Reference frames

Consider a package dropped from airplane (book, Fig 4.16)

$\hat{i}$ - motion

$v_x = v_0 = \text{const}$

$x(t) = v_0 t$

$\hat{j}$ - motion

$v_y(t) = -gt$

$y(t) = h - \frac{g}{2} t^2$

eliminate $t = \frac{x}{v_0}$ $\Rightarrow$ $y(x) = h - \frac{g}{2v_0^2} x^2$

(parabolic orbit)

For the pilot, things look different:

$\hat{i}$ - motion

$v_x = 0$

$x = 0$

$\hat{j}$ - motion

$v_y(t) = -gt$

$y(t) = -\frac{g}{2} t^2$

→ i.e. simple 1D free fall!

In both cases we have $m\vec{a} = \vec{F}$ with $\vec{F} = -mg\hat{j}$

but still the trajectory of the package is different for an observer standing on the ground and for the pilot.

physics statement: motion is viewed from different inertial reference frames

corresponding math statement: $m\vec{a} = -mg\hat{j}$ is solved for different initial conditions (positions and velocities at $t=0$)

Inertial reference frame :⟺ a reference frame that moves with constant velocity or is at rest.

In more general terms:

→ motion of an object relative to two reference frames:

velocity of object wrt ref. frame #1 : $\vec{v}_1$

" " " " ref. frame #2 : $\vec{v}_2$

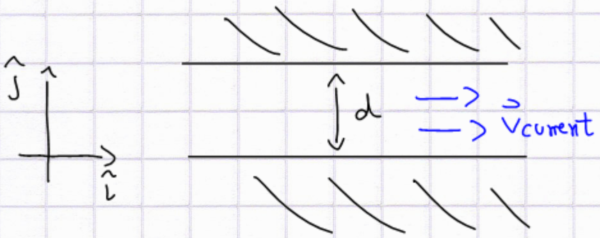
" " ref. frame #2 wrt to ref. frame #1 : $\vec{v}_{rel}$

find ⟶

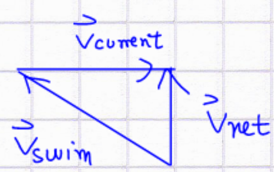
$$\boxed{\vec{v}_1 = \vec{v}_2 + \vec{v}_{rel}}$$

example: Swimming across a river

top view on river



goal: directly cross the river, i.e.



$$\vec{v}_{net} = \vec{v}_{swim} + \vec{v}_{current}$$

decompose velocity addition:

$\hat{i}$: $v_{net,x} = v_{swim,x} + v_{current,x} = v_{swim,x} + v_{current} \overset{!}{=} 0$

$\Leftrightarrow \quad v_{swim,x} = -v_{current}$

$\hat{j}$: $v_{net,y} = v_{swim,y} + \overbrace{v_{current,y}}^{=0} = v_{swim,y}$

$\Leftrightarrow \quad v_{swim,y} = v_{net}$

$\Rightarrow \quad \vec{v}_{swim} = (-v_{current}, v_{net})$

e.g. $\left.\begin{array}{l} v_{current} = 2.0 \text{ m/s} \\ v_{swim} = 3.0 \text{ m/s} \\ d = 20 \text{ m} \end{array}\right\} \Rightarrow \begin{array}{l} v_{swim,x} = -2.0 \text{ m/s} \\ v_{swim,y} = \sqrt{v_{swim}^2 - v_{swim,x}^2} \\ \quad = 2.2 \text{ m/s} = v_{net} \end{array}$

$$t = \frac{d}{v_{net}} = 8.9\,s$$

w/o current the swimmer would need $t = \frac{d}{v_{swim}} = 6.7s$.

Note that even with current he can cross the river in 6.7s, but he would end up some distance downstream!

Relate this problem to reference frames:

ref. frame #1 : fixed at shore $\vec{v}_1 = \vec{v}_{net}$

ref. frame #2 : floating canoe $\vec{v}_2 = \vec{v}_{swim}$

$\vec{v}_{rel} = \vec{v}_{current}$

i.e. $\vec{v}_{net} = \vec{v}_{swim} + \vec{v}_{current}$

$\Leftrightarrow \vec{v}_1 = \vec{v}_2 + \vec{v}_{rel}$ ✓

now differentiate this equation (back to general discussion)

$\Rightarrow \frac{d}{dt}\vec{v}_1 = \frac{d}{dt}\vec{v}_2$ (remember: $\vec{v}_{rel} = const$)

$\Leftrightarrow \vec{a}_1 = \vec{a}_2 = \frac{\vec{F}}{m}$

indeed: same dynamics (same force acting) in all inertial reference frames.

Q: What about accelerating (non-inertial) reference frames?

A: Newton-2 doesn't hold "as is"

(needs to be fixed up by "fictitious forces"; book, Chap 4.5)