PHYS 2030 (Winter 2017) - Midterm Exam

Name & Student number:

Instructions:

- Calculators and computers are not needed for this exam.
- Show all work in order to get full credit. Points can be taken off **if it is not clear** to see how you arrived at your answer (even if the final answer is correct).
- Circle your final answers (where appropriate).
- Please keep your written answers brief; be clear and to the point.
- This test has 6 problems (plus one extra credit problem) for a total of 80 points (with extra credit points possible). It is your responsibility to make sure that you have done all the problems!
- You are allowed a formula sheet to bring with you. It must be a single hard copy sheet of paper (though you can write on front and back). You **must turn such in with your exam**.

1. (*12 points*)
Consider the matrix `A= [1 2;3 4]`.

a. What is `2*A+3`?

b. What is `A^2`?

c. What is `A.^2`?

2. (*18 points*)
Consider the driven van der Pol equation:

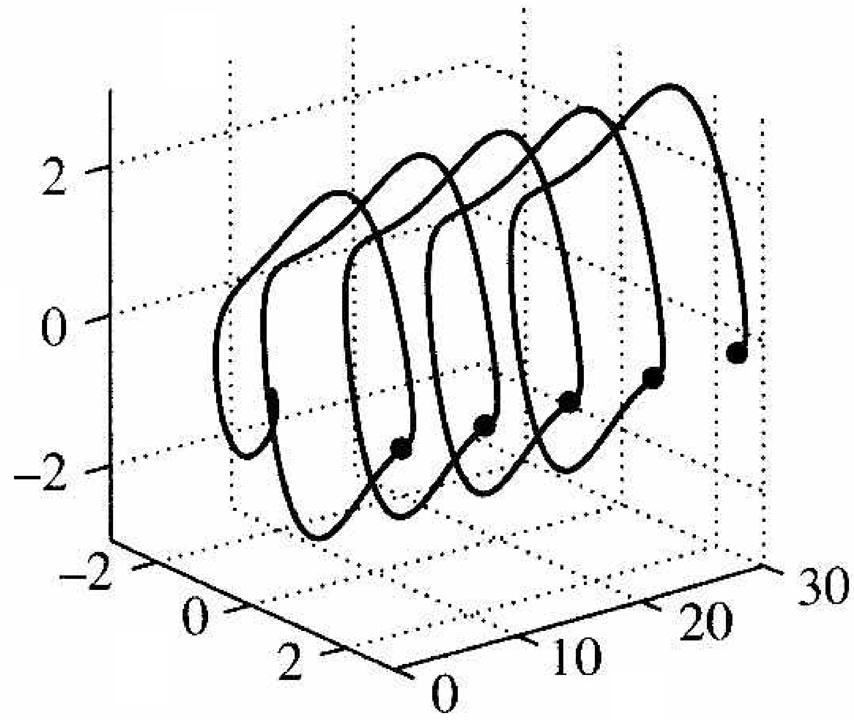
$$\ddot{x} = -x - \epsilon(x^2 - 1)\dot{x} + A\cos(\omega t)$$

a. Briefly indicate how this equation is nonlinear. Also, indicate what happens if ϵ and $A = 0$.

b. Rewrite as a system of first order equations.

c. Sketch a solution in phase space for the autonomous case.

d. Label the axes in the figure shown below.



3. (*12 points*)
Consider the integral

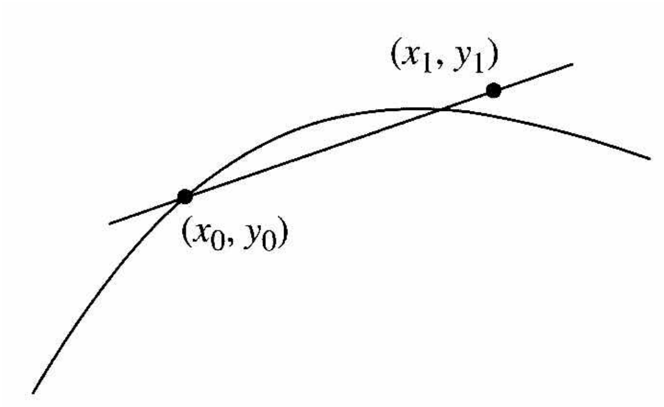
$$\int_0^{\pi} \sin(x)\; dx$$

a. What is the solution to this definite integral?

b. Write a single line of Matlab code that would solve this.

c. Identify the key concept underlying your answer to the last part (i.e., what is the basis for the numerical method employed?).

4. (*12 points*)
Consider the sketch below indicating the *improved Euler method*.



a. What is this methodology generally employed to do?

b. Clearly sketch the basis for how this computation is done (i.e., your answer here is to be *visual*).

5. (*12 points*)
Consider the following equations:

$$
\begin{aligned}
f_0 &= f(x_0, y_0), \\
f_1 &= f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f_0\right) \\
f_2 &= f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f_1\right) \\
f_3 &= f(x_0 + h, y_0 + h f_2).
\end{aligned}
$$

a. What method are these equations associated with?

b. Write down the next equation that would logically follow here.

6. (*14 points*)
Consider the following equations:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots a_{1N}x_N &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots a_{2N}x_N &= b_2 \\ &\vdots \\ a_{N1}x_1 + a_{N2}x_2 + \cdots a_{NN}x_N &= b_N. \end{aligned}$$

a. Are these linear or nonlinear?

b. Briefly indicate two methods by which these equations can be "solved". Also indicate what precisely one is solving for (e.g., are you solving for a_{22}?).

c. Indicate two ways these types of equations arise in the context of PHYS 2030 topics thus far covered.

d. Comment briefly on how these equations are related to the "*Do you speak Matlab?*" tagline (i.e., `x= A\b`).

Extra Credit (4 points)

Briefly explain why or why not Matlab's `polyfit.m` would use `fminsearch.m`.