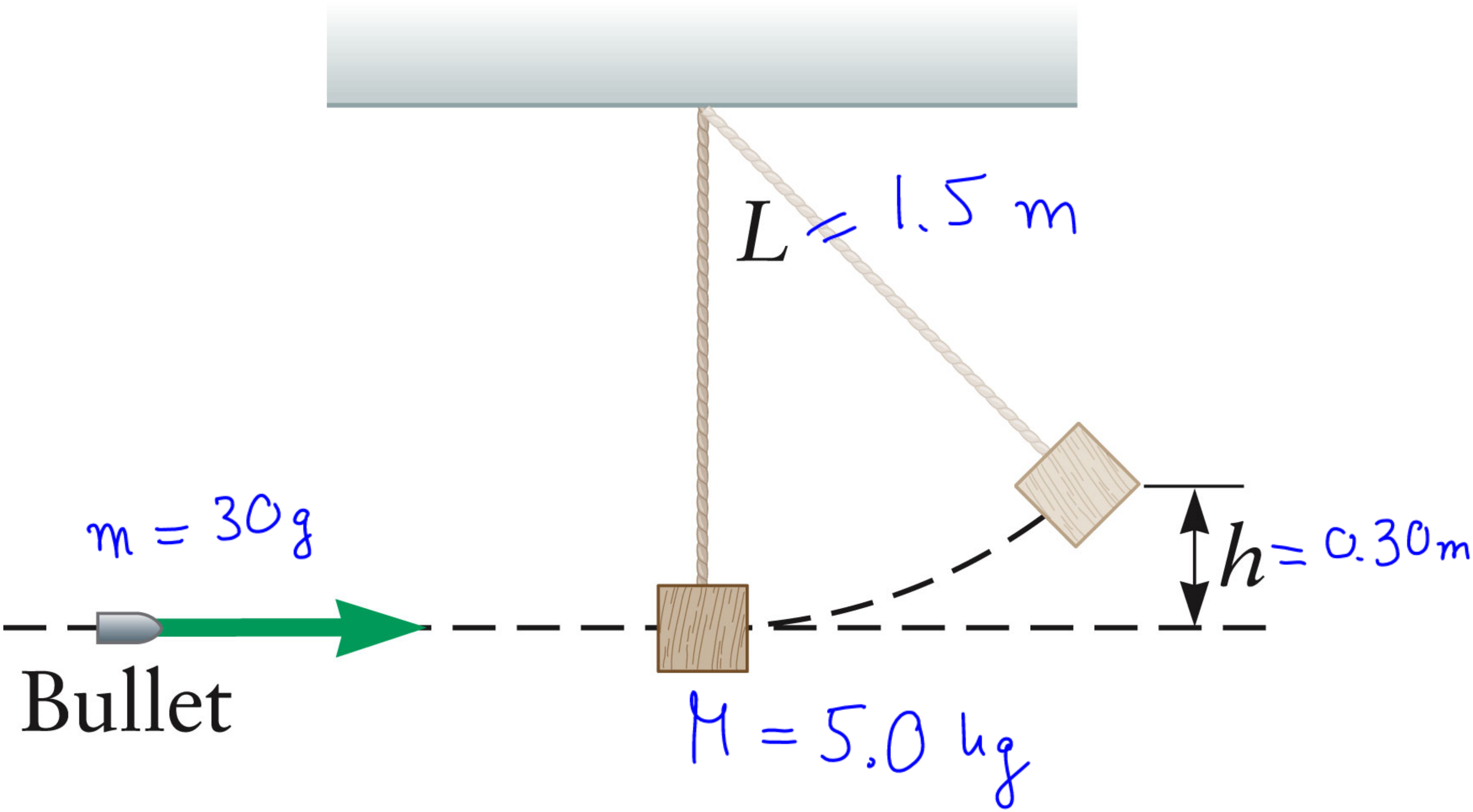
Tutorial Nov 27

Problem 7.52



inelastic (sticky) collision

right after collision

(1D) momentum conservation: $m v_{1ix} = (m+M)\, v_{fx}$ (along $\hat{i}$)

$$\Leftrightarrow \quad v_{1ix} = \frac{m+M}{m} v_{fx}$$

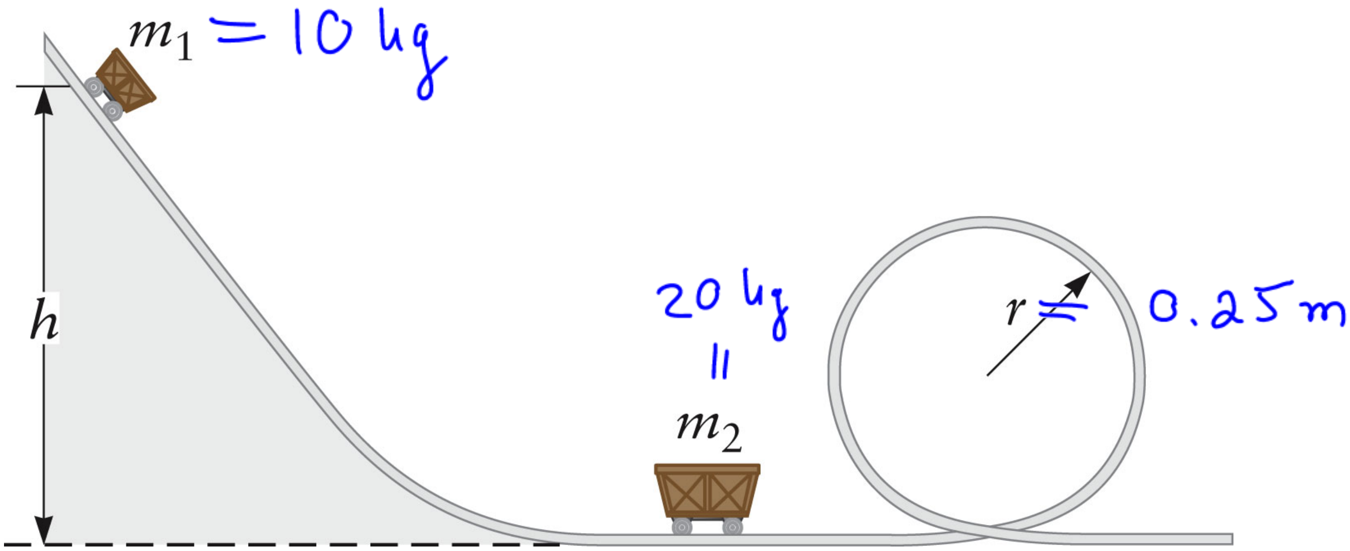
energy conservation (after sticky collision)

$$\frac{\cancel{m+M}}{2} v_{fx}^2 = (\cancel{m+M})\, g h$$

$$\Leftrightarrow \quad v_{fx} = \sqrt{2gh}$$

$$\Leftrightarrow \quad v_{1ix} = \frac{m+M}{m} \sqrt{2gh} = 410 \text{ m/s}$$

Problem 7.66



- energy conservation

this is what we have to find!

Cart #1 before coll.

$$m_1 g h = \frac{m_1}{2} V_{1i}^2 \qquad (1)$$

elastic collision

$$\frac{m_1}{2} V_{1i}^2 = \frac{m_1}{2} V_{1f}^2 + \frac{m_2}{2} V_{2f}^2 \qquad (2)$$

cart #2 after coll.

$$\frac{m_2}{2} V_{2f}^2 = \frac{m_2}{2} V_{loop}^2 + m_2 g (2r) \qquad (3)$$

- momentum conservation (1D)

$$P_{tot,i} = m_1 V_{1i} = m_1 V_{1f} + m_2 V_{2f} = P_{tot,f} \qquad (4)$$

- circular motion of cart #2

$$m_2 a_c = m_2 \frac{V_{loop}^2}{r} = m_2 g + \overset{=0}{N} \qquad (5)$$

0 for minimum velocity to complete loop

unknowns: $h, V_{1i}, V_{1f}, V_{2f}, V_{loop}$

(5) $V_{loop}^2 = gr$ (5')

so there are 4 eqs. left for 4 unknowns — that works!

(2) and (4)

see challenge problem #4

$$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i} \quad (6)$$

$$V_{2f} = \frac{2m_1}{m_1 + m_2} V_{1i} \quad (7)$$

(7) $\Leftrightarrow$ $V_{1i} = \frac{m_1 + m_2}{2m_1} V_{2f}$

use in (1)

C D

$$m_1 g h = \frac{m_1}{2}\left(\frac{m_1 + m_2}{2m_1}\right)^2 V_{2f}^2$$

$$\overset{(3)}{=} \frac{m_1}{m_2}\left(\frac{m_1+m_2}{2m_1}\right)^2\left(\frac{m_2}{2}V_{1ocp}^2 + 2m_2 g r\right)$$

$$\overset{(5)}{=} \frac{(m_1+m_2)^2}{4m_1m_2}\left(\frac{m_2}{2}gr + 2m_2 g r\right)$$

$$= \frac{5}{8}\frac{(m_1+m_2)^2}{m_1\cancel{m_2}}\cancel{m_2} g r$$

$$\Leftrightarrow m_1 g h = \frac{5}{8}\frac{(m_1+m_2)^2}{m_1} g r$$

$$\Leftrightarrow \boxed{h = \frac{5}{8}\left(\frac{m_1+m_2}{m_1}\right)^2 r}$$

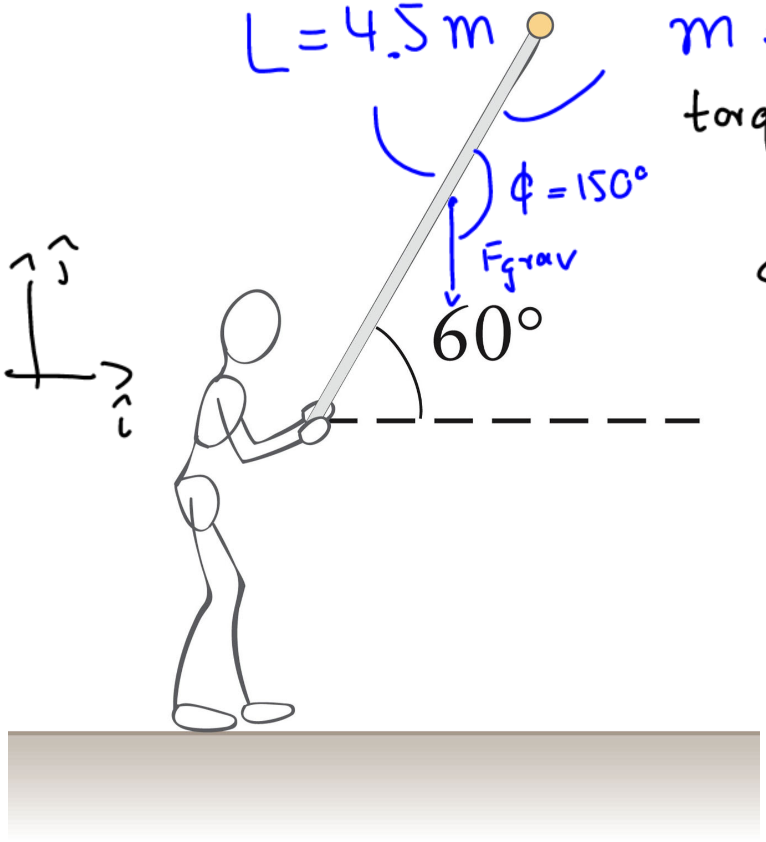
(a) $h = 1.4\ m$

$(m_1 = 10\ kg,\ m_2 = 20\ kg)$

(b) $h = 0.35\ m$

$(m_1 = 20\ kg,\ m_2 = 10\ kg)$

Problem 8.20



$m = 12$ kg

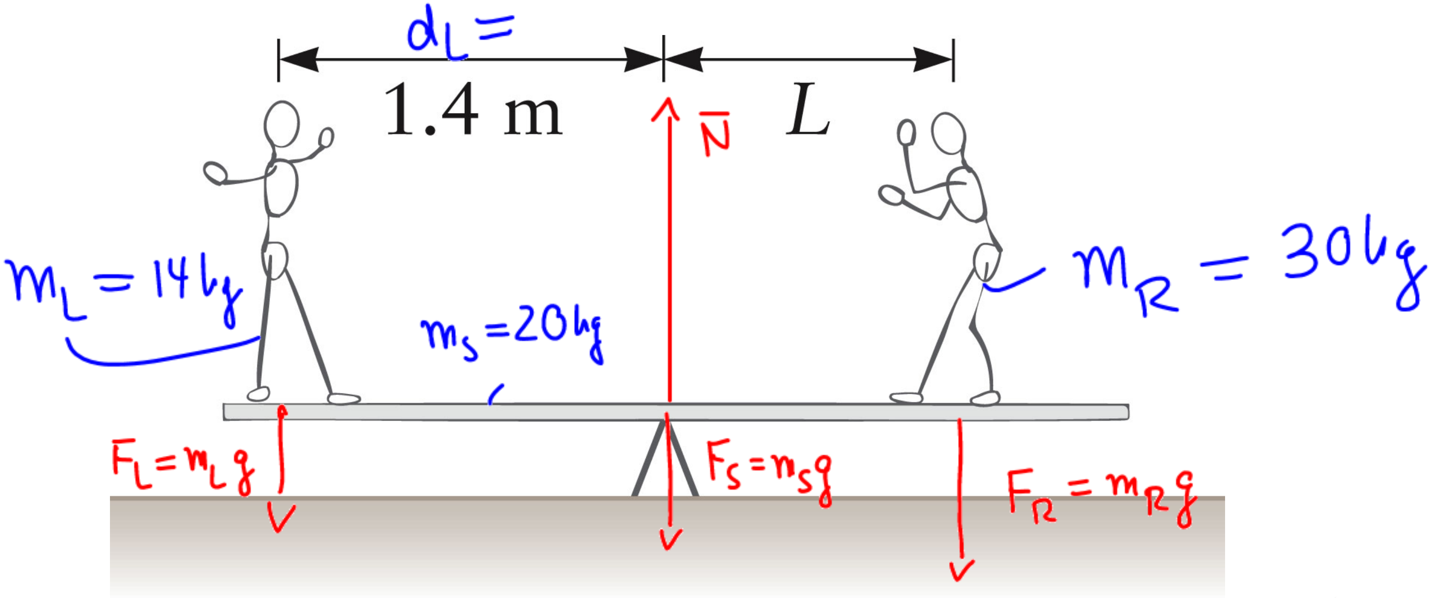
torque on pole : $\vec{\tau} = \vec{r} \times \vec{F}_{grav}$

lever arm : centre of mass (gravity) @ $\frac{L}{2}$

so $\vec{\tau} = (-\frac{L}{2} mg \sin 150)\hat{k}$

$= (-130\ \text{Nm})\hat{k}$

Problem 8.28



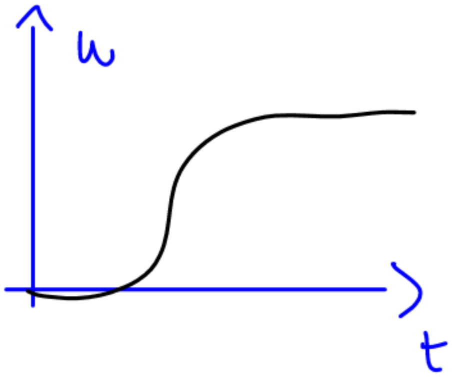
- rotational equilibrium: $\sum \vec{\tau} = 0$; translational eq.: $\sum \vec{F} = 0$

$$\sum \vec{\tau} = \vec{\tau}_L + \vec{\tau}_R = 0$$

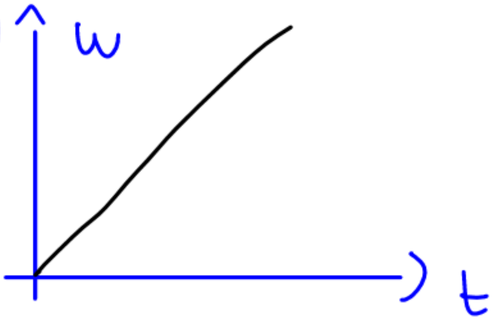
$$\Leftrightarrow \quad m_L g d_L - m_R g L = 0 \quad \Leftrightarrow \quad L = \frac{m_L}{m_R} d_L = 0.65 m$$

Problem 8.56: ω-t graphs

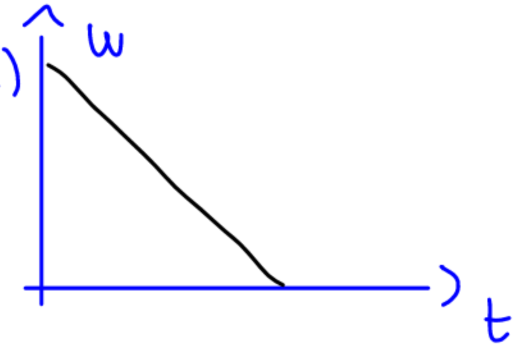
(a)



(b)



(c)



car wheel

use $\omega = \frac{V_{CM}}{r}$