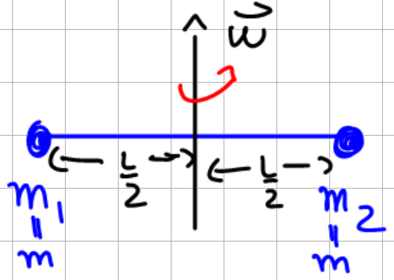
Moment of inertia

so far $I = mr^2$ for single mass point at distance r from rotation axis

generalize to $I = \sum_i m_i r_i^2$ with pieces of mass m_i at distances r_i from rot. axis

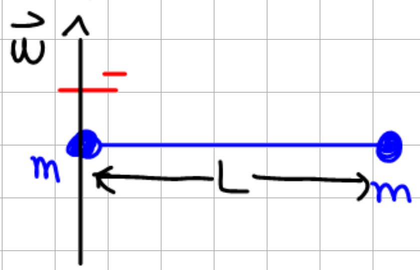
example ①: two balls (connected by massless rod of length L)



$$I = m_1 r_1^2 + m_2 r_2^2 \qquad m_1 = m_2 = m$$

$$= m_1 \left(\frac{L}{2}\right)^2 + m_2 \left(\frac{L}{2}\right)^2 = \frac{mL^2}{2}$$

example ②:



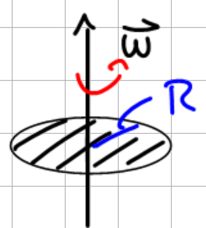
$$I = m_1 r_1^2 + m_2 r_2^2$$

$$= 0 + mL^2 = mL^2$$

example ③: thin hoop of radius R

$$I = \sum_i m_i r_i^2 = \sum_i m_i R^2 = M_{tot} R^2$$

example ④: disk (with homogeneous mass distribution)



$$I = \sum_i m_i r_i^2 = \underbrace{\dots}_{\text{some nontrivial calculation}} = \frac{1}{2} M_{tot} R^2$$

General result for homogeneous mass distributions

$$I = \gamma M_{tot} R^2$$

(see table 8.2 for more examples)

γ = geometric factor, R = characteristic length of object