Biophysics I (BPHS 4080)

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Website: http://www.yorku.ca/cberge/4080W2018.html

York University
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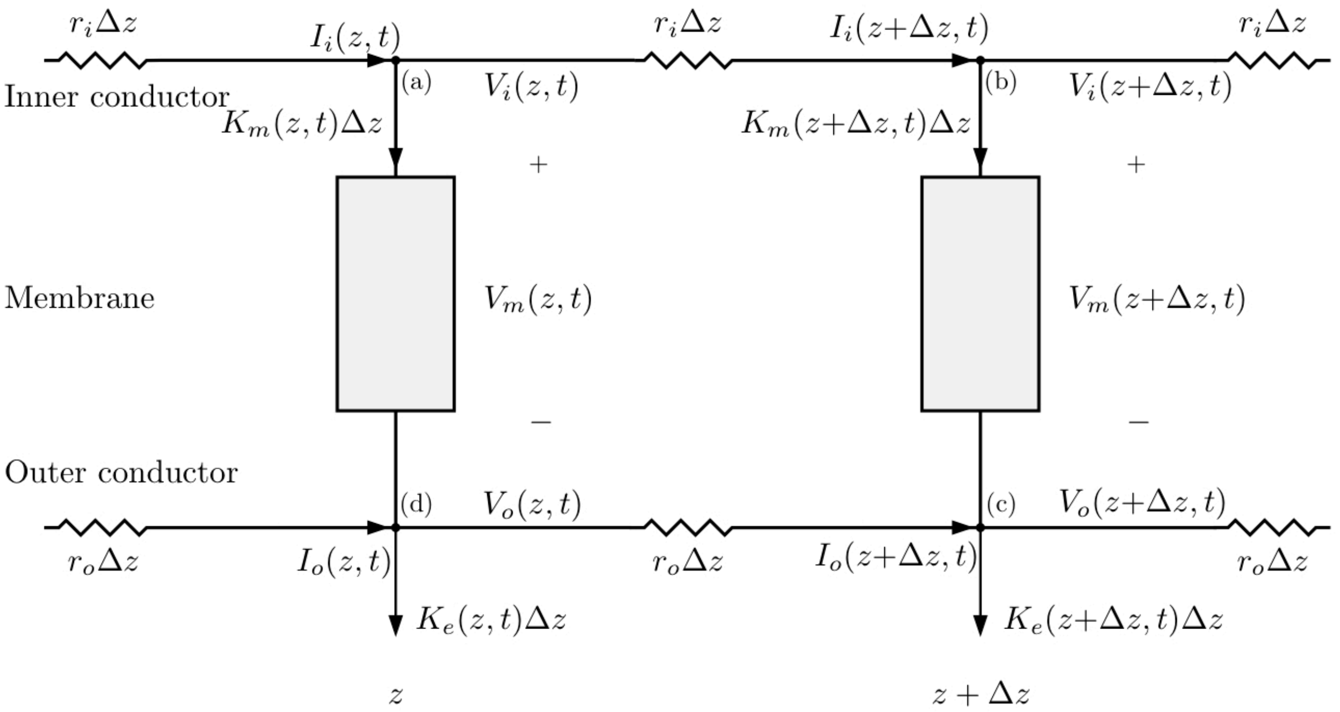
Lecture 18

Reference/Acknowledgement:
- TF Weiss (Cellular Biophysics)
- D Freeman



THE Core – Conductor Equation

Assumptions/geometry above, along with Kirchoff's & Ohm's Laws lead us to the...

$$\frac{\partial^2 V_m(z,t)}{\partial z^2} = (r_o + r_i)K_m(z,t) - r_o K_e(z,t)$$

→ Relates spatial change in transmembrane potential to current flowing through the membrane

Some Implications

Consider no external electrodes
(i.e., $K_e = 0$):

$$K_m(z,t) = \frac{1}{r_o + r_i}\frac{\partial^2 V_m(z,t)}{\partial z^2}$$

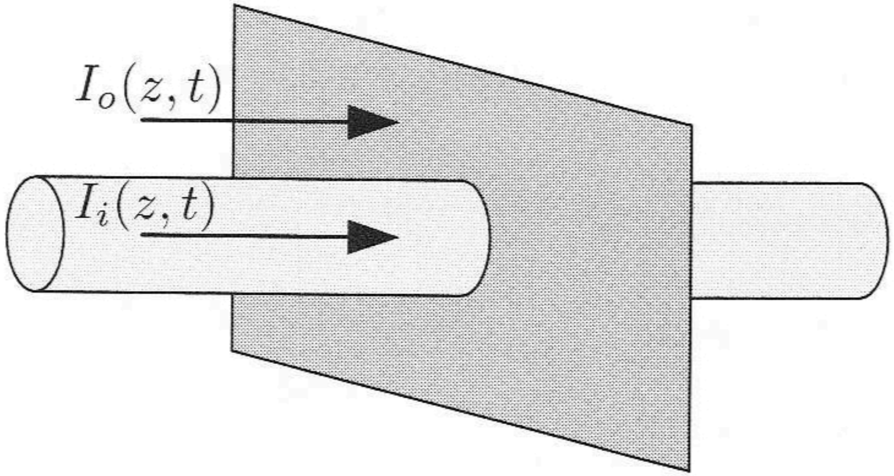


Figure 2.9

Conservation of charge requires: $$I_i(z,t) + I_o(z,t) = 0$$

Core – Conductor Equations

$$\frac{\partial I_i(z,t)}{\partial z} = -K_m(z,t)$$

$$\frac{\partial I_o(z,t)}{\partial z} = K_m(z,t) - K_e(z,t)$$

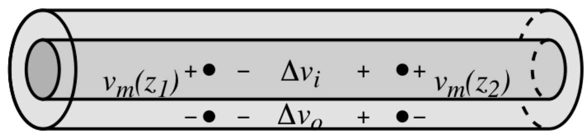
$$\frac{\partial V_i(z,t)}{\partial z} = -r_i I_i(z,t)$$

$$\frac{\partial V_o(z,t)}{\partial z} = -r_o I_o(z,t)$$

$$I_o(z,t) = \frac{1}{r_o + r_i}\frac{\partial V_m(z,t)}{\partial z}$$

Relation between extracellular and intracellular potentials

Some useful interrelationships...



KVL:

$$v_m(z_2) + \Delta v_o - v_m(z_1) - \Delta v_i = 0$$

$$\Delta v_o - \Delta v_i = v_m(z_1) - v_m(z_2)$$

$$\Delta v_i = -\int_{z_1}^{z_2} r_i I_i dz = -r_i \int_{z_1}^{z_2} I_i dz$$

$$\Delta v_o = -\int_{z_1}^{z_2} r_o I_o dz = -r_o \int_{z_1}^{z_2} I_o dz$$

$$I_o = -I_i$$

$$\Delta v_i = -r_i \int_{z_1}^{z_2} I_i dz = -r_i \int_{z_1}^{z_2} (-I_o) dz = -\frac{r_i}{r_o} \Delta v_o$$

$$\Delta v_o - \Delta v_i = \Delta v_o + \frac{r_i}{r_o} \Delta v_o = v_m(z_1) - v_m(z_2)$$

$$\Delta v_o = \frac{r_o}{r_o + r_i} \left(v_m(z_1) - v_m(z_2) \right)$$

let $z_1 \to -\infty$: then $v_o(z_1) \to 0$ and $v_m(z_1) \to V_m^o$

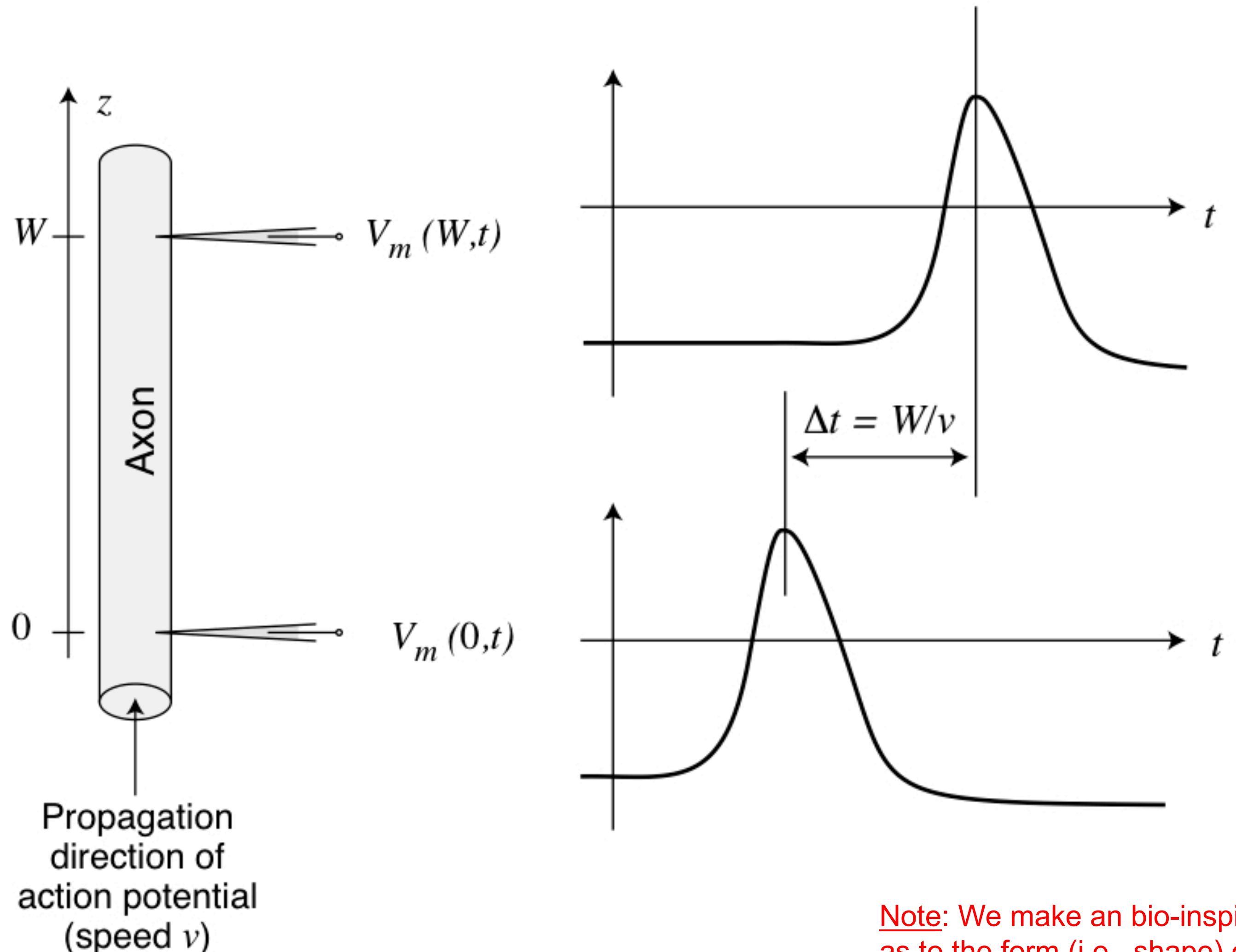
$$\Delta v_o = v_o(z_2) - v_o(z_1) = v_o(z_2) = \frac{r_o}{r_o + r_i} \left(V_m^o - v_m(z_2) \right)$$

$$v_o(z_2) = -\frac{r_o}{r_o + r_i} \left(v_m(z_2) - V_m^o \right)$$

Propagation at *Constant Velocity*

<u>Assumption</u>: Membrane potential behaves in a ***wave-like*** fashion (const. velocity)

$$V_m(z,t) = f\left(t \pm \frac{z}{v}\right)$$

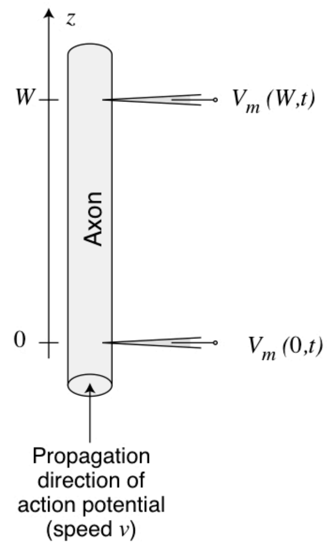


<u>Note</u>: We make an bio-inspired assumption as to the form (i.e., shape) of f here

Propagation at *Constant Velocity*

$$V_m(z,t) = f\left(t \pm \frac{z}{v}\right)$$

"Snapshot" at one specific time
→ ***Spatial dependence***



"Snapshots" at two different spatial locations
→ ***Temporal dependence***

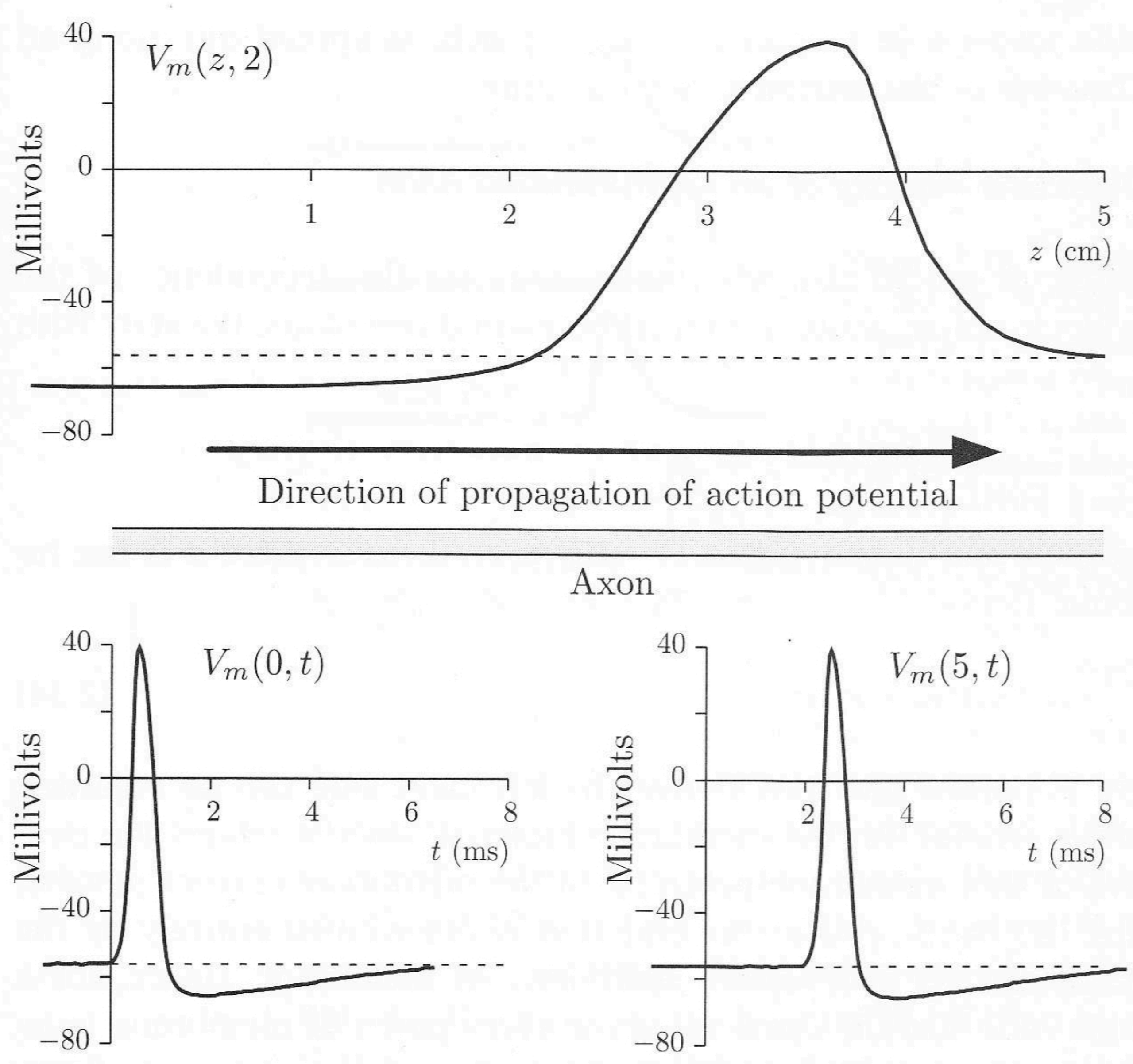


Figure 2.13

Propagation at *Constant Velocity*

$$V_m(z,t) = f\left(t \pm \frac{z}{v}\right)$$

Think carefully about what the diacritical dot means here!

$$\frac{\partial V_m(z,t)}{\partial z} = \pm\frac{1}{v}\dot{f}\left(t \pm \frac{z}{v}\right) \quad \text{and} \quad \frac{\partial V_m(z,t)}{\partial t} = \dot{f}\left(t \pm \frac{z}{v}\right)$$

$$\frac{\partial^2 V_m(z,t)}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 V_m}{\partial t^2}$$

Wave equation
(differential form)

$$I_o(z,t) = \pm\frac{1}{(r_o + r_i)v}\frac{\partial V_m(z,t)}{\partial t}$$

and

$$K_m(z,t) = \frac{1}{(r_o + r_i)\,v^2}\frac{\partial^2 V_m(z,t)}{\partial t^2}$$

→ So when we assume a wave propagating at constant velocity, the core conductor model yields explicit time relationships as well

$$I_o(z,t) = \frac{1}{r_o + r_i}\frac{\partial V_m(z,t)}{\partial z}$$

Connecting action potential-type waves to the core conductor model

$$I_o(z,t) = \pm \frac{1}{(r_o + r_i)\nu} \frac{\partial V_m(z,t)}{\partial t}$$

and

$$K_m(z,t) = \frac{1}{(r_o + r_i)\,\nu^2} \frac{\partial^2 V_m(z,t)}{\partial t^2}$$

$$I_i(z,t) + I_o(z,t) = 0$$

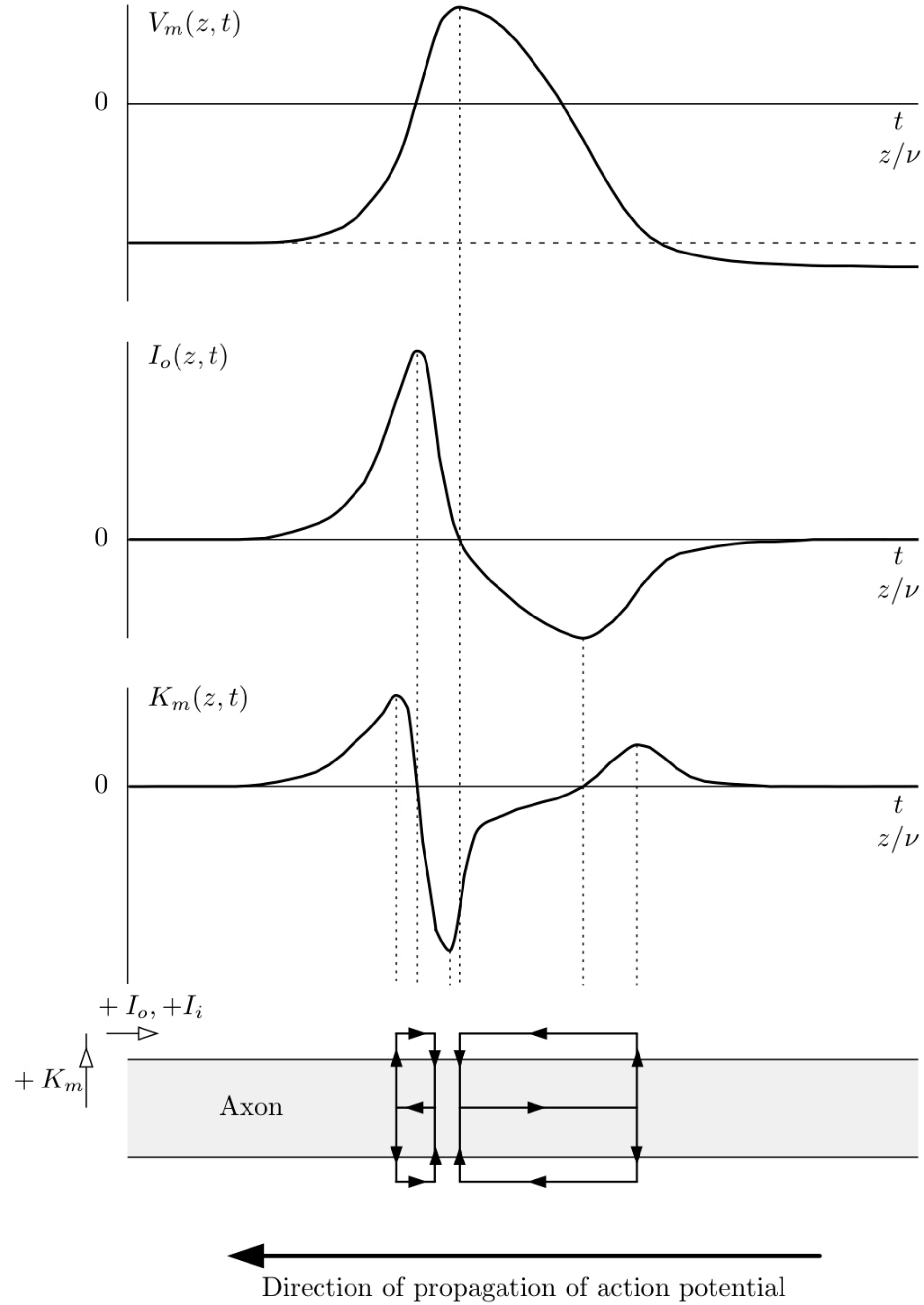


Figure 2.12

Conduction Velocity *(unmyelinated axon)*

$$K_m(z,t) = \frac{1}{(r_o + r_i)\nu^2} \frac{\partial^2 V_m(z,t)}{\partial t^2}$$

$$J_m(z,t) = K_m(z,t)/(2\pi a)$$

$$\frac{\partial^2 V_m(z,t)/\partial t^2}{J_m(z,t)} = 2\pi a(r_o + r_i)\nu^2$$

Left-side: constant, only depends upon electrical properties of membrane per unit area

Right-side: constant, velocity depends only upon axon diameter and fluid resistances

Conduction Velocity *(unmyelinated axon)*

Change r_o

$$\frac{\partial^2 V_m(z,t)/\partial t^2}{J_m(z,t)} = 2\pi a(r_o + r_i)\nu^2$$

Changing resistivity affects conduction velocity

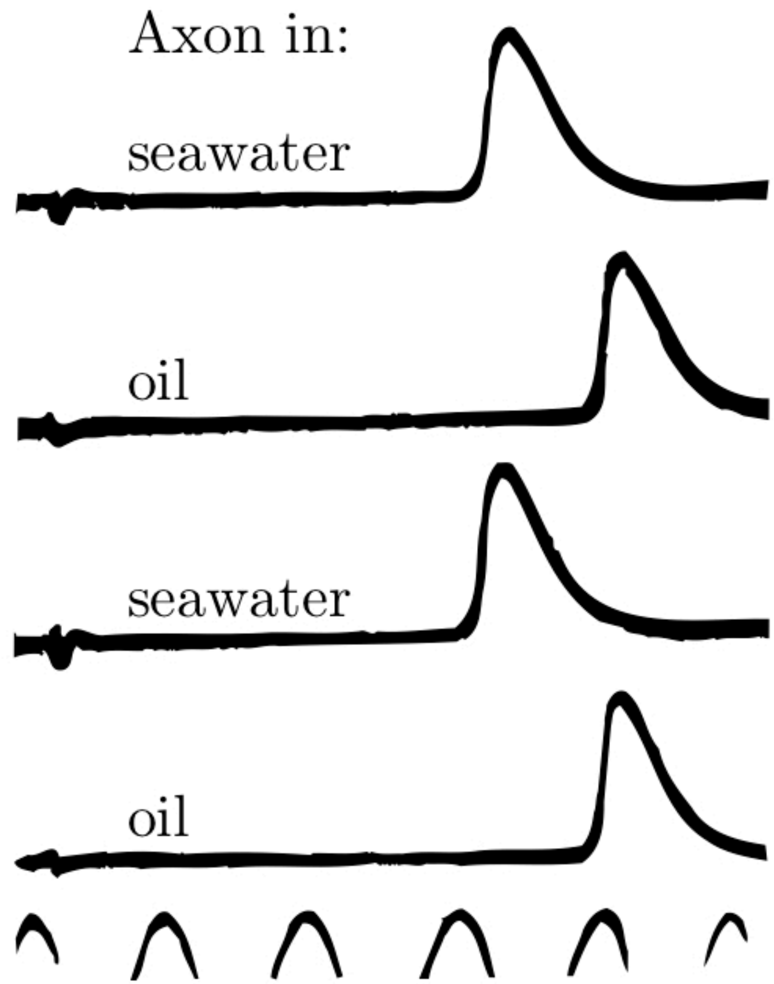


Figure 2.14

Conduction Velocity *(unmyelinated axon)*

Change r_i

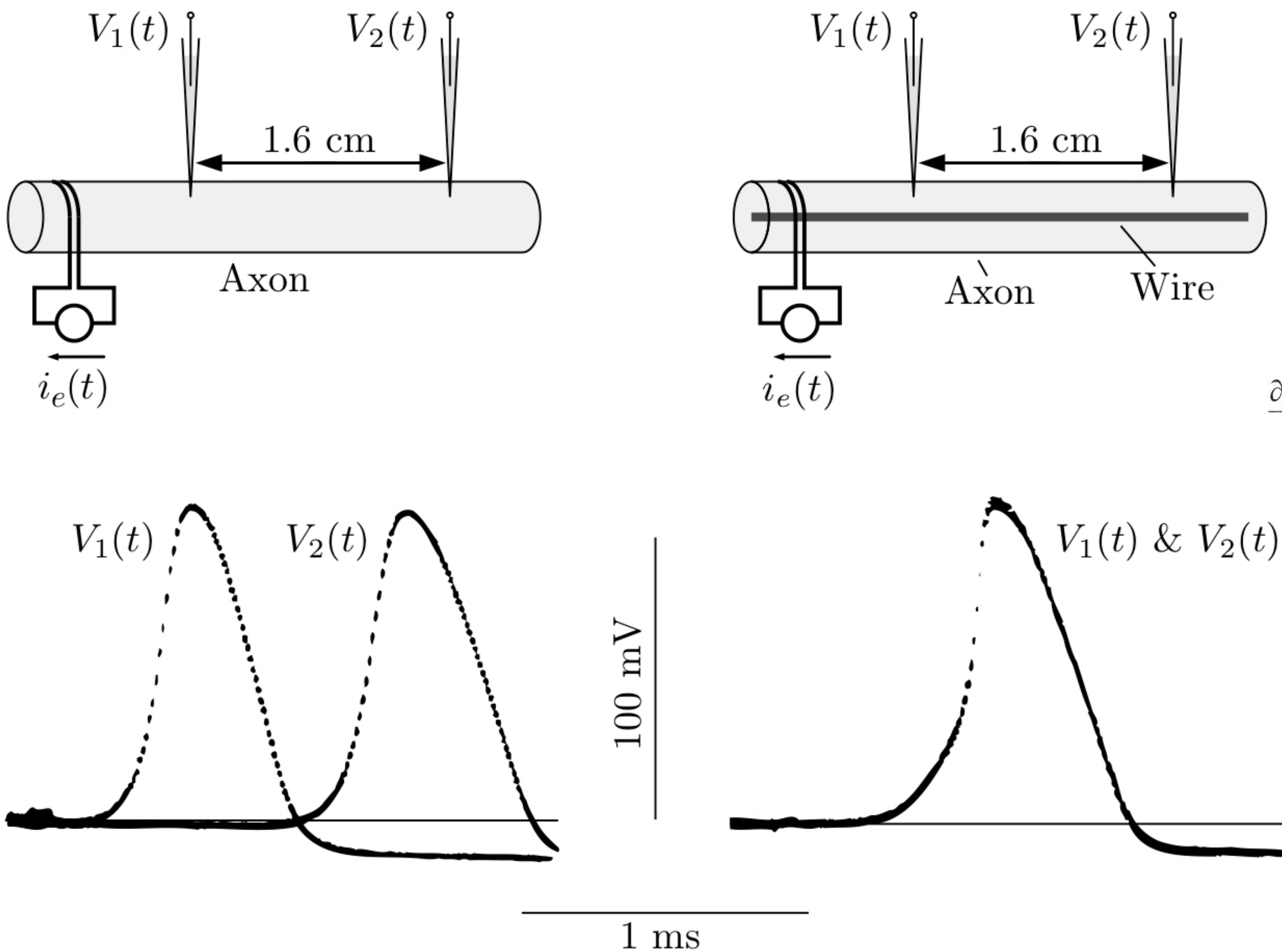


Figure 2.15

Inner wire essentially acts like a short

$$\frac{\partial^2 V_m(z,t)/\partial t^2}{J_m(z,t)} = 2\pi a(r_o + r_i)\nu^2$$

→ 'Space clamp'

Conduction Velocity *(unmyelinated axon)*

Assume $r_i >> r_o$

$$2\pi a r_i v^2 = K_m$$

$$r_i = \frac{\rho_i}{\pi a^2}$$

$$v = \sqrt{\frac{K_m a}{2\rho_i}}$$

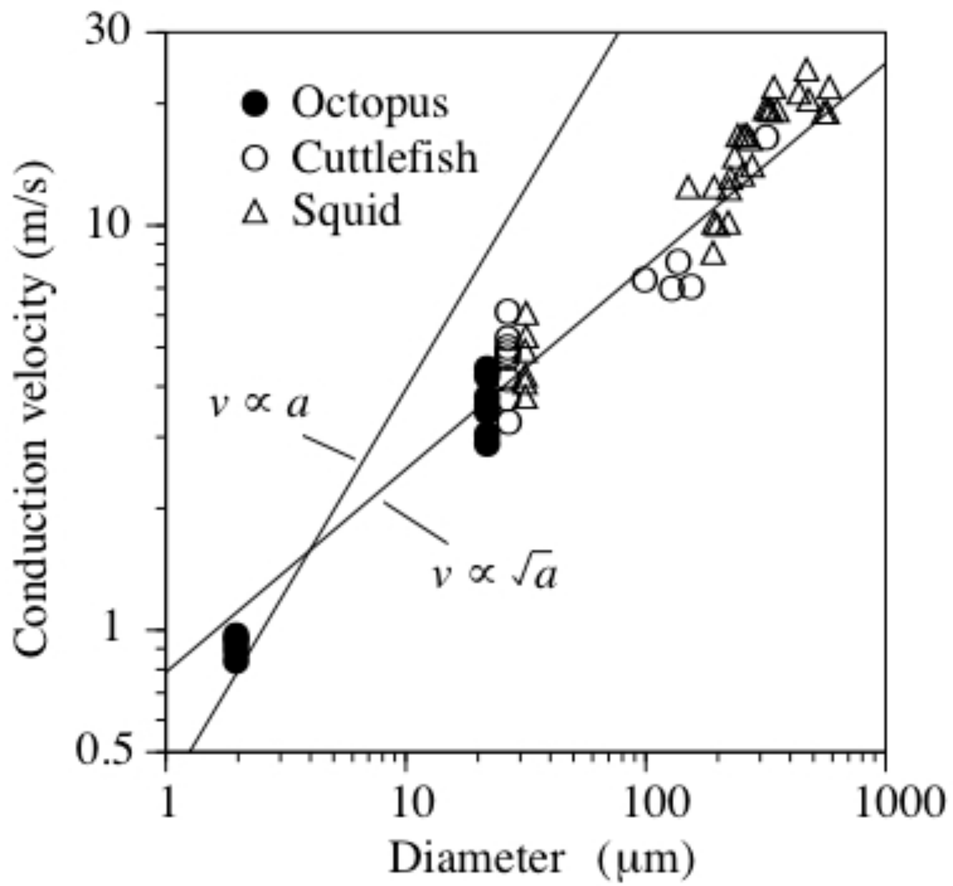


Figure 2.16

→ thicker axons = faster propagation

Core Conductor Model

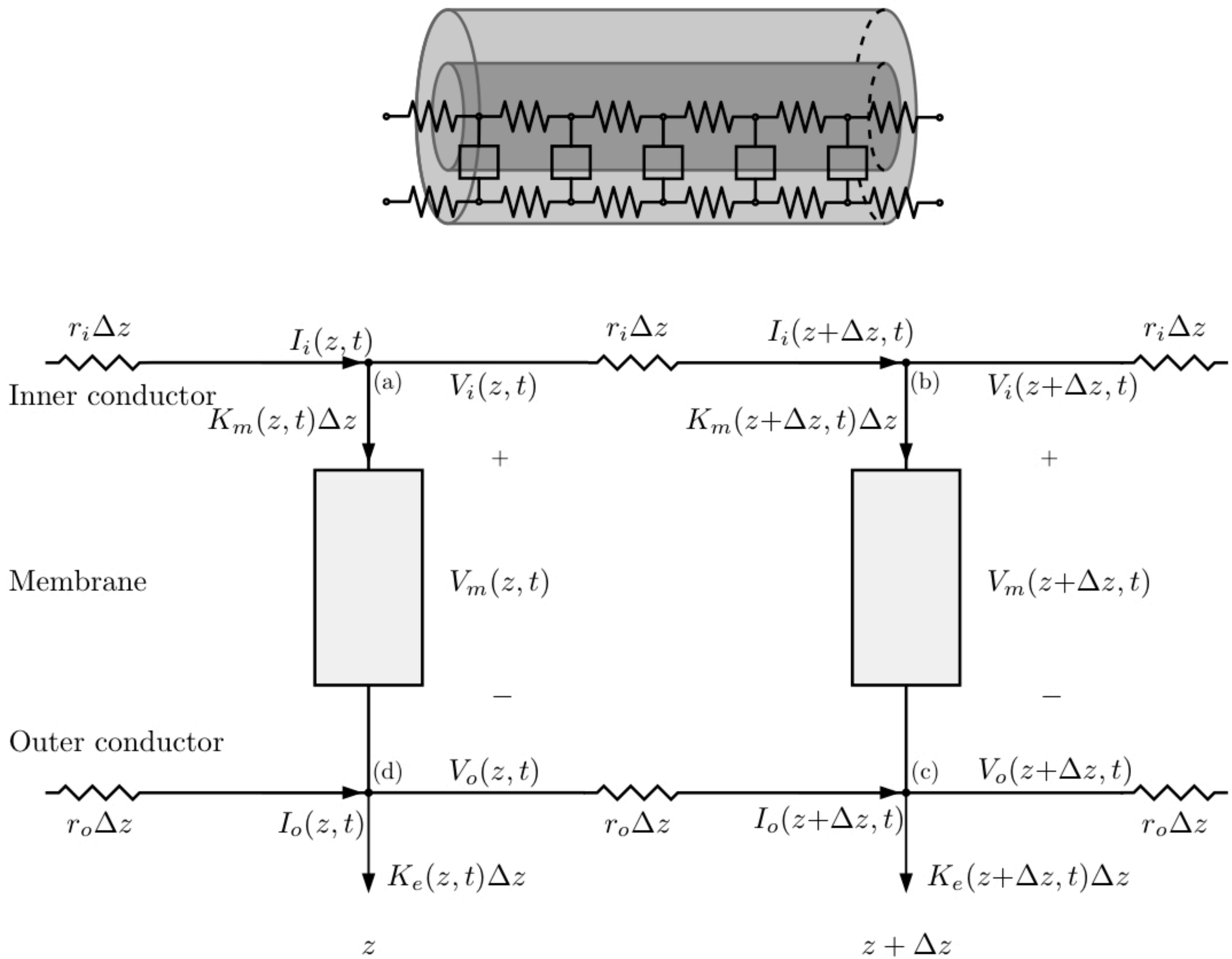
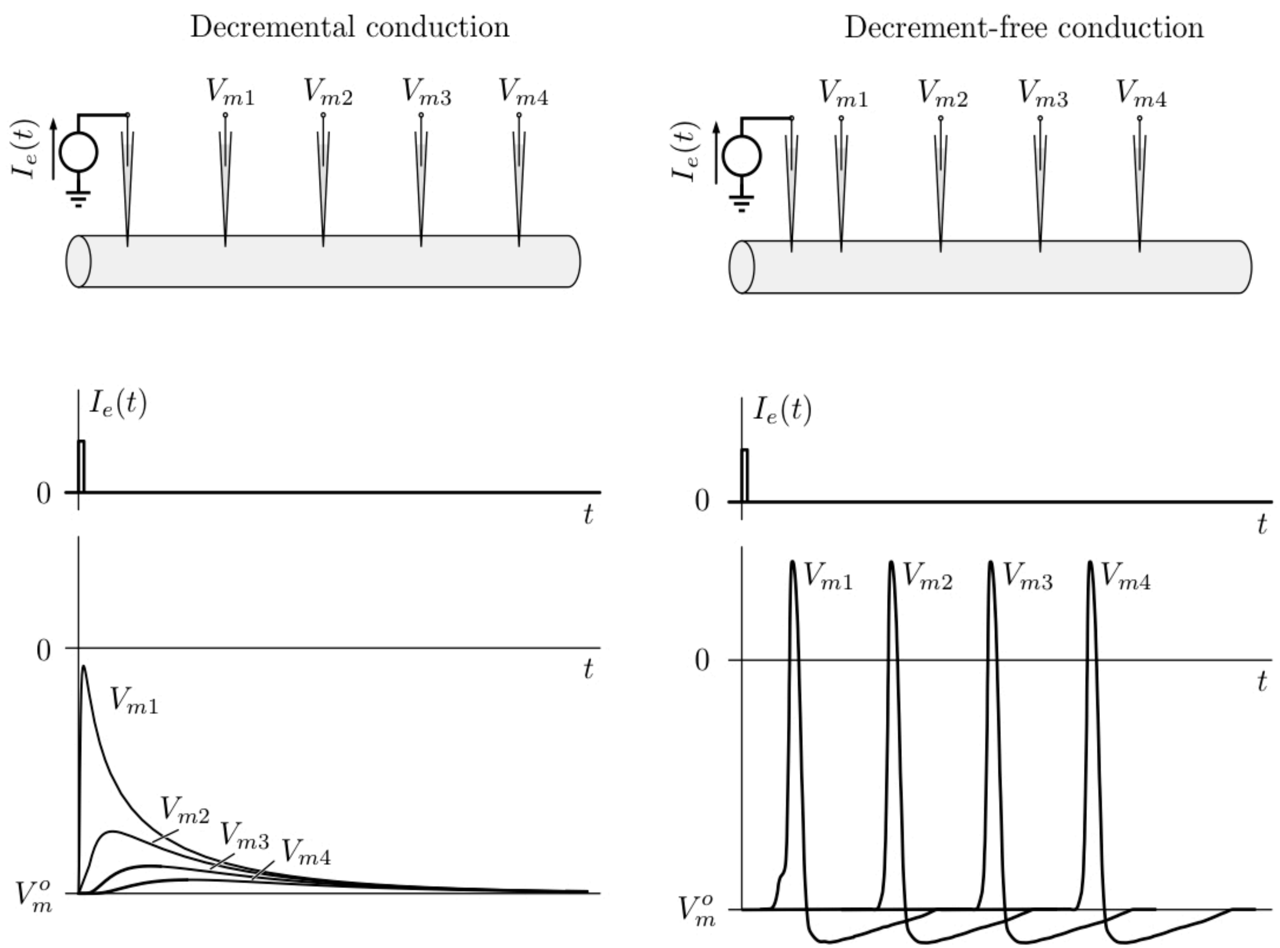
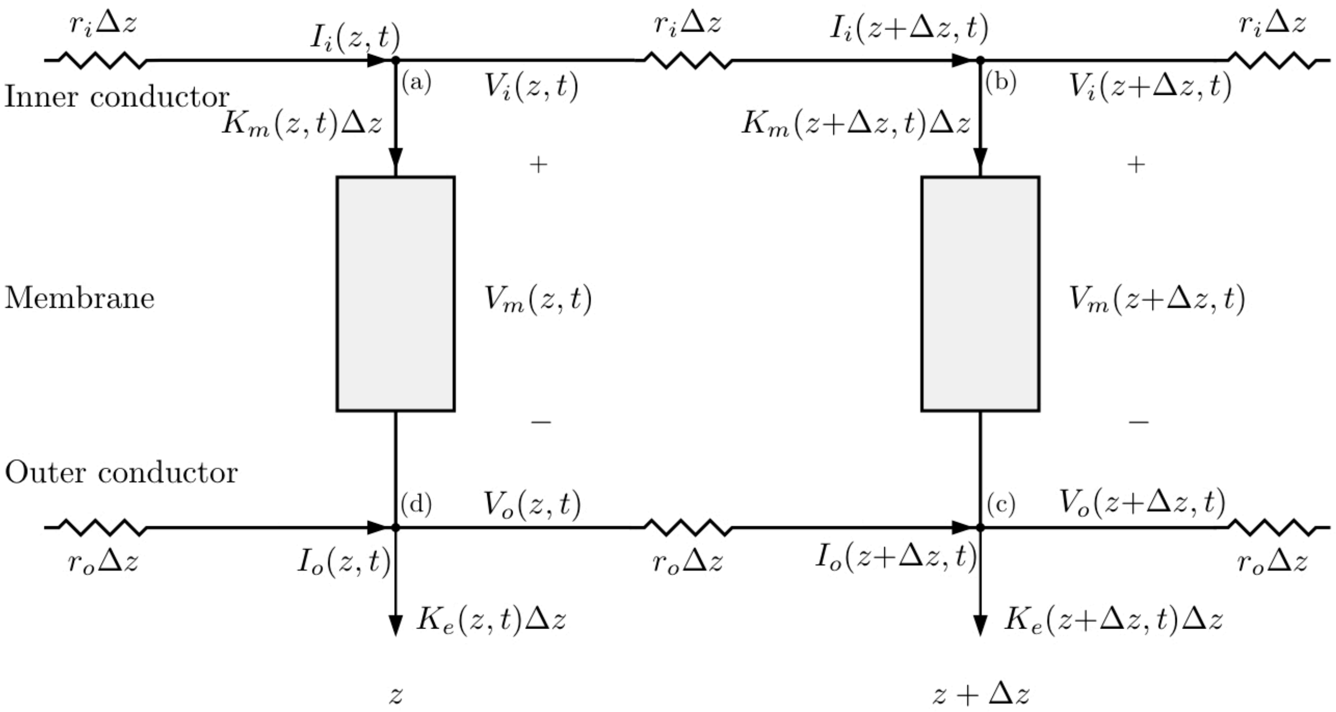


Figure 2.7



Core-Conductor Model (starting point) → Model for electrically large cells



No assumptions made about membrane!

THE Core – Conductor Equation

$$\frac{\partial^2 V_m(z,t)}{\partial z^2} = (r_o + r_i)K_m(z,t) - r_o K_e(z,t)$$