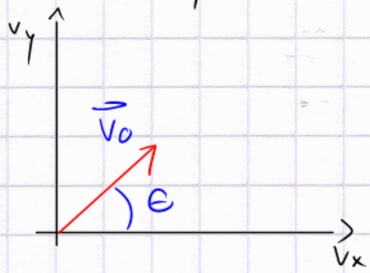
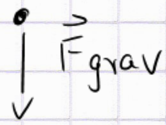
Projectile motion

characterized by:



FBD



$\hat{i}$ - motion

$F_x = 0 = m a_x$

$\Leftrightarrow \quad a_x = 0$

1st integration $\Leftrightarrow \quad v_x = v_{0,x} = \text{const}$

$\hat{j}$ - motion

$F_y = -mg = m a_y$

$\Leftrightarrow \quad a_y = -g$

$\Leftrightarrow \quad v_y(t) = v_{0,y} - gt$

express $v_{0,x}$, $v_{0,y}$ in terms of initial speed v_0 and launch angle Θ.

$v_x = v_0 \cos\Theta \qquad\qquad v_y(t) = v_0 \sin\Theta - gt$

2nd integration $\Leftrightarrow \quad x(t) = x_0 + v_0 t \cos\Theta \qquad\qquad y(t) = y_0 + v_0 t \sin\Theta - \frac{g}{2} t^2$

(general solution)

Discussion

(i) Special case #1: $\Theta = \frac{\pi}{2} \Rightarrow$ recover 1D free-fall problem

(ii) Special case #2: $\Theta = 0,\ y_0 = h \Rightarrow$ ball falling off a table
(see "Forces and Motion")

equations:

$x(t) = v_0 t \qquad\qquad y(t) = h - \frac{g}{2} t^2$

$\Leftrightarrow \quad t = \frac{x}{v_0} \qquad\qquad = h - \frac{g}{2}\left(\frac{x}{v_0}\right)^2 = y(x)$

(parabolic 'orbit')

(iii) General case (e.g. realized in motion of a baseball)

- maximum height

step (i) calculate t_{max} by

$$v_y(t_{max}) = 0 = v_0 \sin\theta - g t_{max}$$

$$\Leftrightarrow \quad t_{max} = \frac{v_0 \sin\theta}{g}$$

step (ii)

$$y_{max} = y(t_{max}) = y_0 + v_0 t_{max} - \frac{g}{2} t_{max}^2$$

$$= h + \frac{v_0^2 \sin^2\theta}{2g}$$

(compare to 1D free fall result!)

note that y_{max} is not influenced by motion in x-direction.

But: speed at y_{max}:

$$v(t_{max}) = v_{0,x} = v_0 \cos\theta$$

- Landing: characterized by

$$y(t_{gr}) = 0 = h + v_0 t_{gr} \sin\theta - \frac{g}{2} t_{gr}^2$$

$$\Leftrightarrow \quad t_{gr}^2 - \frac{2 v_0 \sin\theta}{g} t_{gr} - \frac{2h}{g} = 0$$

for special case $h = 0$ one obtains $t_{gr} = \frac{2 v_0 \sin\theta}{g} = 2 t_{max}$

final speed (just before ball hits the ground)

$$v(t_{gr}) = \sqrt{v_x^2(t_{gr}) + v_y^2(t_{gr})}$$

$$= \sqrt{v_0^2 \cos^2\theta + (v_0 \sin\theta - g t_{gr})^2}$$

$$= \sqrt{v_0^2 \cos^2\theta + v_0^2 \sin^2\theta - \frac{4 v_0^2 g \sin^2\theta}{g} + g^2 \frac{4 v_0^2 \sin^2\theta}{g^2}}$$

$$= v_0 \sqrt{\sin^2\theta + \cos^2\theta} = v_0 \quad \text{(no surprise!)}$$

- range

$$\Delta x = x(t_{gr}) - x_0 = v_0 t_{gr} \cos\theta = \frac{2v_0^2 \sin\theta\cos\theta}{g}$$

$$= \frac{v_0^2 \sin 2\theta}{g}$$

- which angle θ gives the maximum range ?

note that $\sin 2\theta \leq 1$. The maximum is reached for $2\theta = \frac{\pi}{2}$

$\Leftrightarrow$ $\theta = 45°$