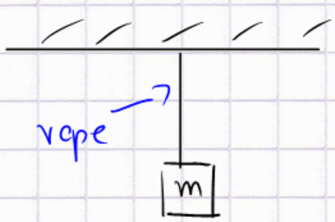
Tension forces and static equilibrium

- tension forces (due to ropes, cables etc) pull on objects <-> normal forces (due to surfaces that are in contact) push objects
- condition for translational equilibrium: $\sum \vec{F} = 0$

if $\vec{v}_0 = 0 \Rightarrow \vec{v} = 0$ for all times: static equilibrium

example 1: mass hanging on rope from ceiling



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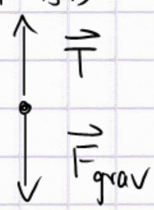


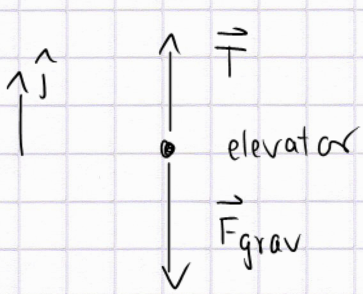
equilibrium condition:

$$\vec{T} + \vec{F}_{grav} = 0$$

$$\Leftrightarrow T = mg$$

example 2: moving elevator compartment (see Fig 3.19 in Giordano)

FBD



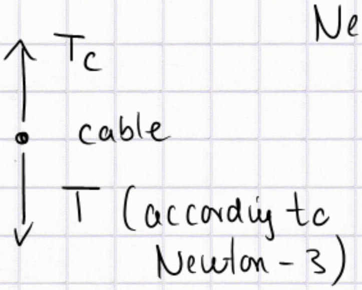
Newton-2: $ma_y = F_{net,y} = -mg + T$

$$\Leftrightarrow \quad T = m(g + a_y) \begin{cases} > mg & \text{if } a_y > 0 \\ = mg & \text{if } a_y = 0 \\ < mg & \text{if } a_y < 0 \end{cases}$$

Analyze forces on cable

case (i): assume $m_{cable} = 0$

FBD



Newton-2: $m_{cable} a_y = 0 = T_c - T$

$$\Leftrightarrow T_c = T$$

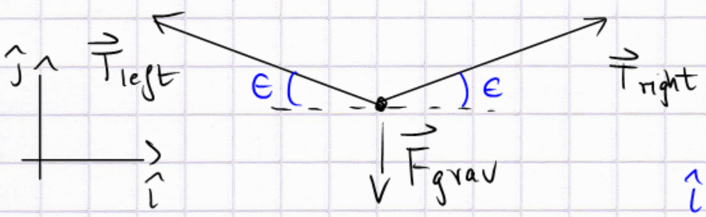
tension is the same everywhere in cable

case (ii): $m_{cable} \neq 0$

$\Rightarrow \quad m_{cable}\, a_y = T_C - T - m_{cable}\, g$

if $a_y = 0 \Rightarrow \quad T_C = T + m_{cable}\, g > T$

example 3: tightrope walker in equilibrium (at midpoint)



equilibrium condition

$$\vec{T}_{left} + \vec{T}_{right} + \vec{F}_{grav} = 0$$

$\hat{\imath}$: $T_{left,x} + T_{right,x} = 0$

$\Leftrightarrow -T_{left}\cos\theta + T_{right}\cos\theta = 0$

$\Rightarrow T_{left} = T_{right} \equiv T$

$\hat{\jmath}$: $T_{left,y} + T_{right,y} - mg = 0$

$\Leftrightarrow \quad 2T\sin\theta = mg$

$\Leftrightarrow \quad T = \dfrac{mg}{2\sin\theta}$

e.g. $m = 60\,kg,\ \theta = 22°$

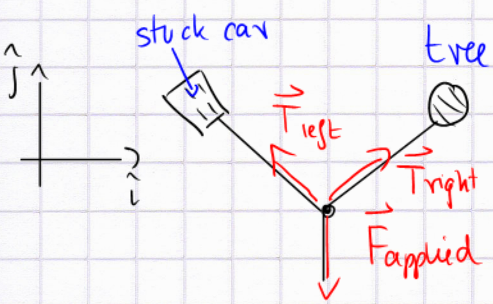
$\Rightarrow T = 800\,N$

$(T > F_{grav}\ !)$

in fact if $\theta \to 0$

$\Rightarrow T \to \infty$

Application: modify your pull (book, ex 4.2)



$\hat{\imath}$: $T_{left} = T_{right} = T$ (see above)

$\hat{\jmath}$: $2T\sin\theta = F_{applied}$

$\Leftrightarrow \quad T = \dfrac{1}{2\sin\theta}\, F_{applied}$

amplification factor for small angles

e.g. $\dfrac{1}{2\sin\theta} = 2.9$ for $\theta = 10°$