**York University BPHS 3090 (Winter 2015) - HW 1**

**Due Date: Jan. 16, 2015 5:30 PM**

# Questions

**1.** Fick's first law says that the flux of a solute is down its concentration gradient. Concentration gradients are well defined for continuous quantities. However, matter is not continuous. The point of this exercise is to understand whether the discrete nature of matter invalidates our formulation of Fick's first law.

a. Assume that the sodium concentration in physical saline is 150 mmol/L. Estimate the dimensions of a cube of physical saline that contains one million sodium molecules.

b. With a few well-chosen sentences, describe how the result in Part a can be used to under- stand how the discrete nature of matter limits our application of Fick's first law.

**2.** Measurements show that the diffusivity of potassium ions in aqueous solutions of KCl changes very little with the concentration of KCL. The diffusivities in 10, 100, and 1000 mmol/L solutions are $1.917 \times 10^{-5}$, $1.844 \times 10^{-5}$, and $1.892 \times 10^{-5}$ cm$^2$/s. Is this consistent or inconsistent with the Random Walk Model of diffusion. Explain with a few well-chosen sentences.

**3.** According to the random walk model, diffusion occurs because thermal agitations cause random motions of solute particles. It is therefore clear that the diffusivity of a solute should depend on temperature. However, diffusivity also depends on the solvent. Is the dependence of diffusivity on solvent consistent or inconsistent with the Random Walk Model of diffusion. Explain.

**4.** A variable $n(t)$ is described by a first-order linear differential equation with constant coefficients

$$\tau \frac{dn(t)}{dt} + n(t) = n_\infty$$

where $\tau$ and $n_\infty$ are constants. Let $n(0) = n_0$.

a. For $t \geq 0$, determine an expression for $n(t)$ in terms of $n_0$, $\tau$ and $n_\infty$.

b. Plot $n(t)$ versus $t$ for the following two cases and explain the difference between the two plots:

- $n_0 = 0$, $n_\infty = 10$, and $\tau = 1$
- $n_0 = 0$, $n_\infty = 10$, and $\tau = 10$

c. Plot $n(t)$ versus $t$ for the following two cases and explain the difference between the two plots:

- $n_0 = 10$, $n_\infty = 0$, and $\tau = 1$
- $n_0 = 10$, $n_\infty = 10$, and $\tau = 1$

d. Plot $n(t)$ versus $t$ for the following two cases and explain the difference between the two plots:

- $n_0 = 10$, $n_\infty = 0$, and $\tau = 1$
- $n_0 = -10$, $n_\infty = 10$, and $\tau = 1$

e. For the above generated graphs, describe a physical system that can be characterized by said plots. Ensure that you explain what each axis represents.

**5.** A general solution to a first-order linear differential equation with constant coefficients can be written as

$$n(t) = n_\infty + (n_0 - n_\infty)e^{-t/\tau}$$

a. Determine the slope $m_0 = dn/dt$ at $t = 0$ in terms of $n_0$, $\tau$ and $n_\infty$.

b. If this slope were extended for $t > 0$ (i.e., $n^*(t) = n_0 + m_0 t$), for what value of $t$ will $n^*(t) = n_\infty$?

c. Plot $n(t)$ and $n^*(t)$ when $n_0 = -10$, $n_\infty = 10$, and $\tau = 1$.

**6.** For the chemical reaction

$$A \underset{k_r}{\overset{k_f}{\rightleftharpoons}} B$$

$c_A$ is the concentration of the reactant $A$, $c_B$ is the concentration of the product $B$, and the $k$s are rate constants for the chemical reaction. Then

$$\frac{dc_A}{dt} = k_r c_B - k_f c_A$$

and

$$\frac{dc_B}{dt} = k_f c_A - k_r c_B$$

The total quantity of product plus reactant is conserved in this reaction so that

$$C = c_A(t) + c_B(t)$$

a. Find the differential equation satisfied by $c_A(t)$.

b. If the initial concentration of $A$ is $c_A(0) = c_{A0}$, determine $c_A$ and $c_B$ for $t \geq 0$ and sketch the results.

**7.** A 10 $m^3$ tank filled with water contains 0.05 $m^3$ of sand. A substance is added to the tank such that its concentration in the water is 10 $M$. The partition coefficient between the sand and water is $K_n = C_{sand}/C_{water} = 30$.

a. Given the partition coefficient of the system, what does this tell us about the tank? What is expected to happen to the added substance? (i.e in which compound is the added substance more soluble?)

b. Allowing the system to reach equilibrium, what will the concentration of the added substance be in the sand?

c. Sketch a graph depicting the concentration of the added substance in the sand as a function of time. Make sure to label the graph accordingly.