Biophysics I  (BPHS 3090)

Instructors: Prof. Christopher Bergevin (cberge@yorku.ca)

Website: http://www.yorku.ca/cberge/3090W2015.html
Steady-State Electrodiffusion through Membranes

Steady-state
\[
\begin{align*}
\frac{\partial c_n(x, t)}{\partial t} &= 0 \\
\frac{\partial J_n(x, t)}{\partial x} &= 0 \\
J_n &= \text{constant}
\end{align*}
\]

Nernst Equilibrium Potential
\[
V_n = \frac{RT}{z_nF} \ln \frac{c_n(d)}{c_n(0)} = \frac{RT}{z_nF} \ln \frac{c^o_n}{c^i_n}
\]
\[
G_n = -\int_0^d \frac{1}{u_n z_n^2 F^2 c_n(x)} \, dx \geq 0
\]

\[J_n = G_n (V_m - V_n)\]
Model of Steady-State Electrodiffusion through Membranes

\[ J_n = G_n (V_m - V_n) \]

if \( V_m - V_n > 0 \), then \( J_n > 0 \)
if \( V_m - V_n < 0 \), then \( J_n < 0 \)

direction of current

Nernst Equilibrium Potential
\[ V_n = \frac{RT}{z_n F} \ln \frac{c_n^0}{c_n^i} \]

Electrical Conductivity
\[ G_n = \frac{1}{\int_0^d \frac{dx}{u_n z_n^2 F^2 c_n(x)}} \geq 0 \]
Temperature-dependence can affect things by several mV (for $z_n = +1$, room temp.).

\[
V_n = \frac{RT}{z_n F \log_{10} e} \ln \left( \frac{c_n^0}{c_n^i} \right) = \frac{RT}{z_n F \log_{10} e} \log_{10} \left( \frac{c_n^0}{c_n^i} \right)
\]

\[\frac{RT}{z_n F \log_{10} e} \sim 59 \text{ mV} \quad \text{for } z_n = +1, \text{ room temp.}\]

\[\rightarrow \text{Temperature-dependence can affect things by several mV}\]

Figure 7.15 Variation with temperature of the factor $RT/(F \log_{10} e)$. 
Resting Potential: Model considering only a single permeant ion

Bernstein’s idea (1902) was that membrane was permeable to potassium only, thereby $K^+$ determined resting potential.

Inside cell: high [K+], low [Na+]
Outside cell: low [K+], high [Na+]

Figure 7.17

Figure 7.18
Resting Potential: Model considering only a single permeant ion

Model does a decent job, but deviations apparent (e.g., low $c_K$, Na$^+$ does matter)
Resting Potential: Model considering only a **multiple permeant ions**

→ What if different ions are able to diffuse?

Figure 7.24
At electrodiffusive equilibrium... (i.e., zero current densities concurrently)

$$\frac{RT}{z_1F} \ln \left( \frac{c_1^o}{c_1^i} \right) = \frac{RT}{z_2F} \ln \left( \frac{c_2^o}{c_2^i} \right) = \cdots = \frac{RT}{z_nF} \ln \left( \frac{c_n^o}{c_n^i} \right)$$

Possible concentrations are constrained!

$$\left( \frac{c_1^o}{c_1^i} \right)^{1/z_1} = \left( \frac{c_2^o}{c_2^i} \right)^{1/z_2} = \cdots = \left( \frac{c_n^o}{c_n^i} \right)^{1/z_n}$$

Nernst potentials must be equal

e.g., $c_K^o/c_K^i = c_{Cl}^i/c_{Cl}^o$ or that $c_K^o c_{Cl}^o = c_K^i c_{Cl}^i$

Donnan relation
More general case

\[ J_m = \sum_n J_n \quad \text{total membrane current is sum of all permeant charged species} \]

\[ \sum_n G_n (V_m^0 - V_n) = 0 \quad \text{‘resting state’ condition such that membrane potential is const. (i.e., no net charge entering/leaving cell)} \]

\[ V_m^0 = \sum_n \frac{G_n}{G_m} V_n \quad \text{where} \quad G_m = \sum_n G_n \quad \text{(total conductance per unit area of membrane)} \]
Model with three relevant “paths”:
1. $K^+$
2. $Na^+$
3. Other/Leakage (e.g., $Cl^-$, $Ca^{++}$)

\[
V_m^o = \sum_n \frac{G_n}{G_m} V_n
\]

<table>
<thead>
<tr>
<th>Ion</th>
<th>$G_n , (S/cm^2)$</th>
<th>$G_n/G_m$</th>
<th>$c_n^o/c_n^i$</th>
<th>$V_n , (mV)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+$</td>
<td>$3.7 \times 10^{-4}$</td>
<td>0.55</td>
<td>0.05</td>
<td>$-72$</td>
</tr>
<tr>
<td>$Na^+$</td>
<td>$1 \times 10^{-5}$</td>
<td>0.016</td>
<td>9.8</td>
<td>$+55$</td>
</tr>
<tr>
<td>leakage</td>
<td>$3.0 \times 10^{-4}$</td>
<td>0.44</td>
<td>—</td>
<td>$-49$</td>
</tr>
</tbody>
</table>

‘resting’ values for squid giant axon (determined empirically)
Multiple ion model seems to take us one step closer, but still doesn’t explain everything....
K⁺ contribution?

$$V_m^o = \frac{RT}{F} \left( \frac{G_K}{G_m} \right) \ln \left( \frac{c^o_K}{c^i_K} \right) + \sum_{n \neq K} \frac{G_n}{G_m} V_n$$

→ Chief issue is deviations at lower K⁺ concentrations (i.e., saturation in resting potential)
What if conductances were voltage-dependent?

i.e., voltage-gated ion channels

(more detail in vol.2 ch.6)

Hodgkin-Huxley model conductances

Figure 7.32

Figure 7.28
\[ \sum_{n} G_n (V_m^o - V_n) = 0 \]