PHYS 2030 (Winter 2018) - HW 3

Due Date: Jan. 31, 2018   11:30 AM

Questions

1. Consider the differential equation

$$\frac{dy}{dx} = y^2 - c$$

where c is a constant greater than zero.

a. Determine all equilibrium solutions and their stability.

b. Solve this equation analytically to obtain an expression for $y(x)$. Your answer should depend upon c and contain an arbitrary constant.

c. Write a code to solve the equation numerically using Euler's method on the interval $x \in [0, 5]$ for the initial condition $y(0) = 0$ and with $c = 4$. On a single figure, plot your estimated solution curve using the following step sizes for Δx: 0.5, 0.2, 0.1, 0.05, and 0.01. Make clear which curve corresponds to each step-size. How does the solution depend upon Δx?

d. Using $\Delta x = 0.01$, find solution curves for different initial conditions $y(0) = y_o$. How do the solutions depend upon y_o?

e. Explain your answer to the last part in terms of your analytic solution. Are the two results consistent?

f. What is the effect of varying c? Explain in the contexts of both your analytical answer and numerical simulations. Do both agree?

2. Consider the following problem dealing with the "Gompertz equation", which is a simple model for tumor growth:

a. Write a code to solve the system of ODEs listed in the top line (i.e., solve for N and γ). Make clear what approach you took to solve the ODEs (e.g., RK4, `ode45`). Along with your code, include several plots to indicate the general dynamics of the tumor growth.

b. Carry through the calculation as described in the "figure". That is, clear show how the system can be reduced to two different forms of a single 1st order ODE.

c. Modify your code from the first part to also independently solve these two other versions of the equations. Are they all equivalent? Show at least one plot comparing all three to argue your case, indicating/explaining any differences.

A solid tumor usually grows at a declining rate because its interior has no access to oxygen and other necessary substances that the circulation supplies. This has been modeled empirically by the Gompertz growth law,

$$\frac{dN}{dt} = \gamma N \qquad \text{where} \quad \frac{d\gamma}{dt} = -\alpha\gamma.$$

γ is the effective tumor growth rate, which will decrease exponentially by this assumption. Show that equivalent ways of writing this are

$$\frac{dN}{dt} = \gamma_0 e^{-\alpha t} N = (-\alpha \ln N)N.$$

Figure 1: From the book *Mathematical models in biology* by Leah Edelstein-Keshet (2005). Here, N represents tumor cell population and γ is tumor growth rate.

d. Comment briefly/concisely on how some form of drug or chemotherapy would affect certain parameters of this model.

3. Write a Matlab script[1] that does the following:

a. Records a snippet of audio that captures your voice saying your name. Make a plot of the recorded time waveform. Use a sample rate (SR) of 22050 Hz (or something close to it). Note that you will need to use some sort of microphone as an A/D converter.

b. Repeat the last item, but with a lower sample rate. Make a plot of the waveform with several different sample rates to compare, commenting on how they differ visually. Also, pick a small time interval focused at one point in time (e.g., the start of you saying your last name) and show the recorded waveforms of varying SR all superimposed atop one another.

c. Playback the recorded audio, including that for lower sample rates, and listen. Comment on how the sound differs with respect to sample rate, and how such relates back to the plot in the last part.

d. Repeat the last two parts, but instead of varying the SR, change the bit depth. How low can you go before your speech becomes unrecognizable upon playback?

[1]If Matlab does not work for you in this regard, you can also use something else like Audacity