

## PHYS 2030 (Winter 2018) - HW 7

Due Date: Apr.6, 2018 11:30 AM

### Questions

1. [2 points] Write a code to animate the build-up of a square wave by successively adding in higher and higher order harmonics. You should turn in your code, as well as at least three snapshots from your animation.

2. [3 points] Nonlinear systems produce “distortion”, such as a cheap speaker due to the lack of linearity in the response stemming from shoddy materials. Your job here is to demonstrate how different types of nonlinearity give rise to harmonic and intermodulation distortions. Consider an input signal ( $I$ ) such as the sum of two (“primary”) sinusoids:

$$I(t) = A_1 \cos(2\pi f_1 t + \phi_1) + A_2 \cos(2\pi f_2 t + \phi_2) \quad (1)$$

Now consider some mapping function that takes the input signal and “maps” it into a new output signal. For a linear mapping, the amplitude and phase at a given frequency might be affected, but additional frequency components are not created. For example consider

$$x(I) = I^3 \quad (2)$$

Such would have the effect of cubing two sinusoids and thereby create new frequencies, which would manifest as harmonics of the two primaries, as well as intermodulations (e.g.,  $2f_1 - f_2$ ).

- Consider  $A_2 = 0$  and  $x(I) = I^3$ . Briefly explain why new frequencies would arise (e.g.,  $3f_1$ ) would arise by virtue of the relevant trig identity.
- Write a code to demonstrate the spectral amplitude of two quantized tones ( $A_1 = A_2$  and  $f_2 = 1.2 * f_1$ ) for the cubic mapping. Your code should produce a plot showing both the input spectra and the output. Briefly describe what distortion frequencies that are created.
- Now repeat for a quartic nonlinearity:  $x(I) = I^4$ . Show the spectrum and describe differences, explaining briefly. What happens at the primary frequencies?
- Lastly consider a hyperbolic tangent:  $x(I) = \tanh(I)$ . Show the spectrum and describe differences, explaining briefly. What happens at the primary frequencies?

You should submit your code, as well as an images created.

3. [3 points] Unlike the previous problem where a simple nonlinear mapping gave rise to distortions whose properties were directly tied to the nature of the nonlinearity (e.g., a cubic created 3rd harmonic distortion only), generation of distortion products in a differential system is more nuanced. Consider the driven Duffing oscillator:

$$m\ddot{x} = -kx - \beta x^3 - b\dot{x} + A \sin(\omega t)$$

where  $\alpha$ ,  $\beta$ ,  $\delta$ , and  $\omega$  are constants.

- a. First, solve the linear case to demonstrate a lack of distortion. Solve the differential equation numerically using a fourth order Runge-Kutta (RK4) method on the interval  $x \in [0, 1000]$ . Note that you'll need to make some choice for the values of the initial conditions. For the parameters, use  $m = 1$ ,  $k = 1$ ,  $\beta = 0$ ,  $b = 0.3$ ,  $A = 1.5$ , and  $\omega = 3.0/(2\pi)$ . Make a plot of the associated time waveform (i.e.,  $x$  versus  $t$ ) and phase space (i.e.,  $\dot{x}$  versus  $x$ ). Also plot the amplitude of the spectra after the system has come to a steady-state (i.e., initial transients have settled down).
- b. Second, the nonlinear case to demonstrate even and odd distortions for a harmonic input. Solve as before, but use  $m = 1$ ,  $k = 1$ ,  $\beta = 10$ ,  $b = 0.3$ ,  $A = 1.5$ , and  $\omega = 1.5/(2\pi)$ . Similarly, produce the same three plots as before. Describe any distortions you see.
- c. Third, repeat the last but with  $b = 0.1$ . How has the spectrum changed?
- d. Lastly, show the Duffing system can behave chaotically. Use  $m = 1$ ,  $k = -1$ ,  $\beta = 1$ ,  $b = 0.3$ ,  $A = 0.5$ , and  $\omega = 1.2/(2\pi)$ . [Hint: You should get a phase plane portrait similar to that shown in Fig.1.] What does the spectrum look like now?

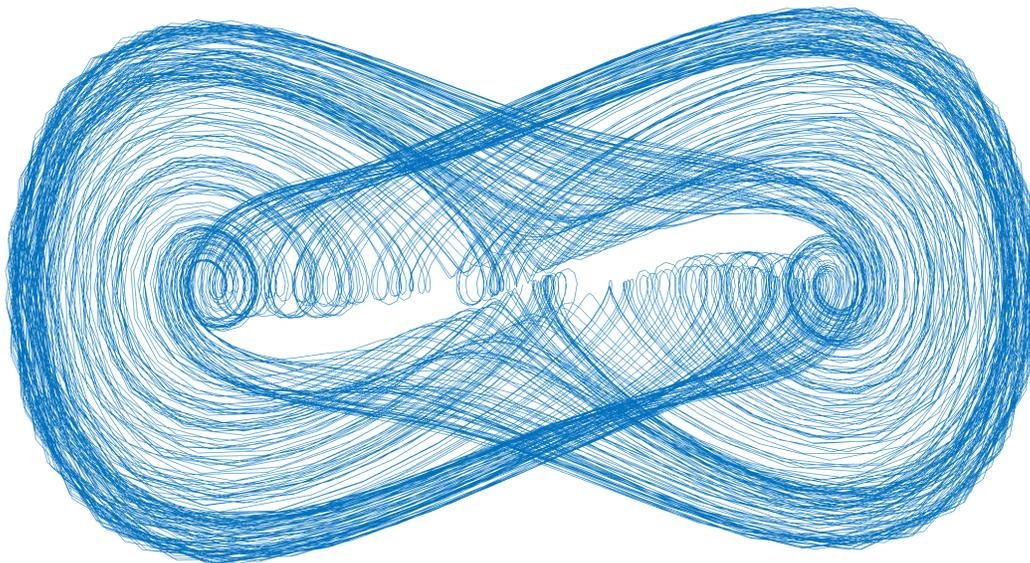


Figure 1: Chaotic phase plane portrait for the Duffing oscillator.

You should submit your code, as well as an images created.

4. [2 points] Cross-correlation is a powerful means to analyze and “process” signals. For this problem, you will create an echoed version of you saying your own name, and then subtract out that echo via cross-correlation<sup>1</sup>.

- Record a short audio segment of you saying your name.

<sup>1</sup>May be helpful to find informative/reliable resources online, as this problem has some components we have not explicitly covered in class (e.g., digital filtering). For example: <https://www.mathworks.com/help/signal/ref/xcorr.html>. It is okay to “use” other’s code to help guide you, but you must ultimately write your own code! Instrumental to this is adding in appropriate comments, where you use your own words to clearly describe the key computational steps “used” as best you can.

- Create a modified version that includes an “echo”, where the original is delayed by 400 ms and attenuated (uniformly) by 70%.
- Perform an autocorrelation on the echoed version of the signal. Plot the resulting autocorrelation. Briefly describe any salient features (e.g., is there a peak somewhere? if so, where?).
- Use a digital filter to get rid of the echo. Note that this sort of digital signal processing (DSP) is a bit beyond the scope of the course (i.e., it is okay to implement this in a bit of a “black box” fashion). Make a plot of all three versions of the time waveform, and include your code for all of these steps.
- Briefly comment on how the three different version sound upon audio playback.

You should submit your code, as well as an images created.