Biophysics I (BPHS 4080)

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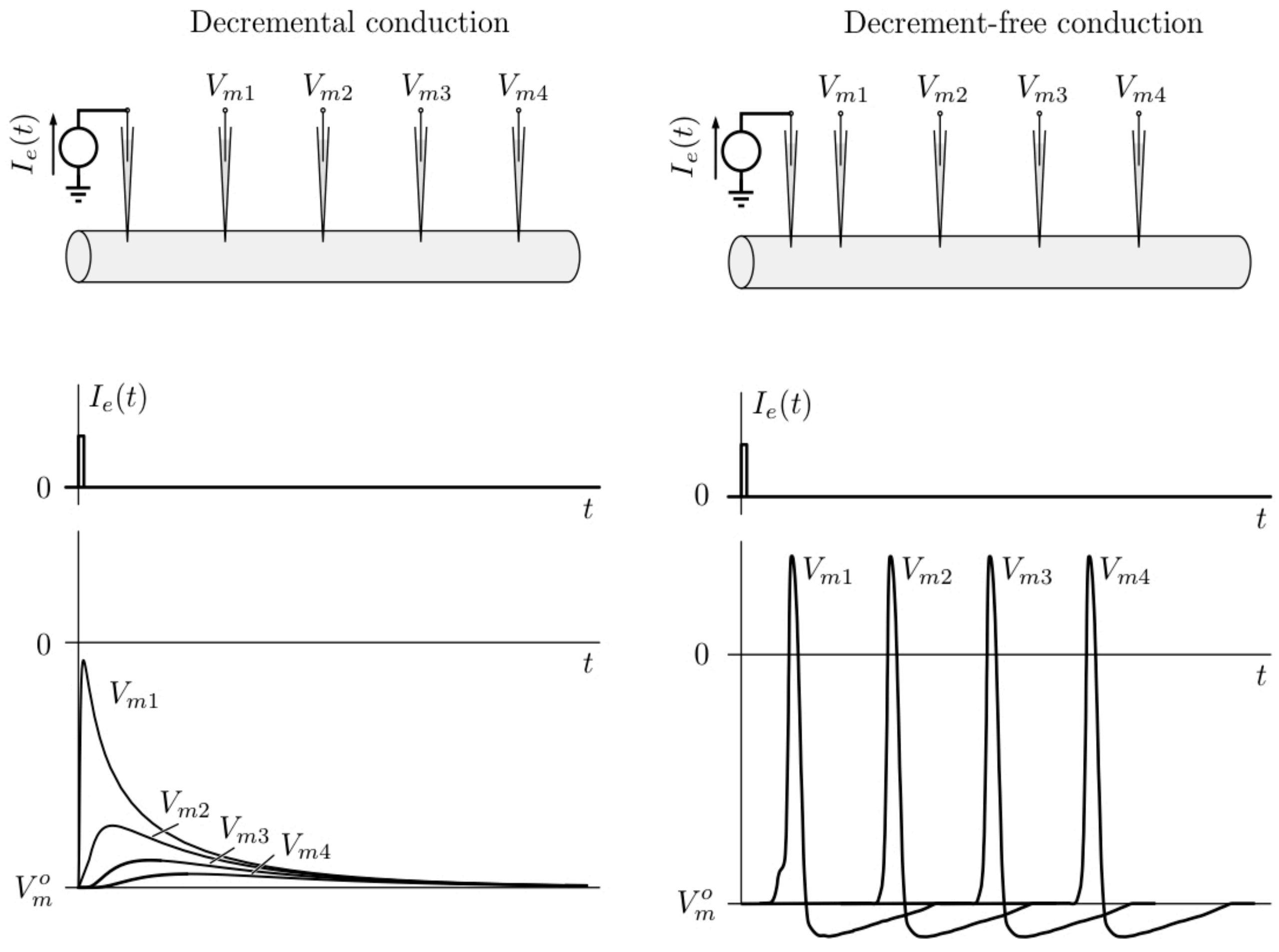
Website: http://www.yorku.ca/cberge/4080W2018.html

York University
Winter 2018
Lecture 17

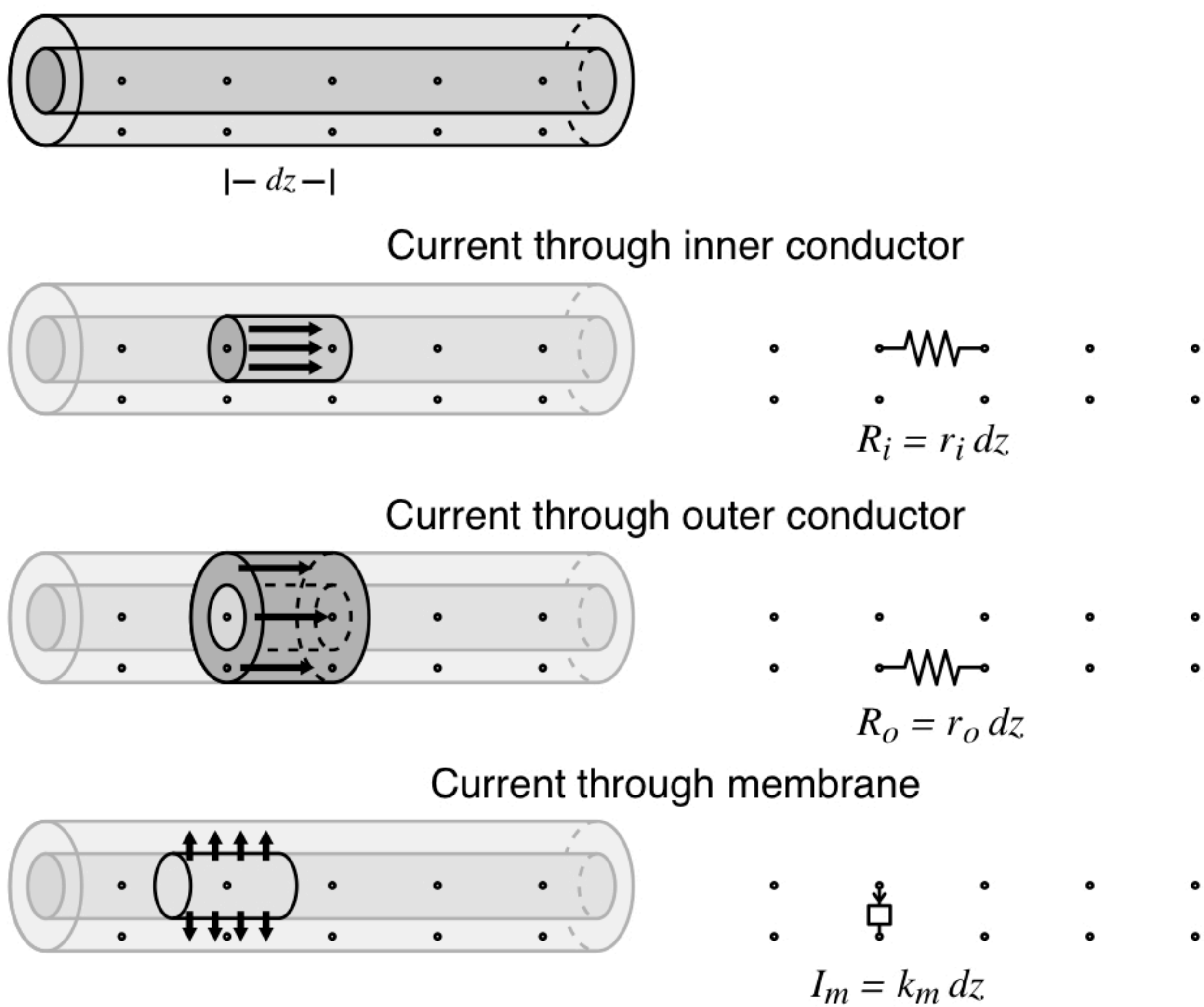
Reference/Acknowledgement:
- TF Weiss (Cellular Biophysics)
- D Freeman



Model for electrically large cells → Core-Conductor Model (starting point)

Note: Applicable regardless of whether or not cell is electrically excitable

Core Conductor Model



Core Conductor Model

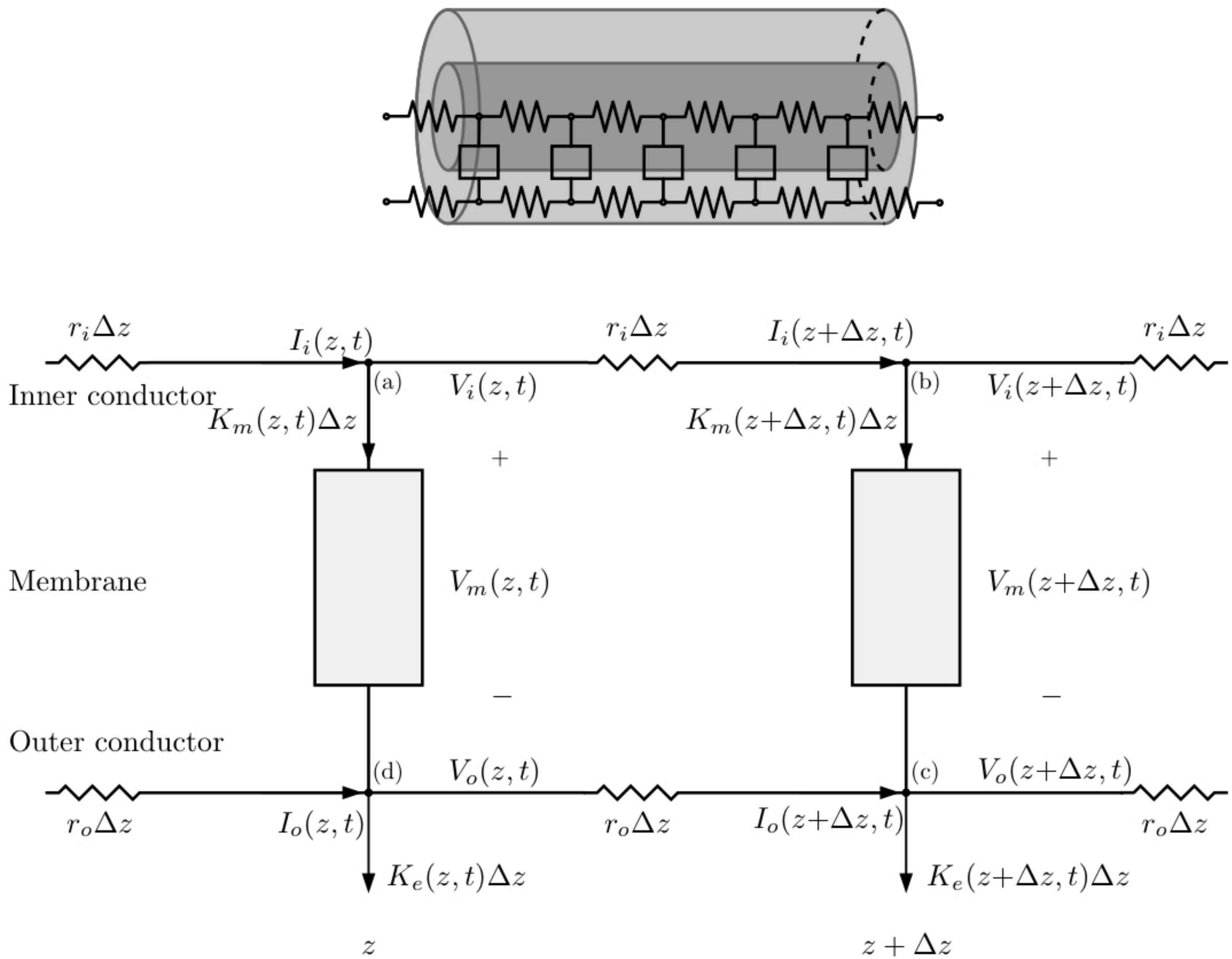


Figure 2.7

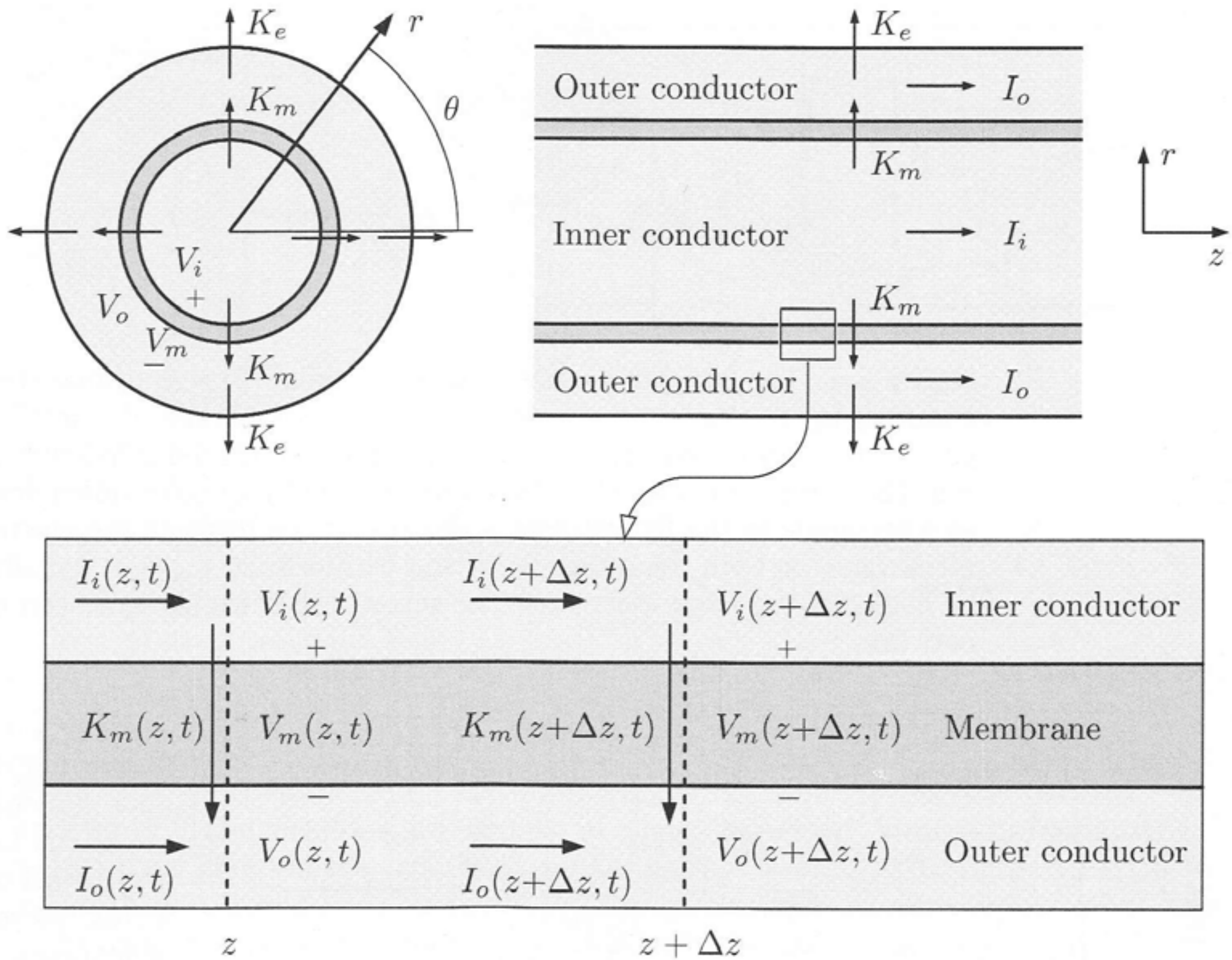


Figure 2.6 Geometry of the core conductor model of a cylindrical cell.

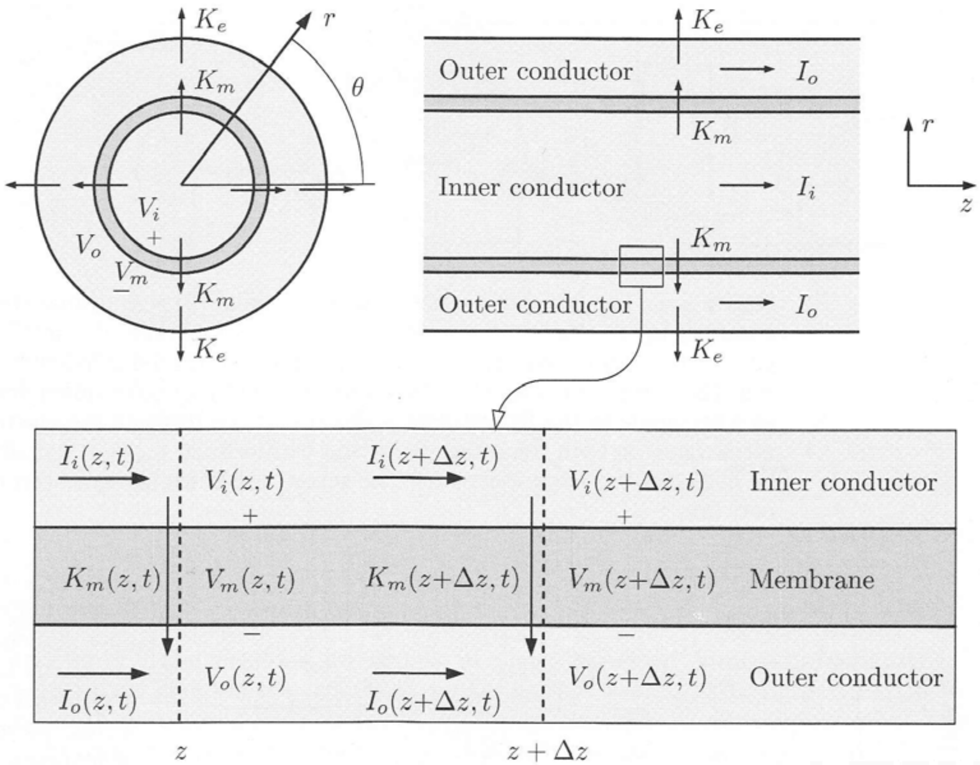


Figure 2.6 Geometry of the core conductor model of a cylindrical cell.

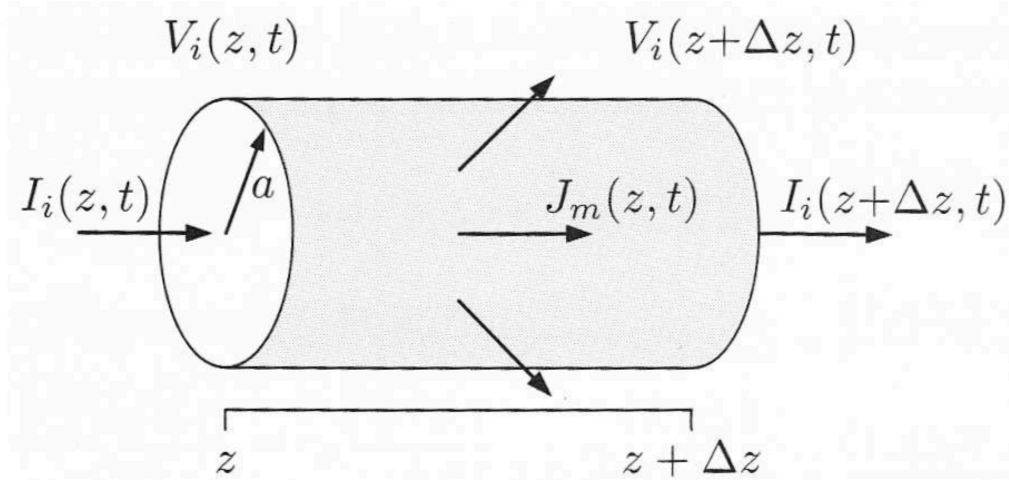


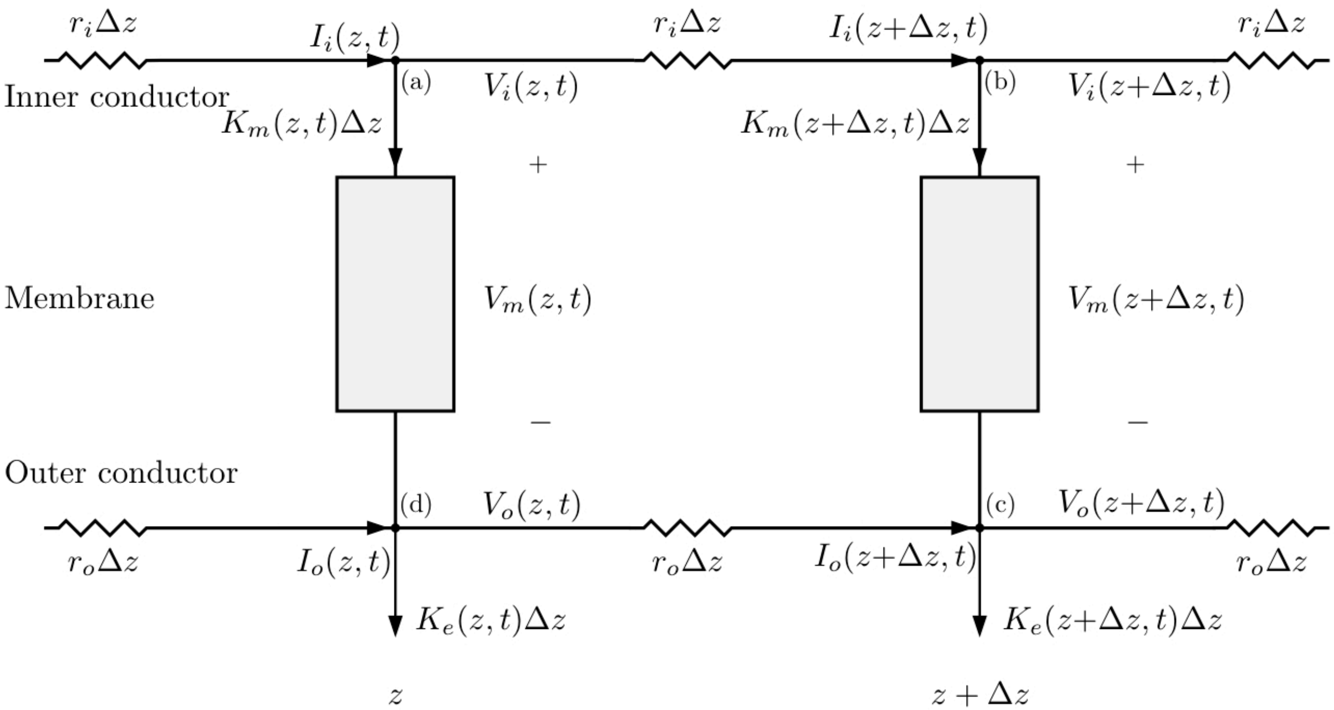
Figure 2.8

Assumption & Variables in the Core Conductor Model

- The cell membrane is a cylindrical boundary that separates two conductors of electric current, the intracellular and extracellular solutions, which are assumed to be homogeneous and isotropic and to obey Ohm's law.
- All the electrical variables have cylindrical symmetry, i.e., all the electrical variables are independent of θ.
- A circuit theory description of currents and voltages is adequate. That is, the quasi-static terms of Maxwell's equations are sufficient, and electromagnetic radiation effects are negligible.
- Currents in the inner and outer conductors flow in the longitudinal direction only. Current flows through the membrane in the radial direction only.
- At a given longitudinal position along the cell, the inner and outer conductors are equipotentials, so the only variation in potential in the radial direction, r, occurs across the membrane.

The variables used to describe the electrical properties of a cylindrical cell are defined as follows:

- $I_o(z,t)$ is the total longitudinal current flowing in the positive z-direction in the outer conductor (A).
- $I_i(z,t)$ is the total longitudinal current flowing in the positive z-direction in the inner conductor (A).
- $J_m(z,t)$ is the membrane current density flowing from the inner conductor to the outer conductor (A/m^2).
- $K_m(z,t)$ is the membrane current per unit length flowing from the inner conductor to the outer conductor (A/m).
- $K_e(z,t)$ is the current per unit length due to external sources applied in a cylindrically symmetric manner (A/m). Inclusion of this current allows us to represent the current applied through external electrodes to the cell surface. A similar term could be added to represent the current supplied by an internal electrode (see Problem 2.6).
- $V_m(z,t)$ is the membrane potential, which is a positive quantity when the inner conductor has a positive potential with respect to the outer conductor (V).
- $V_i(z,t)$ is the potential in the inner conductor (V).
- $V_o(z,t)$ is the potential in the outer conductor (V).
- r_o is the resistance per unit length of the outer conductor (Ω/m).
- r_i is the resistance per unit length of the inner conductor (Ω/m).
- a is the radius of the cylindrical cell.

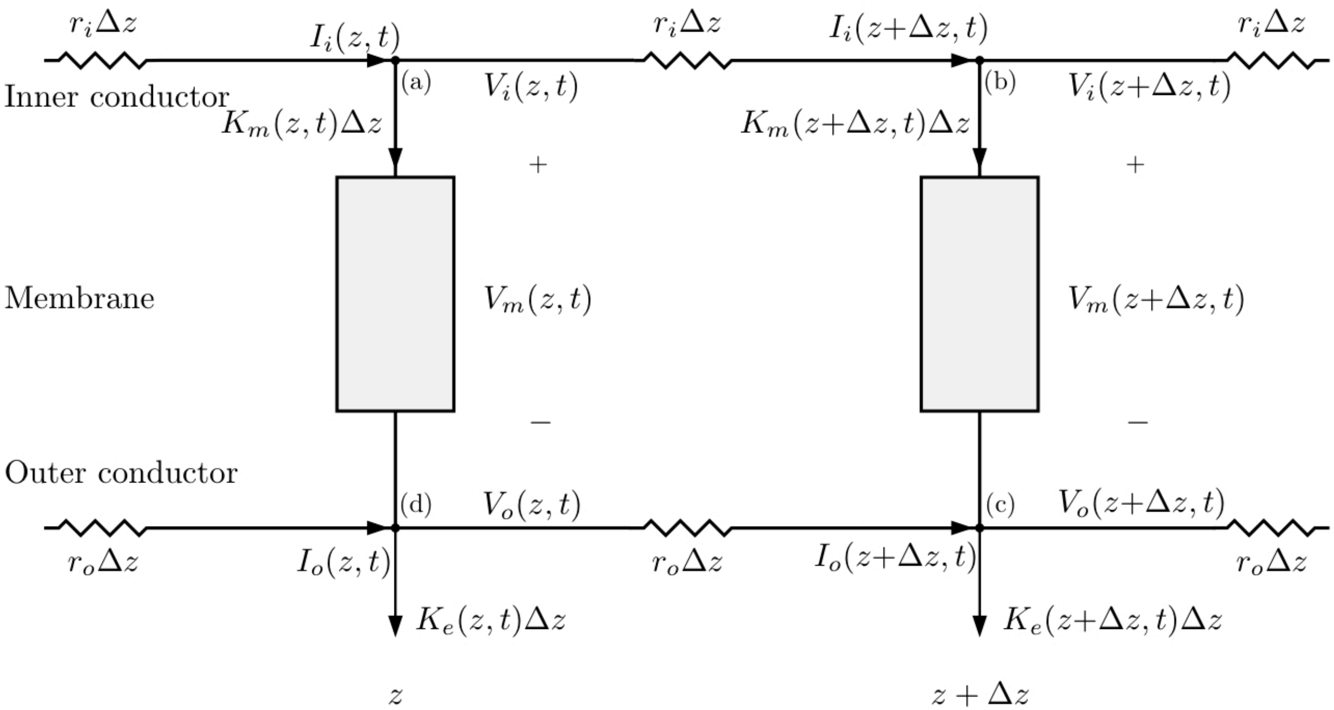


$$\textbf{KCL at (a)}: \quad I_i(z,t) - I_i(z+\Delta z,t) - K_m(z,t)\Delta z = 0$$

$$\textbf{KCL at (d)}: \quad I_o(z,t) - I_o(z+\Delta z,t) + K_m(z,t)\Delta z - K_e(z,t)\Delta z = 0$$

$$\textbf{Ohm's law at (a)} - \textbf{(b)}: \quad V_i(z,t) - V_i(z+\Delta z,t) = r_i\Delta z I_i(z+\Delta z,t)$$

$$\textbf{Ohm's law at (c)} - \textbf{(d)}: \quad V_o(z,t) - V_o(z+\Delta z,t) = r_o\Delta z I_o(z+\Delta z,t)$$



$$\frac{I_i(z+\Delta z,t)-I_i(z,t)}{\Delta z} = -K_m(z,t) \rightarrow \frac{\partial I_i(z,t)}{\partial z}$$

$$\frac{I_o(z+\Delta z,t)-I_o(z,t)}{\Delta z} = K_m(z,t) - K_e(z,t) \rightarrow \frac{\partial I_o(z,t)}{\partial z}$$

$$\frac{V_i(z+\Delta z,t)-V_i(z,t)}{\Delta z} = -r_i I_i(z+\Delta z,t) \rightarrow \frac{\partial V_i(z,t)}{\partial z}$$

$$\frac{V_o(z+\Delta z,t)-V_o(z,t)}{\Delta z} = -r_o I_o(z+\Delta z,t) \rightarrow \frac{\partial V_o(z,t)}{\partial z}$$

Core – Conductor Equations

$$\frac{\partial I_i(z,t)}{\partial z} = -K_m(z,t)$$

$$\frac{\partial I_o(z,t)}{\partial z} = K_m(z,t) - K_e(z,t)$$

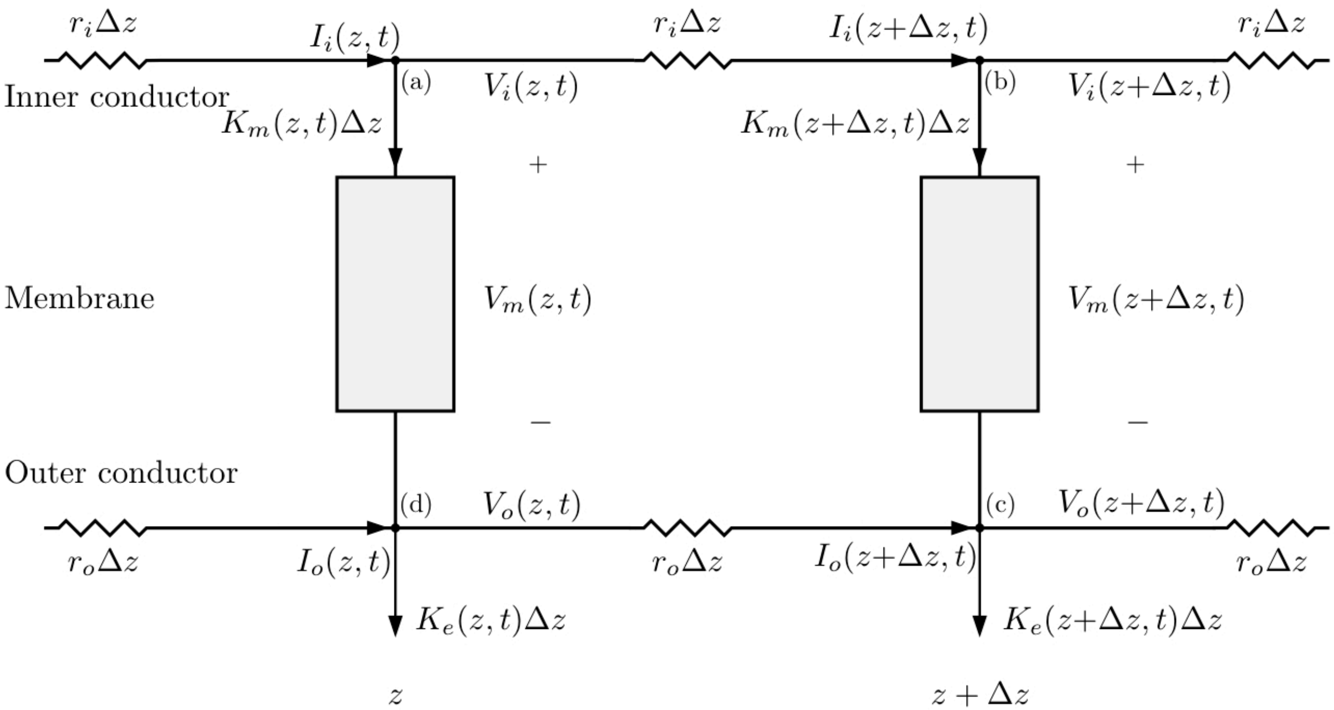
$$\frac{\partial V_i(z,t)}{\partial z} = -r_i I_i(z,t)$$

$$\frac{\partial V_o(z,t)}{\partial z} = -r_o I_o(z,t)$$

$$V_m(z,t) = V_i(z,t) - V_o(z,t)$$

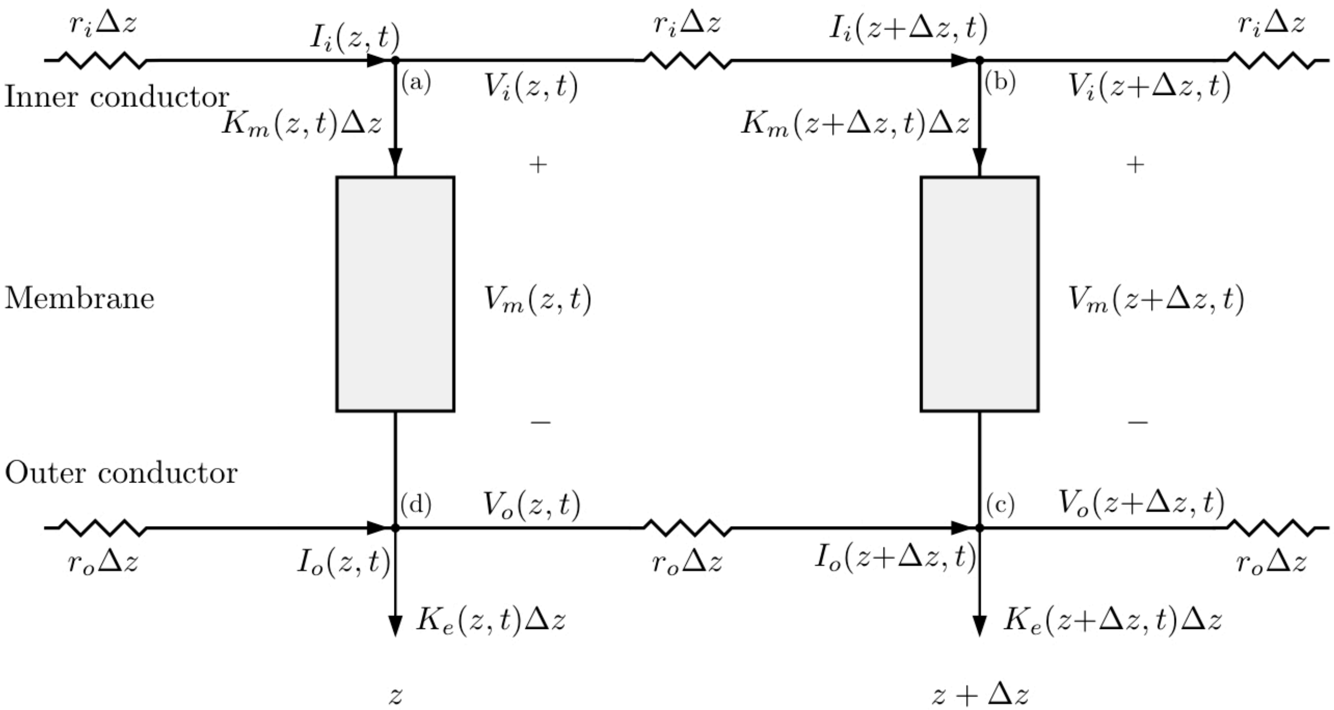
$$\frac{\partial V_m(z,t)}{\partial z} = \frac{\partial V_i(z,t)}{\partial z} - \frac{\partial V_o(z,t)}{\partial z} = -r_i I_i(z,t) + r_o I_o(z,t)$$

$$\frac{\partial^2 V_m(z,t)}{\partial z^2} = -r_i \frac{\partial I_i(z,t)}{\partial z} + r_o \frac{\partial I_o(z,t)}{\partial z} = r_i K_m(z,t) + r_o (K_m(z,t) - K_e(z,t))$$



THE Core – Conductor Equation

$$\frac{\partial^2 V_m(z,t)}{\partial z^2} = (r_o + r_i)K_m(z,t) - r_o K_e(z,t)$$



THE Core – Conductor Equation

Assumptions/geometry above, along with Kirchoff's & Ohm's Laws lead us to the...

$$\frac{\partial^2 V_m(z,t)}{\partial z^2} = (r_o + r_i)K_m(z,t) - r_o K_e(z,t)$$

→ Relates spatial change in transmembrane potential to current flowing through the membrane

Some Implications

Consider no external electrodes (i.e., $K_e = 0$):

$$K_m(z,t) = \frac{1}{r_o + r_i} \frac{\partial^2 V_m(z,t)}{\partial z^2}$$

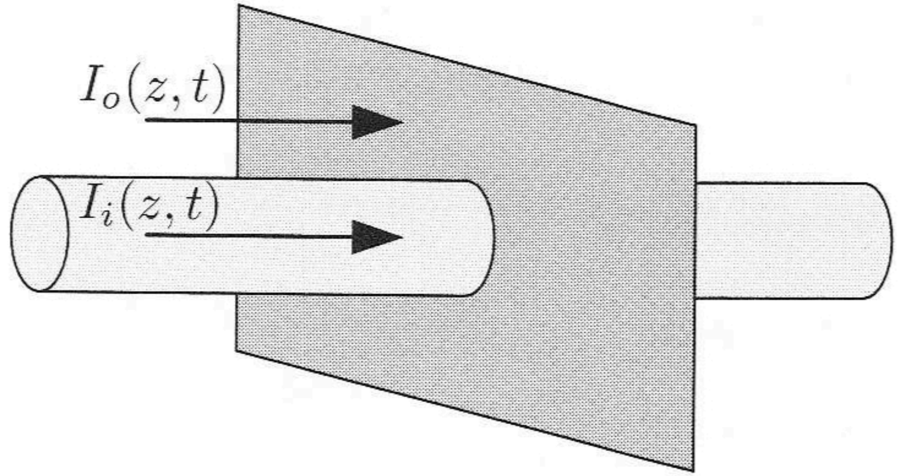


Figure 2.9

Conservation of charge requires: $I_i(z,t) + I_o(z,t) = 0$

Core – Conductor Equations

$$\frac{\partial I_i(z,t)}{\partial z} = -K_m(z,t)$$

$$\frac{\partial I_o(z,t)}{\partial z} = K_m(z,t) - K_e(z,t)$$

$$\frac{\partial V_i(z,t)}{\partial z} = -r_i I_i(z,t)$$

$$\frac{\partial V_o(z,t)}{\partial z} = -r_o I_o(z,t)$$

$$I_o(z,t) = \frac{1}{r_o + r_i} \frac{\partial V_m(z,t)}{\partial z}$$