PHYS 1410 6.0: PHYSICAL SCIENCE (Fall 2011)

Class test #2 **Nov 16, 1:30 p.m. – 2:20 p.m. = 50 min**

NAME: STUDENT NR:

Formulae at the end and calculators = only aid; total = 21 points (**20 points = 100 %**)

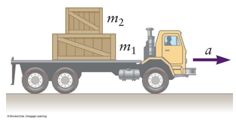
Note on units: You don't have to write out the units in intermediate steps as long as you are working consistently in the SI system. Give your final answers in SI units (where appropriate).
Note on format: to earn full marks you have to support your results by explicit calculations and/or convincing arguments, i.e, just providing final equations and plugging in numbers does not yield full marks.

1. (1 point) A boy swings on a tire swing. When is the tension in the rope the greatest?

   ○ at the highest point of the swing
   ○ midway between the highest and lowest points
   ○ at the lowest point of the swing
   ○ it is constant

   Give reasons for your answer!

2. Two crates of mass $m_1 = 32$ kg and $m_2 = 16$ kg are stacked on the back of a truck as sketched in the figure. The frictional forces are strong enough that the crates do not slide off the truck. Assume the truck is accelerating with $a = 1.5$ m/s^2.



   (a) (2 points) Provide free-body diagrams for both crates.

   (b) (5 points) Find the values of all the forces in your diagrams.

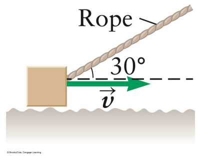
3. Consider uniform circular motion.

   (a) (3 points) Derive the formula for the centripetal acceleration $a_c = v^2/R$ from the position vector

   $$\vec{r}(t) = R(\cos\omega t\hat{\imath} + \sin\omega t\hat{\jmath}).$$

   (b) (2 points) Calculate Earth's (linear) speed in its motion around the Sun starting from the law of gravitation and Newton's second law. Assume for the distance between Earth and Sun $d_{SE} = 1.5 \times 10^{11}$ m and $M_S = 2.0 \times 10^{30}$ kg.

4. A block of mass $m = 8.5$ kg is pulled along a rough horizontal surface by a rope as sketched in the figure. The tension in the rope is $T = 35$ N, and the coefficient of kinetic friction between the block and the surface is $\mu_K = 0.43$.
   (a) (1 point) Provide a free-body diagram for the block.
   (b) (3 points) Calculate the components of the net force in the horizontal and vertical directions.
   (c) (1 point) If the block travels a distance $\Delta x = 7.2$ m along the surface, what is the work done by the rope?
   (d) (1 point) What is the work on the block done by friction?
   (e) (1 point) What is the work done by the net force?
   (f) (1 point) If the block's initial speed is $v_i = 1$ m/s, what is its kinetic energy after it has travelled the distance Δx?



FORMULAE

$$\frac{d}{dt}t^n = nt^{n-1} \; ; \; \frac{d}{dt}\sin t = \cos t \; ; \; \frac{d}{dt}\cos t = -\sin t \; ; \; \frac{d}{dt}\exp t = \exp t$$

$$\frac{d}{dt}\left(f(t)+g(t)\right) = \frac{df}{dt} + \frac{dg}{dt} \; ; \qquad \frac{d}{dt}(\alpha f(t)) = \alpha\frac{df}{dt} \quad \text{for any } \alpha \in \Re$$

$$\text{product rule}: \; \frac{d}{dt}(fg) = \frac{df}{dt}g + f\frac{dg}{dt} \; ; \qquad \text{chain rule}: \; \frac{d}{dt}[f(x(t))] = \frac{df}{dx}\frac{dx}{dt}$$

$\frac{d}{dt}\vec{r}(t) = \vec{v}(t)$; $\frac{d}{dt}\vec{v}(t) = \vec{a}(t)$; $m\vec{a} = \vec{F}_{\text{net}}$

$F_G = G\frac{m_1 m_2}{r^2}$; $g = \frac{GM_E}{R_E^2} = 9.8$ m/s^2; $R_E = 6370$ km; $G = 6.67 \times 10^{-11}\frac{\text{Nm}^2}{\text{kg}^2}$; $M_E = 5.98 \times 10^{24}$ kg

$F_k = \mu_k N$; $F_s \leq \mu_s N$; $\mu_k < \mu_s$; $F_{\text{drag}} = \frac{1}{2}\rho A v^2$

circular motion: $\vec{r}(t) = R\cos\theta(t)\hat{\imath} + R\sin\theta(t)\hat{\jmath}$; $\vec{a}(t) = \vec{a}_c(t) + \vec{a}_\perp(t)$ with $\vec{a}_c \cdot \vec{a}_\perp = 0$; $a_c = \omega^2 R$; $a_\perp = \alpha R$; $\alpha = \dot{\omega} = \ddot{\theta}$; $v = \omega R$

$KE = \frac{m}{2}v^2$, $W = \vec{F}\cdot\vec{r} = F\Delta r\cos\theta$ (for constant forces); $W = \int_{x_i}^{x_f} F(x)dx$ (for 1D problems); $W = \Delta KE$; conservative systems: $W = -\Delta PE$; in 1D: $PE \equiv V(x)$ characterized by: $F(x) = -\frac{dV}{dx}$