PHYS 2030 (Winter 2018) - HW 6

Due Date: Mar.19, 2018 11:30 AM

Questions

1. [2 points] Write a Matlab code that produces an binary (i.e., two-tone) image of a circle. Your code should allow for the user to specify:

- Width of the image (in # of "pixels")
- The radius of the circle
- Whether the circle is filled in or not (i.e., it could be an annulus)
- If an annulus, how "thick" it is
- Option to shift the circle around within the image
- Invert the image (i.e., black becomes white and vice versa)

You should submit your code, as well as an example image created.

2. [3 points] The "diffusion equation" is a partial differential equation used in a wide variety of context and has deep historical connection points throughout physics, mathematics, engineering, biology, etc....

a. Write down the 1-D diffusion equation and explain what the various terms represent.

b. Consider a cylindrical metal bar of cross-sectional area A and length L. The bar is initially at uniform temperature θ_o. The bar is then heated instantaneously along its length by the addition of an amount of energy H (i.e., all energy is delivered in a single instant at $t = 0$ to a single cross-sectional slice along the length of the bar at $x = L/2$ where x is position along the bar and $x \in [0\ L]$). If we neglect heat loss to the environment, the subsequent temperature of the bar (relative to θ_o) as a function of t and x is given by

$$\theta(x,t) = \frac{H}{c_p A\sqrt{4\pi Dt}}\, e^{-x^2/4Dt} \tag{1}$$

where c_p and D are the metal's specific heat capacity per unit volume and thermal diffusivity, both assumed to be constant with respect to temperature. Show that this equation satisfies the diffusion equation.

c. Plot $\theta(x,t)$ as a function of x (for ± 5 cm) at three different times ($t \in [0.01, 0.1, 1]$ s) for both copper and iron[1]. Let $A = 1.0 \times 10^{-4}$ m^2 and $H = 1.0 \times 10^3$ J. Also assume $\theta_o = 300$ K. Your answer should include all six plots (created together in one figure; copper in a left column and iron in the right, each row being a different time) and your code to generate such.

[1] Hint: c_p is 3.45e7 J m^{-3} K^{-1} for Cu and 3.50e7 for Fe. D should be easy to look up...

d. Similar to the last problem, plot $\theta(x, t)$ as a function of t (for $t \in [0\, 10$ s) at three different positions ($x \in [0.005, 0.01, 0.05]$ m) for both copper and iron.

e. Lastly, make a "3-D" plot that shows $\theta(x, t)$ as a function of both x and t such that is appears as a "heat map"[2].

f. Which of the two metals is a better conductor? Explain within the context of your figure generated in the last plot.

3. [2 points] Create two different fractals based upon the Julia set. You should turn in both your code and the images, along with a brief description as to how you generated the two different patterns.

4. [2 points] Consider the quadratic function $f(x) = Ax^2 + Bx + C$ over the interval $-\pi < x < \pi$. A, B and C are constants.

a. Expand $f(x)$ as a Taylor series on this interval (show your work!).

b. Expand $f(x)$ as a Fourier series on this interval (show your work!).

5. [1 points] Using `EXspecREP3.m`[3], for one or more sinusoids, specify a frequency higher than twice the sample rate and demonstrate the variety of aliasing effects that arise. Explain briefly how/why this happens.

[2] `meshgrid.m` is likely to be useful here, as is the commands `surf.m`, `colormap jet` and `view([0 90])`

[3] Or better yet, write your own code from scratch. Though it's okay to perform the spectral analysis using either `fft.m` or `rfft.m` (i.e., no need to create your own FFT algorithm!).