PHYS 2030 (Winter 2018) - HW 4

Due Date: Feb. 26, 2017 11:30 AM

Questions

1. [2 points] The following image was found on a greeting card. Write a Matlab code to reproduce the figure as closely as possible. Your answer should include your code, as well as the produced figure. Your code should plot all of them in the same `Figure` window (i.e., `subplot` might be useful!). Also provide a brief explanation as to how such curves might be produced through mechanical means[1].

2. [2 points] Write a code that reproduces the figure shown below. For reference, the associated equations are given subsequently[2]. Your answer should include your code, as well as the produced figures. Your code should plot all of them in the same (like the last problem).

3. [3 points] Write a Matlab code that does the following:

   a. Generates a noisy set of linear data. Each point should also have a random uncertainty associated with it.

   b. Determine the appropriate least-squares linear fit to the data that accounts for the associated uncertainty. This should be done in two ways. First, use the exact solution as provided in the class notes. Second, perform grid-search method that minimizes χ^2 directly.

   c. Provide a brief description as to why including the uncertainty is beneficial in determining the best possible fit.

You need to write your own code, but it is okay to reference elements from the code provided in class. Your answer should include both the associated code as well as a plot showing how the various methods compare.

4. [3 points] This problem deals with Anscombe's quartet (e.g., `https://en.wikipedia.org/wiki/Anscombe's_quartet`).

   a. Find the original article from which these data derive from. To verify, state the words comprising the top line of the left column on the second page (pg.18). Briefly explain how you acquired the article.

   b. Determine the values associated with Anscombe's quartet, for which, provide a clear explanation and/or screenshot indicating your approach. Feel free to use `deplot.m` as an option.

   c. Determine the least-squares linear fit for each version of the four groups (i.e., 8 total "fits"). For each, determine the associated correlation (e.g., "*Coefficient of determination*") and show that it is (roughly) the same for all cases.

   d. Provide a brief description as to why linear fits to three (of the four) cases are somewhat inane.

[1] If you are unsure how to start, perhaps look up a bit of history on *Nathaniel Bowditch*.

[2] Note that you might need to be a bit clever w/ regard to the δ function.

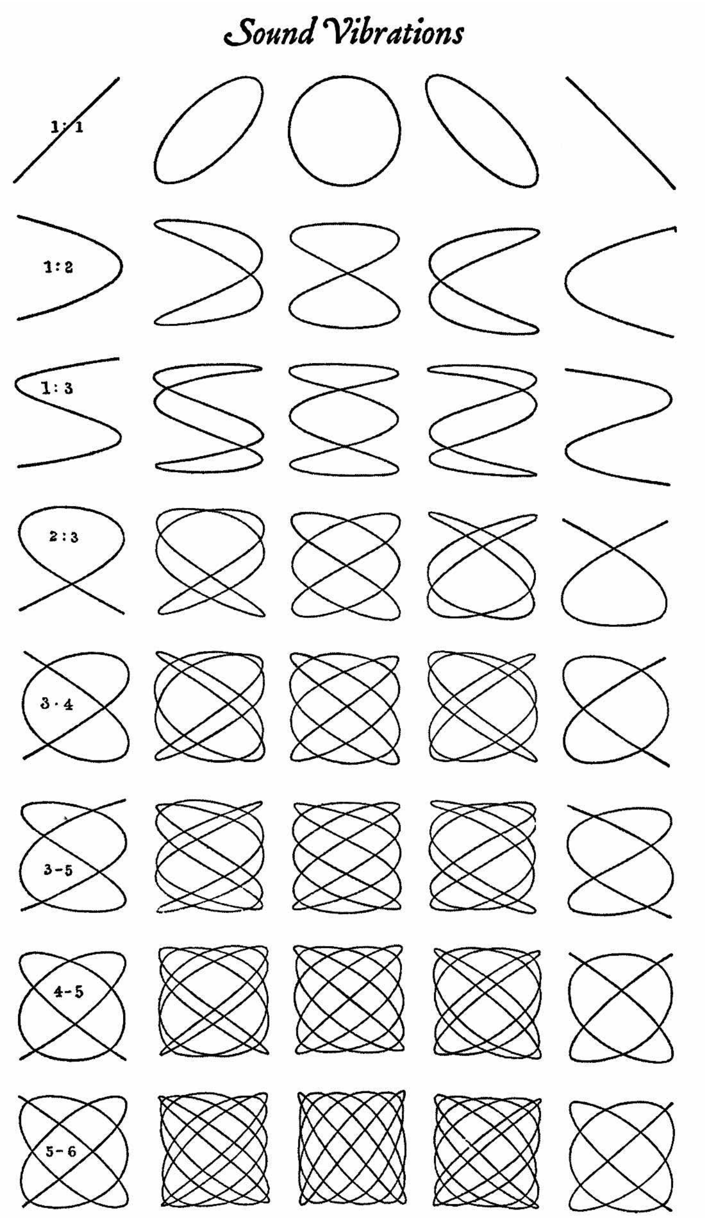

Figure 1: Problem 1

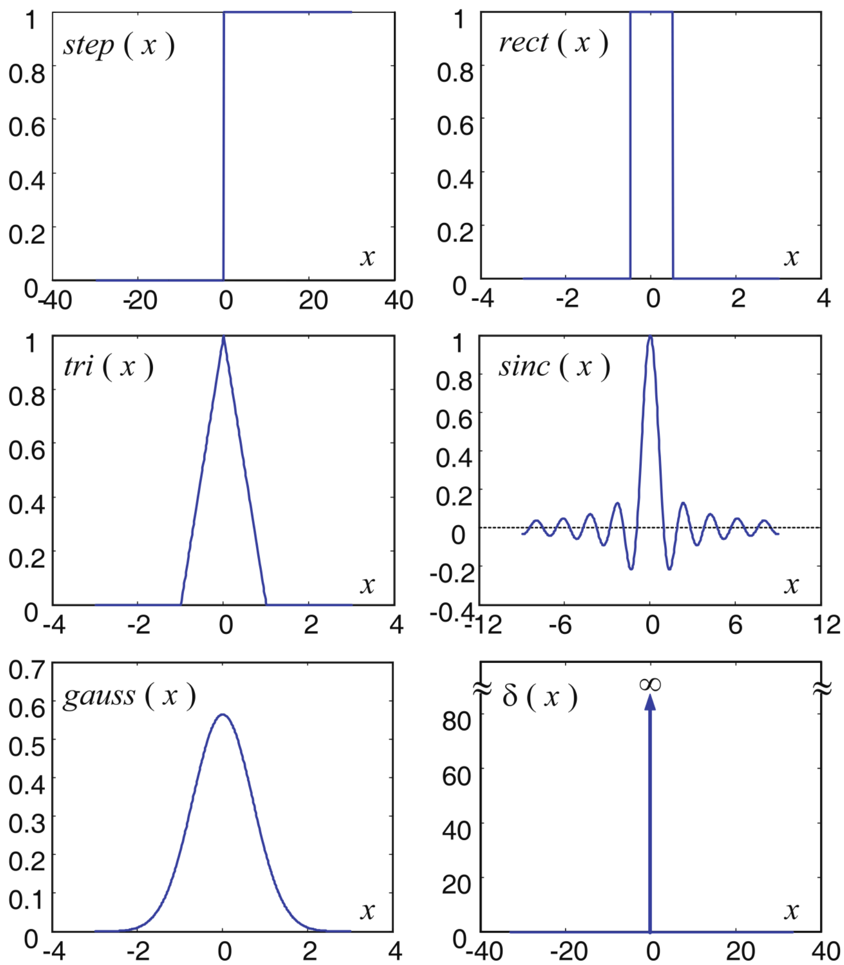

Fig. 4.1. Illustration of some of the fundamental functions of signal processing. By analyzing the changes that these signals experience within linear and complex systems, conclusions can be drawn about the systems themselves. Dirac's *delta* "function" is of particular importance. It is connected to the "impulse response" of the system, which will be used frequently in later chapters. Due to the importance of the *delta* impulse, which is actually a distribution, this "function" is described in detail in ▸ Sects. 4.6–4.7

Figure 2: Taken from *Computed Tomography* by TM Buzug (2008; Springer).

Heaviside step function	$\mathrm{step}(x) = \begin{cases} 1 & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$
Rectangular function	$\mathrm{rect}(x) = \begin{cases} 1 & \lvert x \rvert \leq 1/2 \\ 0 & \text{otherwise} \end{cases}$
Triangle function	$\mathrm{tri}(x) = \begin{cases} 1 - \lvert x \rvert & \lvert x \rvert \leq 1 \\ 0 & \text{otherwise} \end{cases}$
Sinc function	$\mathrm{si}(x) \equiv \mathrm{sinc}(x) = \dfrac{\sin(\pi x)}{\pi x}$
Gaussian function	$\mathrm{gauss}(x) = \dfrac{1}{\sqrt{\pi}} \mathrm{e}^{-x^2}$

Taken from *Computed Tomography* by TM Buzug (2008; Springer).

Delta distribution

$$\int_{-\infty}^{+\infty} f(x)\delta(x - x_0)\,\mathrm{d}x = f(x_0)$$

or formally,

$$\delta(x - x_0) = \begin{cases} 0 & x \neq x_0 \\ \infty & x = x_0 \end{cases} \quad \text{and} \quad \int_{-\infty}^{+\infty} \delta(x - x_0)\,\mathrm{d}x = 1$$