Momentum

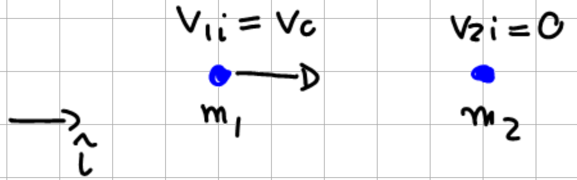
- define (linear) momentum of an object:

$$\vec{p} = m\vec{v} \qquad [p] = kg \cdot \frac{m}{s}$$

- define (linear) momentum of a system of N objects:

$$\vec{P}_{total} = \vec{p}_1 + \vec{p}_2 + \ldots + \vec{p}_N = \sum_{i=1}^{N} \vec{p}_i = \sum_i m_i \vec{v}_i$$

example ①: two billiard balls



- before collision (@ t_i):

$$\vec{p}_1 = m_1 \vec{v}_{1i} = m_1 v_0 \hat{i}, \quad \vec{p}_2 = 0$$

$$\hookrightarrow \vec{P}_{total,i} = \vec{p}_1 + \vec{p}_2 = m_1 v_0 \hat{i}$$

- after collision (@ t_f) one observes

$$\left.\vec{p}_1 = 0, \quad \vec{p}_2 = m v_0 \hat{i} \quad \text{if} \quad m_1 = m_2 = m \right\} \text{we will show this later!}$$

i.e. $\vec{P}_{total,f} = m v_0 \hat{i} = \vec{P}_{total,i}$

Individual particle momenta change, but the total momentum does not. If $m_1 \neq m_2$ one get different results for individual momenta, but the total momentum is still the same before and after the collision.

We will see later that total momentum is conserved for any closed[*] particle system. At this point, this is just meant to be taken as an indication that the concept of total momentum is a useful one (in contrast to total velocity $\sum_i \vec{v}_i$).

[*] Particle system is called closed if there are no external forces acting.

example 2 : two colliding cars (book, ex 7.2)

$m_1 = 1200\ kg$ $\quad$ $m_2 = 2000\ kg$

$v_1 = (+20\hat{i})\ m/s$ $\quad$ $v_2 = (-15\hat{i})\ m/s$

$$\vec{P}_{total,i} = m_1\vec{v}_1 + m_2\vec{v}_2 = (24000 - 30000)\hat{i}\ \frac{kg\cdot m}{s} = -6000\hat{i}\ \frac{kg\cdot m}{s} = \vec{P}_{total,f}$$

(also in this case, even though the situation is obviously more involved since damage is done to cars.)

Note that $\quad P_{total} = |\vec{P}_{total}| = 6000\ \frac{kg\cdot m}{s}$

$$\neq |\vec{p}_1| + |\vec{p}_2|$$

(i.e. signs matter in 1D!)