Tutorial Nov 13

Challenge problem # 4

For the motion of an object one finds (through observation) the position-time law

$$x(t) = A\sin(\omega t),$$

where A and ω are positive constants.

1. Show that the magnitude of the force F_x that accelerates the object is proportional to $|x|$.
2. Show that the potential energy function associated with this force has the form $V(x) = cx^2$. Is c a positive or a negative constant?
3. Calculate the work necessary to displace the object from $x_i = 0$ to $x_f = A$. Does the object speed up or slow down?

① $v(t) = \dot{x} = A\omega\cos(\omega t)$, $a(t) = \dot{v} = -A\omega^2\sin(\omega t)$

$$= -\omega^2 x(t) = \frac{F_x}{m}$$

so $F_x = -m\omega^2 x \Rightarrow |F_x| \propto |x|$ ✓

② check $F_x = -\dfrac{dV}{dx} = -\dfrac{d}{dx}\left(cx^2\right) = -2cx \overset{!}{=} -m\omega^2 x$

$$\Leftrightarrow \quad c = \frac{m\omega^2}{2} > 0$$

③ $W = PE_i - PE_f = V(x_i=0) - V(x_f=A) =$

$$= 0 - cA^2 = -\frac{m\omega^2}{2}A^2$$

negative work slows object down.

Problem 6.24

Skydiver jumps from airplane

$m = 70\ \text{kg}$

$h = 1500\ \text{m}$

$V_{term} = 8\ \text{m/s}$

(a) $W_{grav} = \vec{F}_{grav} \cdot \Delta \vec{r} = mgh$

$= 1.0 \times 10^{6}\ \text{J}$

(b) $F_{drag}^{ave} = \dfrac{W_{drag}}{\Delta y}$

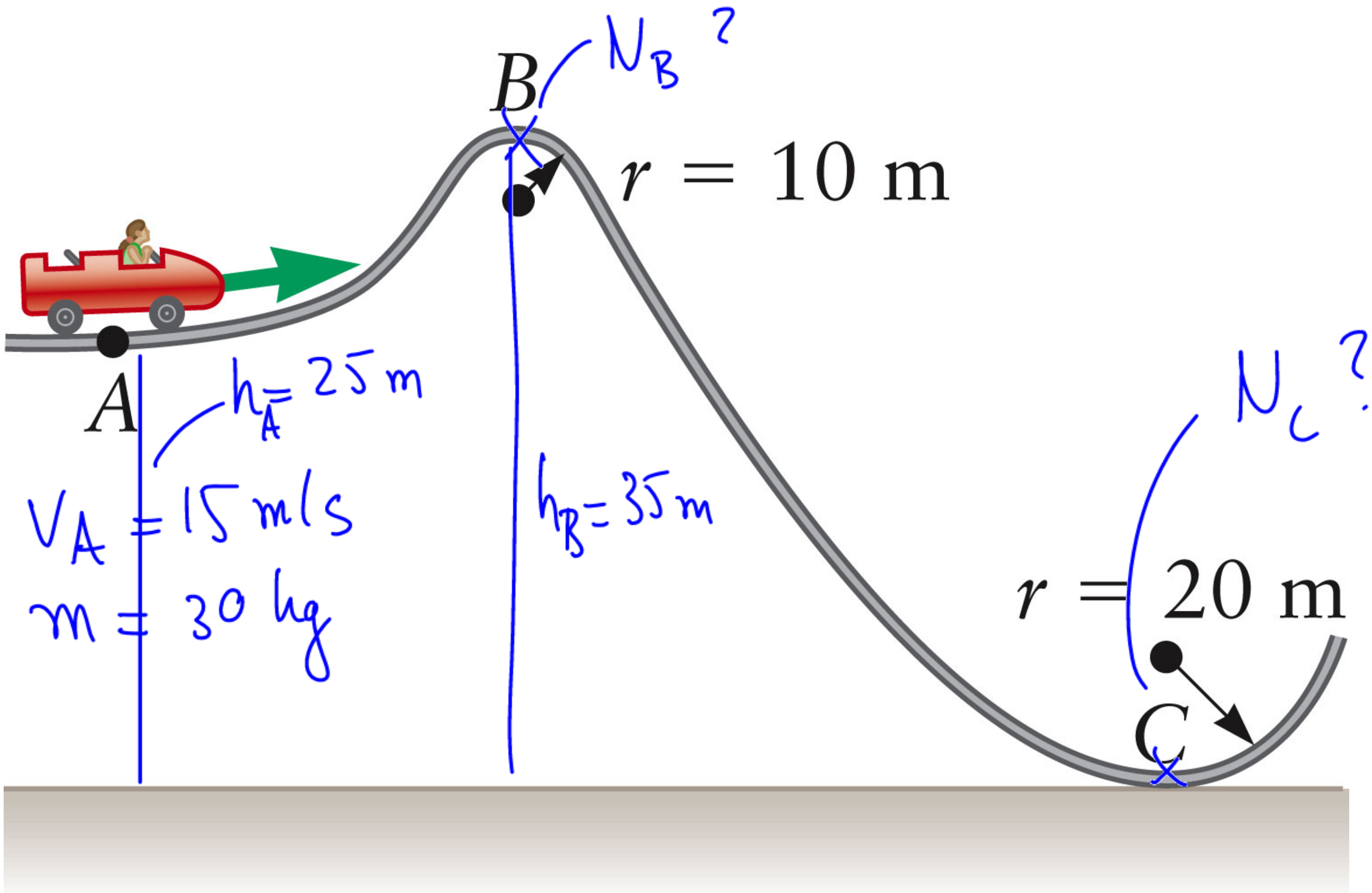
$$W_{tot} = W_{grav} + W_{drag} = \Delta KE = \frac{m}{2}\left(v_f^2 - \overbrace{v_i^2}^{=0}\right)$$

$\uparrow \hat{j}$

$$= \frac{m}{2} v_f^2 = \frac{m}{2} v_{term}^2 = 2240\ \text{J} \Rightarrow W_{drag} \overset{= \Delta KE - W_{grav}}{=} -1.0 \times 10^{6}\ \text{J}$$

$$W_{drag} = F_{drag}^{ave} \Delta y \Leftrightarrow F_{drag}^{ave} = -\frac{W_{drag}}{h} = +670\ \text{N}$$

Problem 6.36



· Derive eqs for N_B and N_C (circular motion)

@ B : $$m a_B = \frac{m v_B^2}{r_B} = F_{net} = mg - N_B$$

$$\Leftrightarrow N_B = m\left(g - \frac{v_B^2}{r_B}\right) = 210\ \text{N}$$

@ C : $$m a_C = \frac{m v_C^2}{r_C} = F_{net} = N_C - mg$$

$$\Leftrightarrow N_C = m\left(g + \frac{v_C^2}{r_C}\right) = 1400\ \text{N}$$

· Determine v_B, v_C from $\Delta TE = 0 \Leftrightarrow (TE)_A = (TE)_B = (TE)_C$

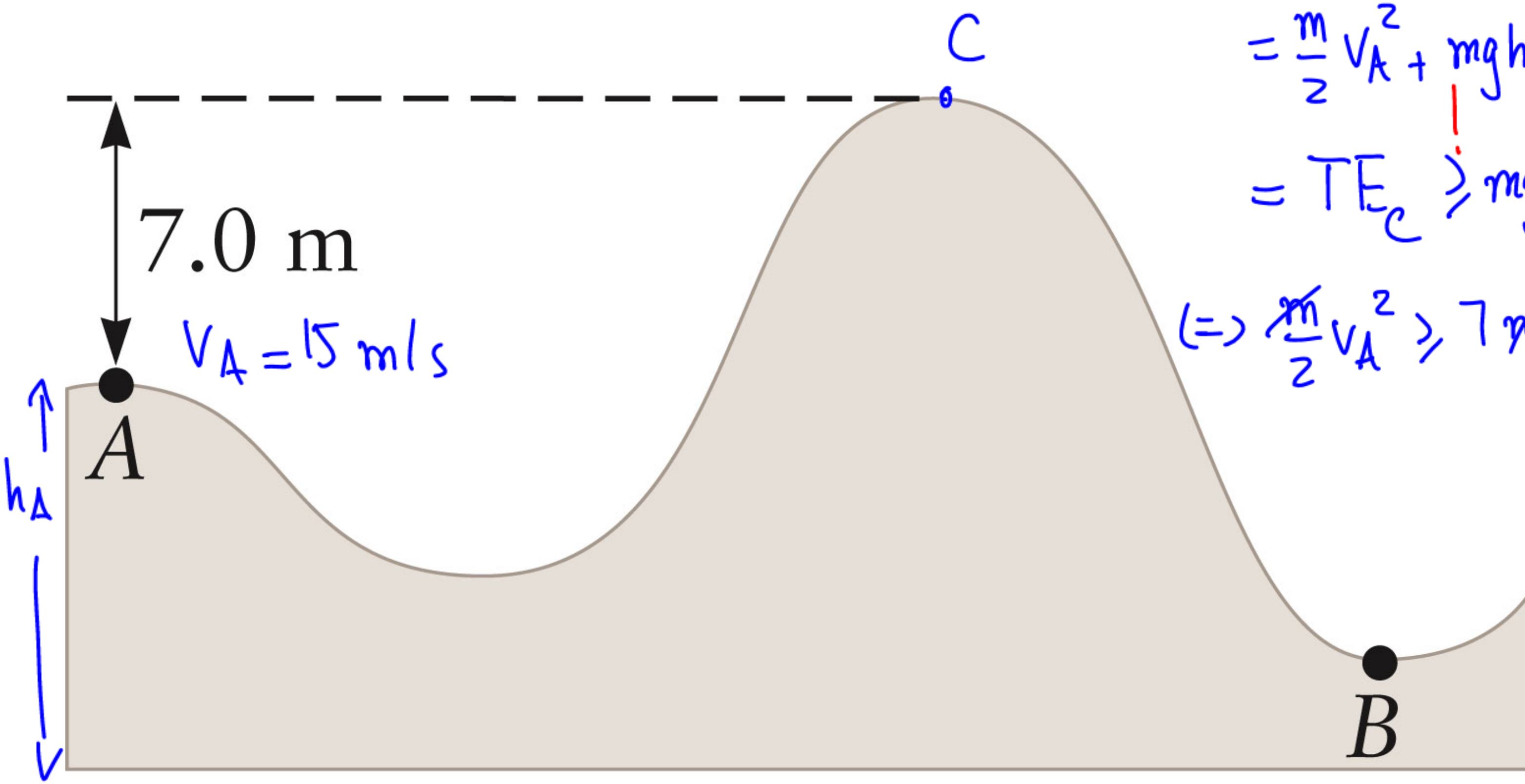
@ B $$\frac{m}{2} v_B^2 + m g h_B = \frac{m}{2} v_A^2 + m g h_A$$

$$\Leftrightarrow v_B^2 = v_A^2 + 2g(h_A - h_B) = 29\ \frac{m^2}{s^2}$$

@ C $$v_C^2 = v_A^2 + 2g h_A = 715\ \frac{m^2}{s^2}$$

Problem 6.38

Will skateboarder reach B ?



$TE_A = KE_A + PE_A$

$= \frac{m}{2} v_A^2 + mgh_A$

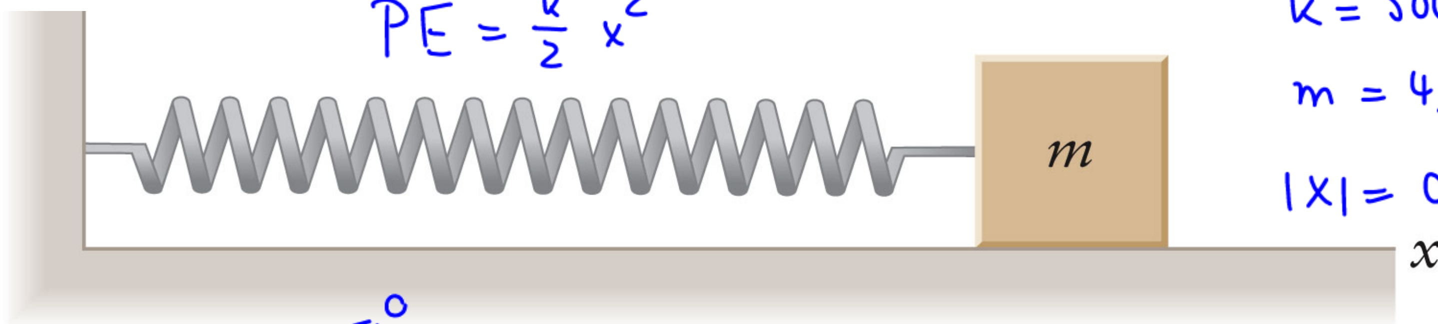
$= TE_C \overset{!}{\geq} mg(h_A + 7)$

$\Leftrightarrow \frac{\cancel{m}}{2} v_A^2 \geq 7\cancel{m}g$ ✓

Problem 6.58

Compress spring and release it

$PE = \frac{k}{2} x^2$



$k = 500 \ \mathrm{N/m}$

$m = 4.5 \ \mathrm{kg}$

$|x| = 0.35 \ \mathrm{m}$

$$(TE)_i = \frac{m}{2}\overbrace{v_i^2}^{=0} + \frac{k}{2} x_i^2 = \frac{k}{2} x^2$$

$$= (TE)_f = \frac{m}{2} v_f^2 \iff v_f = \sqrt{\frac{k}{m}}\, x = 3.7 \ \mathrm{m/s}$$