Impulse

◇ motion of an object with constant force acting;

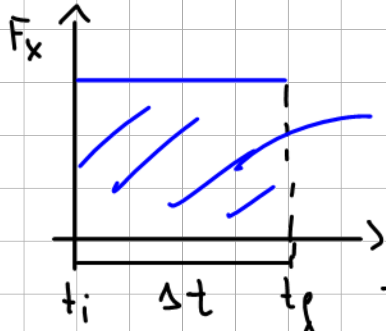
we know that $\vec{v}_f - \vec{v}_i = \vec{a}\,\Delta t \quad (\Delta t = t_f - t_i)$

$$\Leftrightarrow \quad m(\vec{v}_f - \vec{v}_i) = \vec{p}_f - \vec{p}_i = \Delta\vec{p}$$
$$= m\vec{a}\,\Delta t = \vec{F}\,\Delta t$$

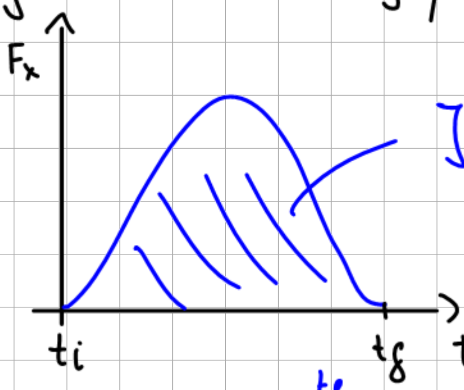
define impulse: $\vec{J} = \vec{F}\,\Delta t$

↳ impulse-momentum theorem: $\vec{J} = \Delta\vec{p}$

visualize J_x (similarly for J_y, J_z)



Hence, generalize definition to deal with non-constant forces accordingly:



J_x = area under F_x-t curve

$$= \int_{t_i}^{t_f} F_x(t)\,dt$$

(similarly for J_y, J_z)

↳ $$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\,dt = \left(\int_{t_i}^{t_f} F_x(t)\,dt\right)\hat{\imath} + \left(\int_{t_i}^{t_f} F_y(t)\,dt\right)\hat{\jmath} + \left(\int_{t_i}^{t_f} F_z(t)\,dt\right)\hat{k}$$

Q: Is the impulse - momentum theorem still valid ?

A: assume "yes" :
$$\int_{t_i}^{t_f} \vec{F}(t)\, dt = \vec{p}_f - \vec{p}_i$$
$$= \vec{p}(t_f) - \vec{p}(t_i) = \vec{p}(t)\Big|_{t_i}^{t_f}$$
$$\Rightarrow \frac{d}{dt}\vec{p} = \vec{F}$$

is this true ?

$$LS = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a} \overset{\text{Newton-2}}{=} \vec{F} = RS \quad \text{yes!}$$

One can turn argument around and infer work-impulse theorem from Newton-2, i.e. $\vec{J} = \Delta\vec{p}$ holds in general !

application ①: calculate / estimate average forces $\vec{F}_{ave}$ for forces that act only for a short time and are difficult to measure

$\vec{F}_{ave}$ is defined via
$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\, dt = \vec{F}_{ave}\,\Delta t$$

example : baseball bat on ball (book, ex 7.4)

$m = 0.14\ kg$, $\Delta t = 0.001\ s$, $\vec{v}_i = (-45\hat{\imath})\frac{m}{s}$, $\vec{v}_f = (+50\hat{\imath})\frac{m}{s}$

$$\Rightarrow \Delta\vec{p} = m(\vec{v}_f - \vec{v}_i) = 13\ kg\cdot\frac{m}{s}$$
$$|\vec{F}_{ave}| = \frac{\Delta p}{\Delta t} = 1.3 \times 10^4\ N \quad (\text{big!})$$

application ②: air bags: increase interaction time Δt to reduce F_{ave} for given Δp.