad v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

$\rightarrow$ v_f can be calculated from momentum conservation alone.

calculate $\Delta KE = KE_f - KE_i$

$$= \frac{m_1 + m_2}{2} v_f^2 - \frac{m_1}{2} v_{1i}^2 - \frac{m_2}{2} v_{2i}^2 < 0$$

(KE is converted into deformation, heat and sound)

Related example: explosion

split a resting object into two pieces

Momentum conservation (in 1D) applies:

$$P_{tot,i} = 0 = P_{tot,f} = m_1 v_{1f} + m_2 v_{2f}$$

$$\Leftrightarrow \quad v_{1f} = -\frac{m_2}{m_1} v_{2f}$$

$$= - v_{2f} \quad \text{if} \quad m_1 = m_2$$