PHYS 2030 (Winter 2018) - HW 2

Due Date: Jan. 19, 2018 11:30 AM

Questions

1. As motivated in the Jan.10, 2018 lecture, determine the Q_{ERB} value for the provided auditory nerve fiber tuning curve. Make sure to include your code, a brief explanation of your approach, and the determined value. [Hint: You should find something on the order of 10^1]

2. Consider the integral

$$\int_{-\pi}^{2} e^{-cx}\,dx$$

where c is a constant greater than zero.

a. Make a sketch of the integrand and indicate the integral as an *area* to compute. Make sure to clearly label things!

b. Can you find an explicit solution for this integral? If so, give it. If not, explain.

c. Assume $c = 2$. Solve the definite integral numerically (by hand) using the left-hand rule (LEFT), right-hand rule (RIGHT), the midpoint rule (MID), the trapezoid rule (TRAP) and Simpson's rule (SIMP). Do this for both $N = 2$ and $N = 3$ (where N is the number of divisions over the interval of integration). How does your estimate compare to the exact answer? How does it change with N?

d. Which of the values you computed are underestimates? Overestimates? Explain briefly.

e. Write a Matlab script (i.e., a .m file) that does the computations for you. In your code, you should be able to readily change N and c (as well as your end points).

f. Show that your code returns the same estimates as computed by hand for $N = 2$ and $N = 3$. How does the estimate change for different values of N (e.g. 5,10, 100,10000)? As N increases, how does it compare to your exact answer?

g. What is the effect of varying c? Explain in the contexts of both your analytical answer and numerical simulations. Do both agree?

h. What happens when you change the end points (e.g., $\int_{-\pi}^{2} \rightarrow \int_{0}^{\pi/2}$)? Does this make sense? Explain.

3. Consider the integral

$$\int_{-\pi}^{2} e^{-cx^2}\,dx$$

where c is a constant greater than zero.

a. Make a sketch of the integrand and indicate the integral as an *area* to compute. How does this differ from that in the first question?

b. Can you find an explicit solution for this integral? If so, give it. If not, explain.

c. Modify your code from the first question to numerically estimate this integral (again assuming $c = 2$). How does your estimate vary with N?

d. What is the effect of varying c? Explain in the contexts of both your analytical answer and numerical simulations. Do both agree?

e. Let $c = 1$. Suppose you wanted to know $\int_{-\infty}^{\infty} e^{-x^2}\,dx$. How might you go about computing this? Given sufficiently large N, what value do you find? Using google or wikipedia (or any other source of information at your disposal), search *normal distribution* as well as *error function*. Based upon those resources, can you determine explicitly what the value of estimate approaches? (Hint: it has π in it)