Biophysics I (BPHS 4080)

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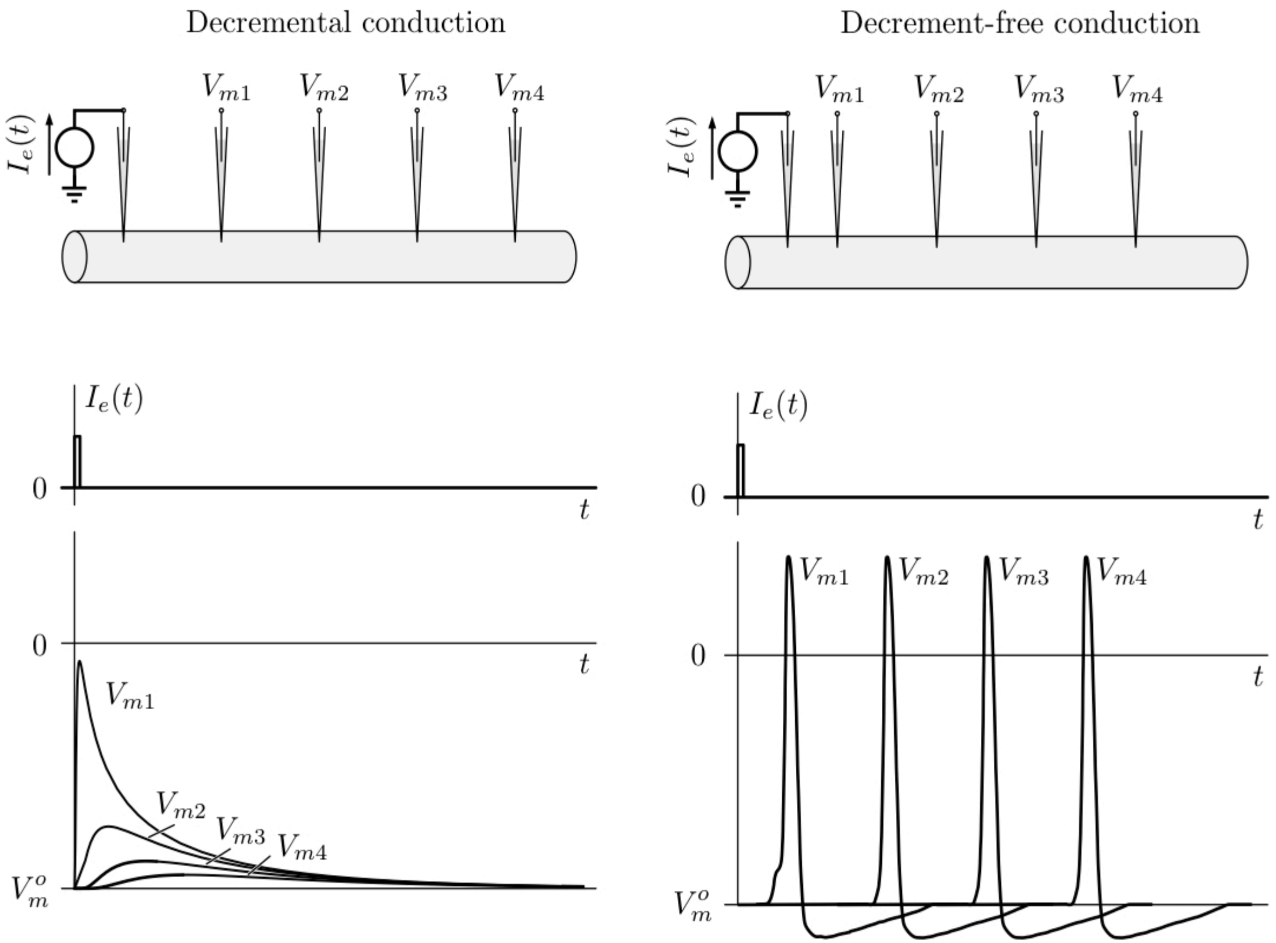
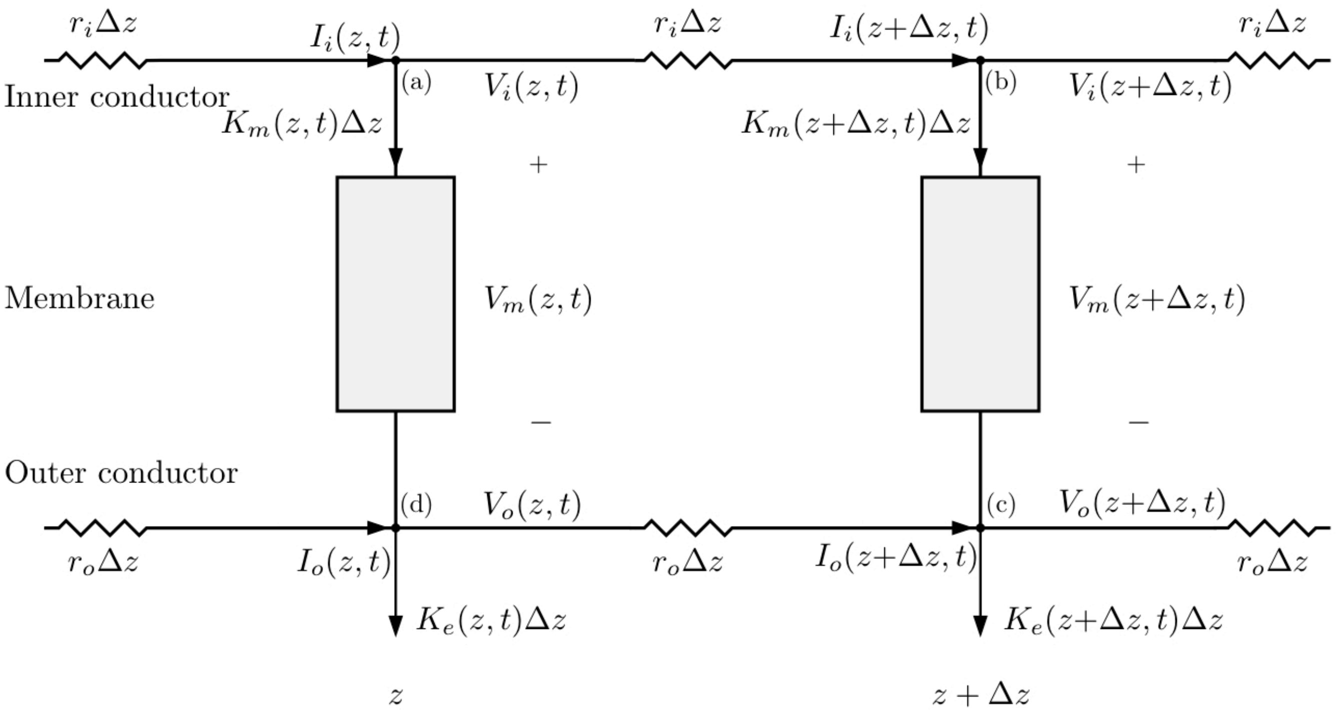
Website: http://www.yorku.ca/cberge/4080W2018.html

York University
Winter 2018
Lecture 19

Reference/Acknowledgement:
- TF Weiss (Cellular Biophysics)
- D Freeman



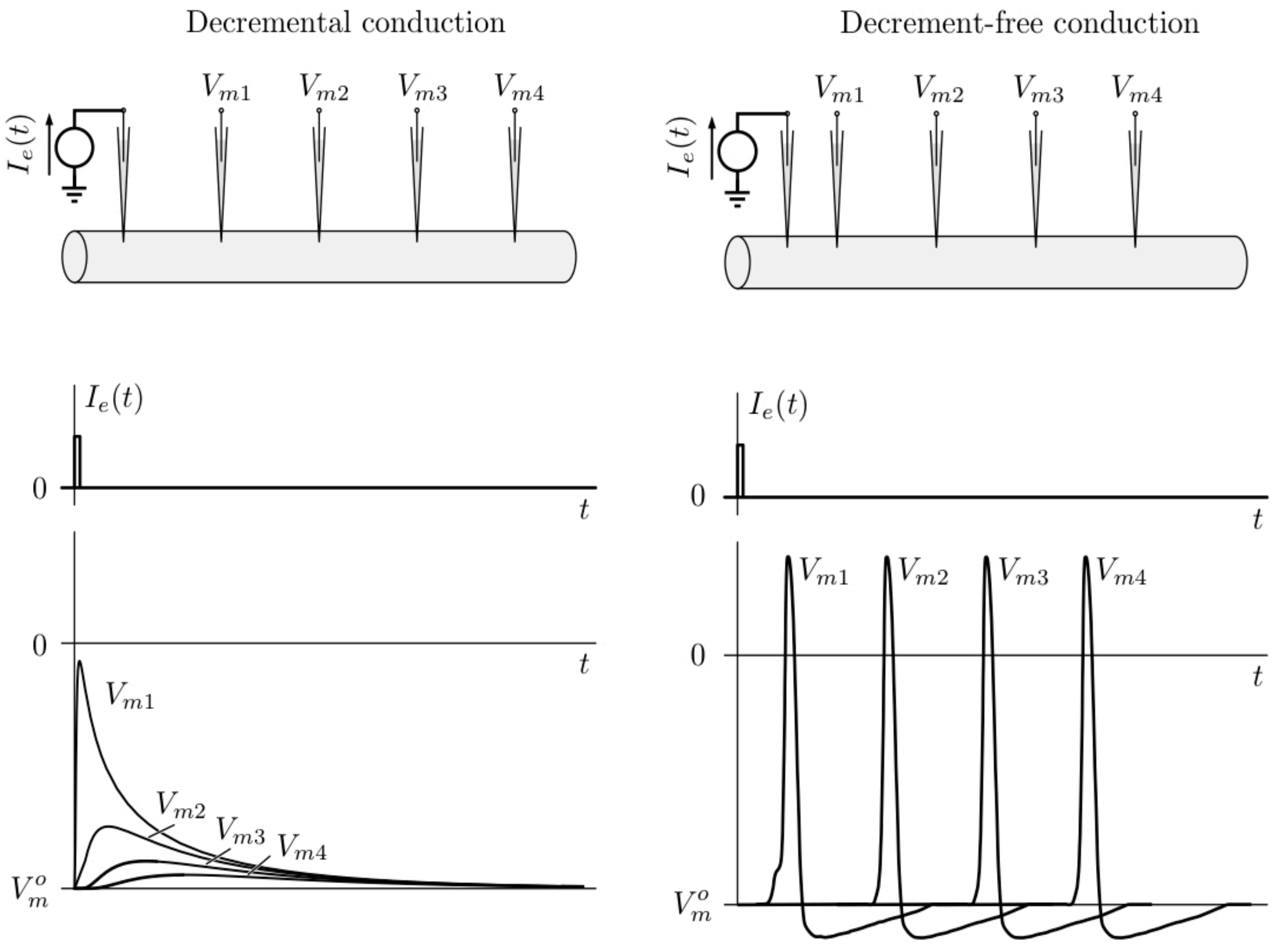
Core-Conductor Model (starting point) → Model for electrically large cells



No assumptions made about membrane!

THE Core – Conductor Equation

$$\frac{\partial^2 V_m(z,t)}{\partial z^2} = (r_o + r_i)K_m(z,t) - r_o K_e(z,t)$$



Note dynamics of response....

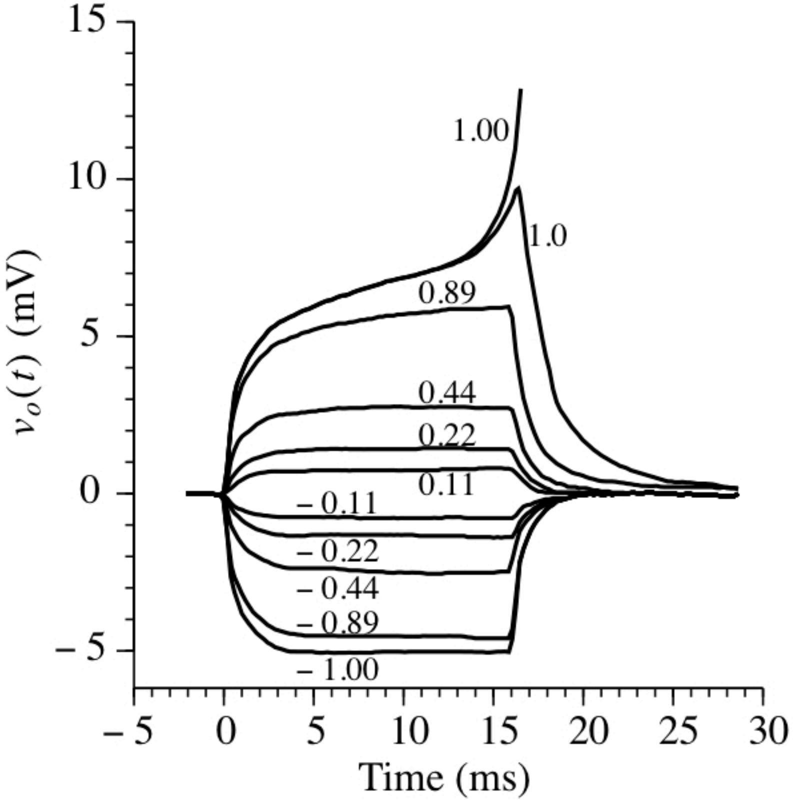


Figure 3.1

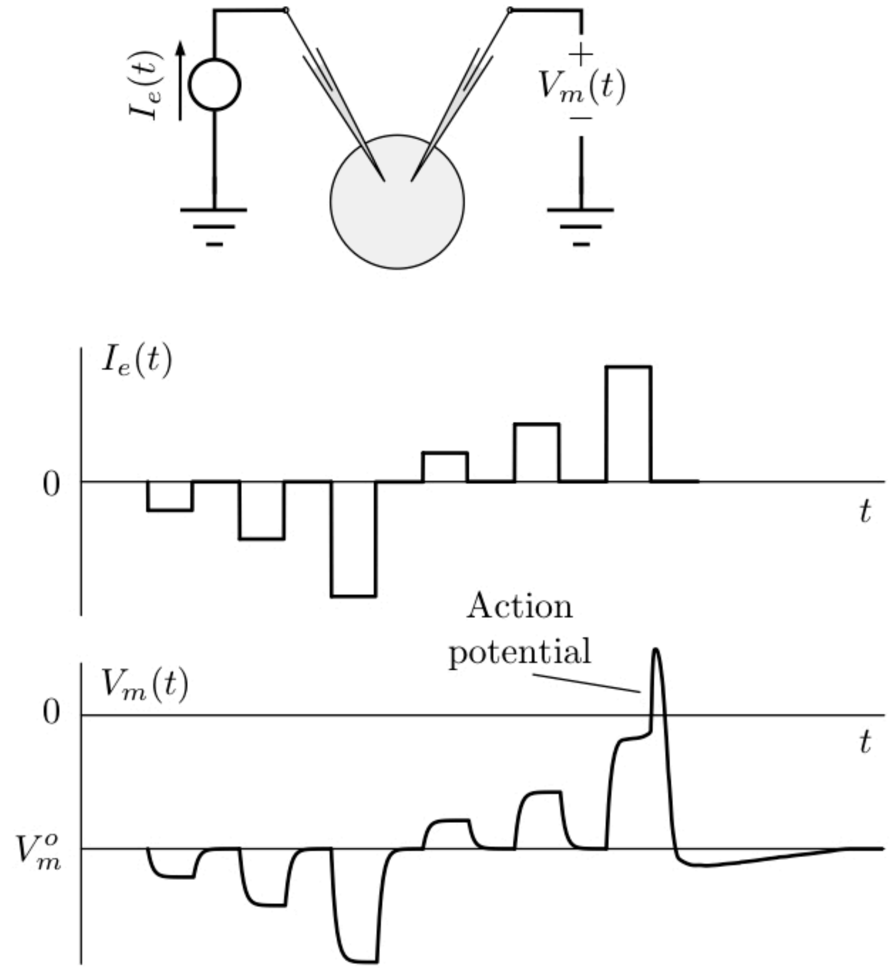


Figure 1.8

1. Linear (to a point)

2. Delay apparent

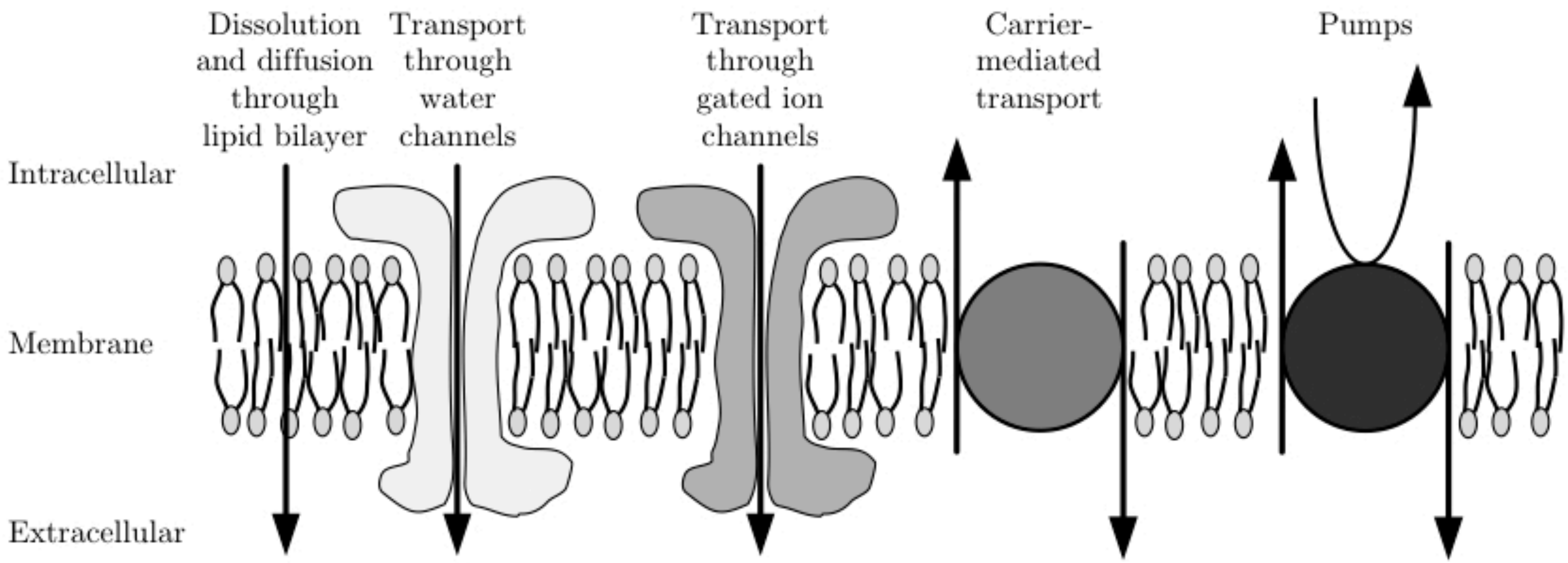
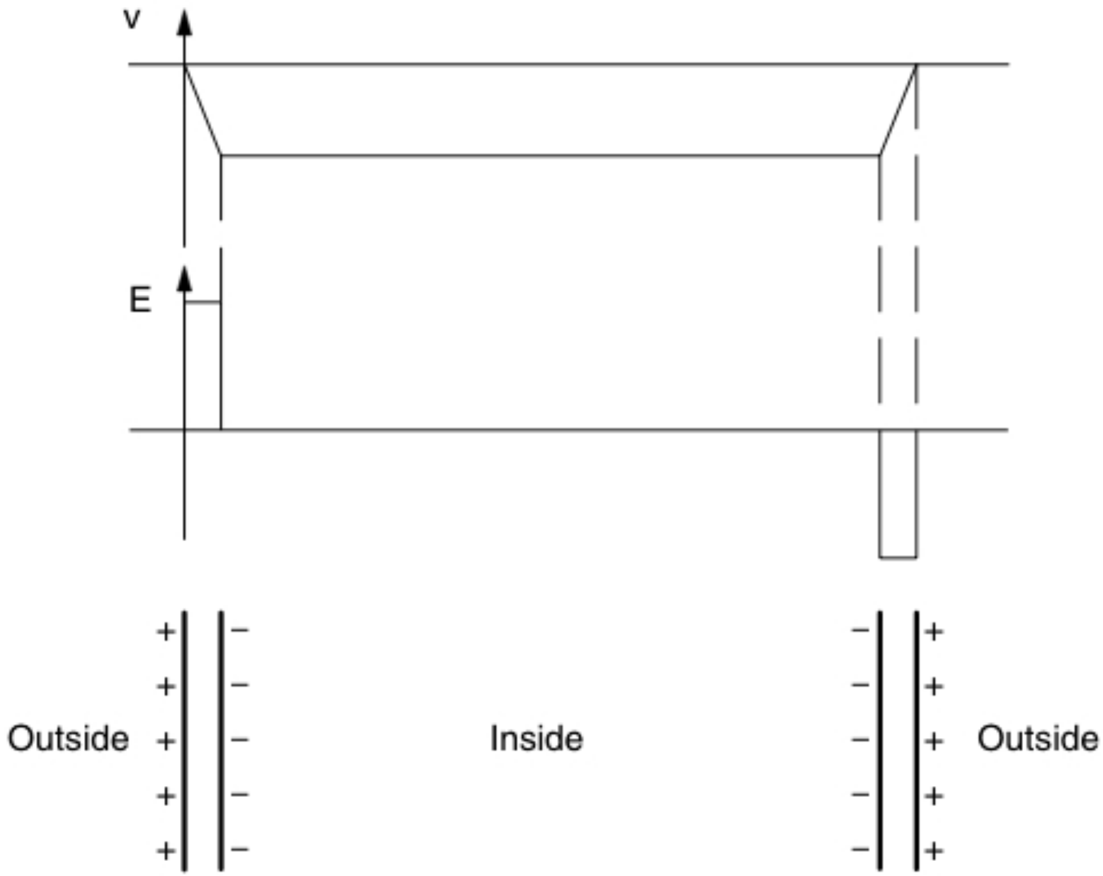


Figure 2.19

<u>Idea:</u> Membrane not only allows for charge transport, but also charge separation

Cell Membrane = Capacitor

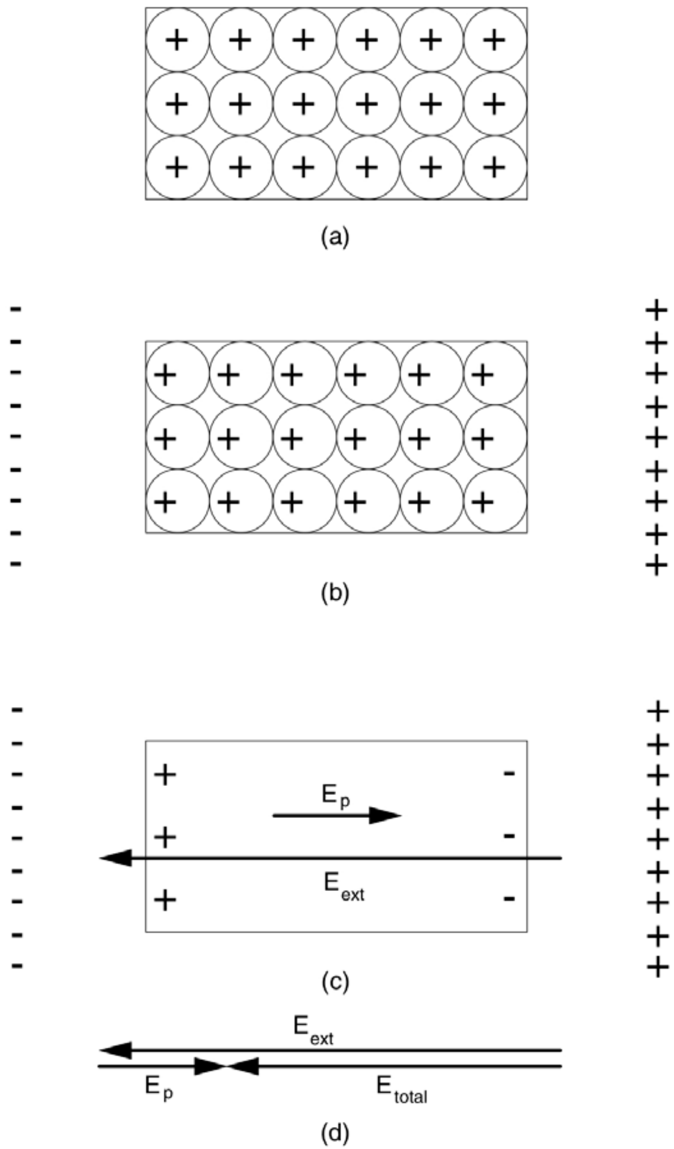


- Steady-state electrodiffusion cause charge buildup on both sides of membrane

- Charge separation acts like parallel-plate capacitor (C ~ 1 μF/cm^2)

Hobbie & Roth

Lipid Bilayer = Dielectric

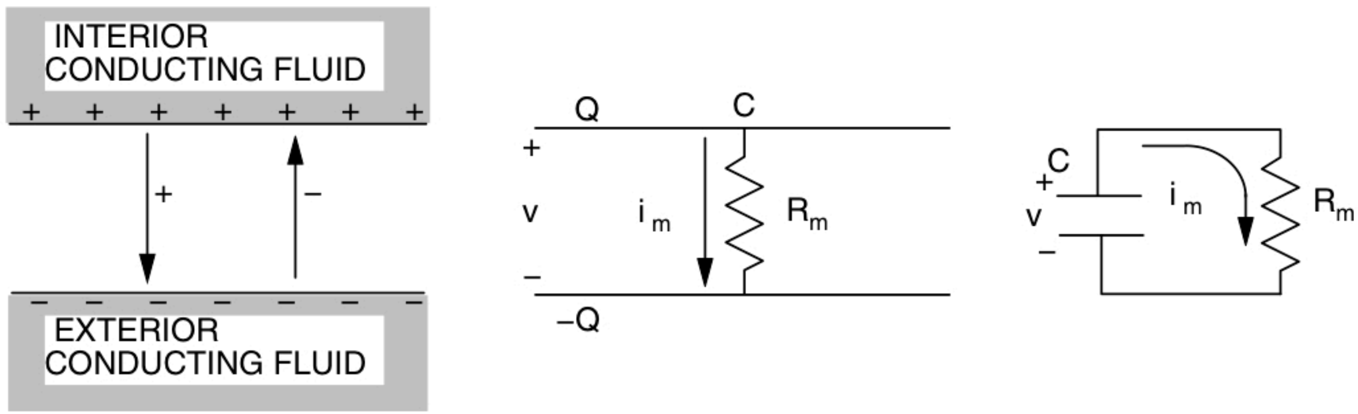


- Lipid bilayer is an insulator (i.e., acts as a dielectric w/ const. κ)

- κ~3-7, meaning more charge separation can occur (higher capacitance)

Hobbie & Roth

Circuit Representation



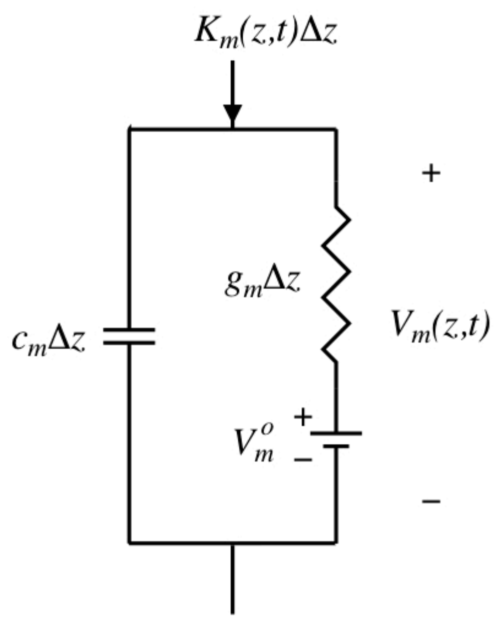


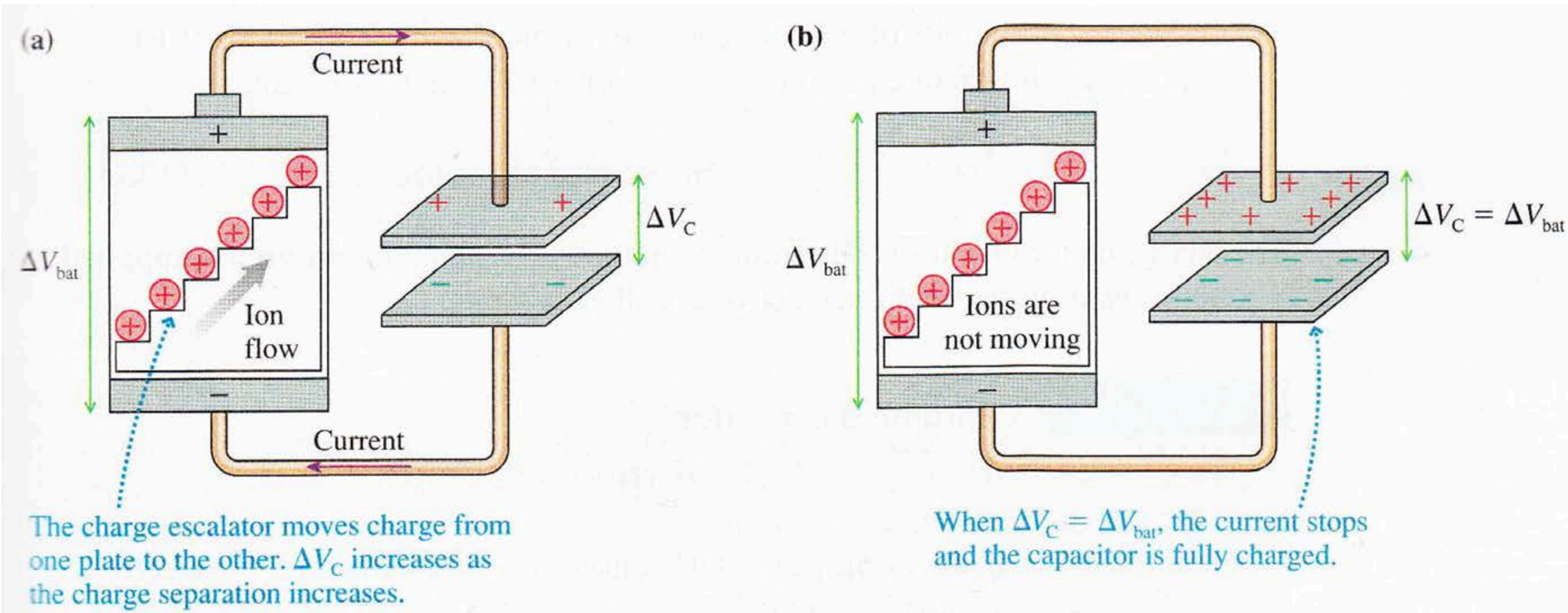
Figure 3.6

Resistor and capacitor in series
→ RC time constant

Hobbie & Roth
Weiss

Review: Capacitance

- Charging a parallel-plate capacitor



$$Q = C\Delta V_C \qquad \text{(charge on a capacitor)}$$

→ Stored charge is proportional to potential difference. Constant of proportionality is characterizes the “capacitance”

Knight

Review: RC Circuits

KVL (combined w/ Ohm's law):

$$\Delta V_{cap} + \Delta V_{res} = \frac{Q}{C} - IR = 0$$

$$I = -\frac{dQ}{dt}$$

Negative because resistor current removes charge from capacitor

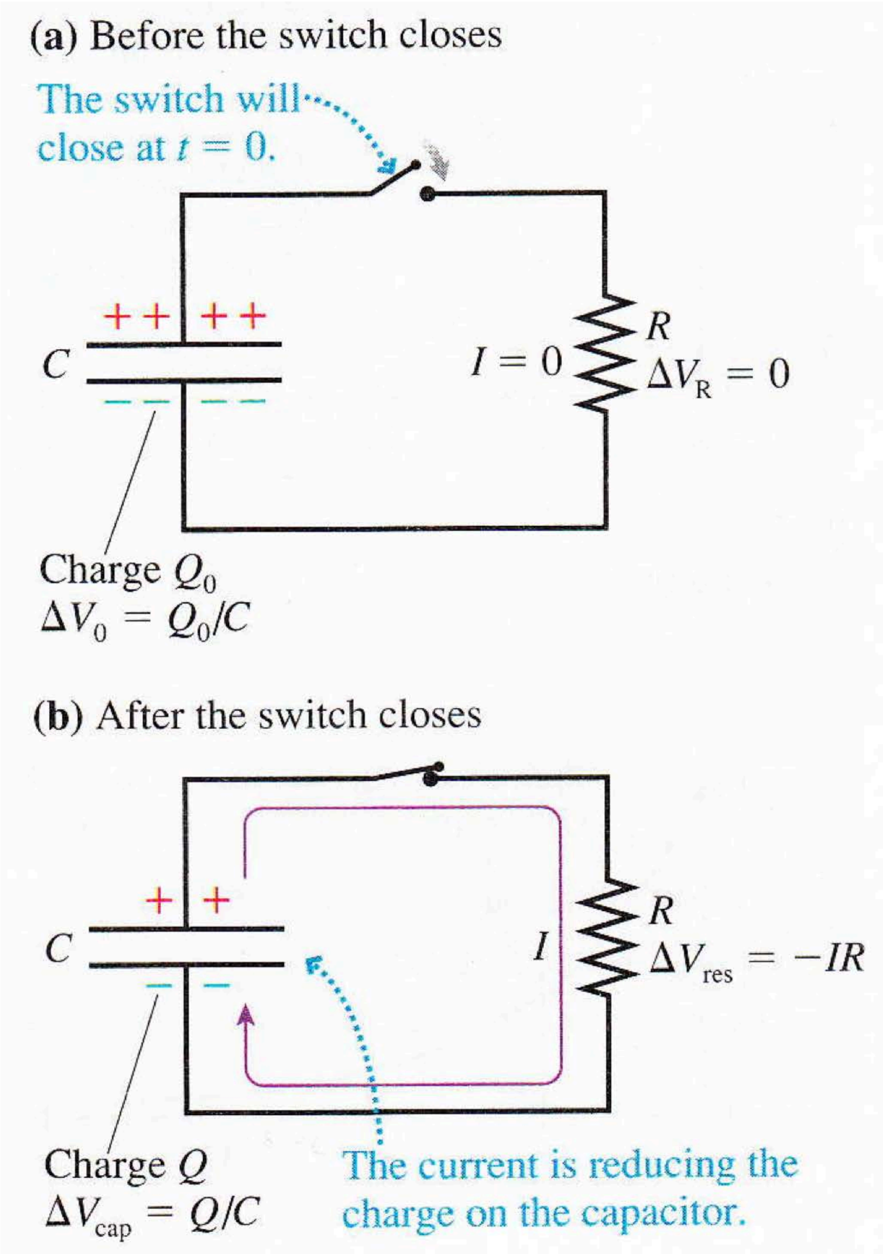
$$\frac{dQ}{dt} + \frac{Q}{RC} = 0$$

$$\frac{dQ}{Q} = -\frac{1}{RC}dt$$

$$\int_{Q_0}^{Q} \frac{dQ}{Q} = -\frac{1}{RC}\int_0^t dt$$

$$\ln Q \Big|_{Q_0}^{Q} = \ln Q - \ln Q_0 = \ln\left(\frac{Q}{Q_0}\right) = -\frac{t}{RC}$$

$$Q = Q_0 e^{-t/RC}$$



Knight

Review: RC Circuits

$$Q = Q_0 e^{-t/RC}$$

$$Q = C\,\Delta V_C$$

$$\tau = RC$$

"RC time constant"

$$\Delta V_C = \Delta V_0 e^{-t/\tau}$$

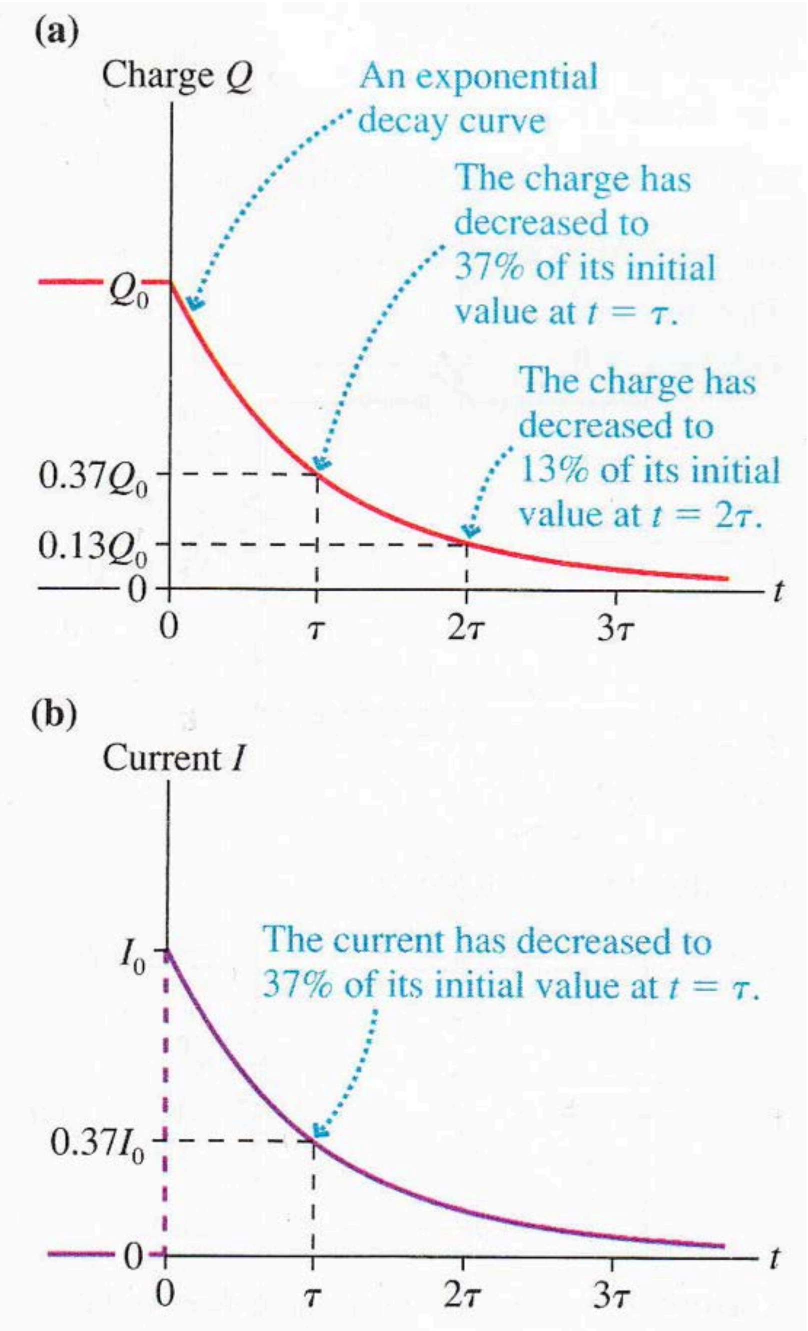
→ Resistor dissipates energy stored in the capacitor

Current through the capacitor?

$$I = \frac{dQ}{dt}$$

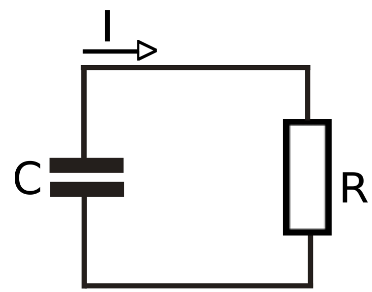
$$I_C = C\frac{dV_C}{dt}$$

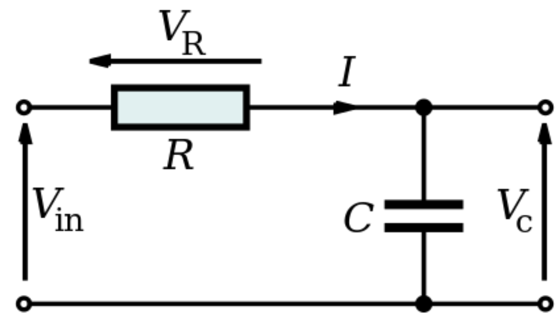
$$Q = C\,V_C$$



Knight

Review: RC Circuits





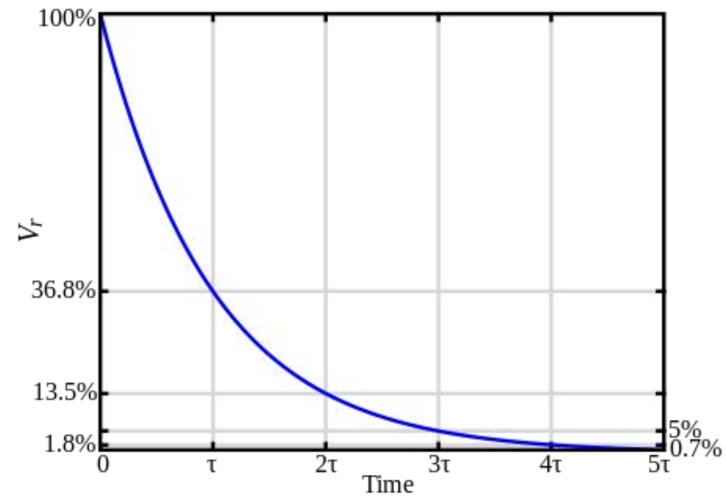
DC (some energy initially stored via charged capacitor) → KCL

$$C\frac{dV}{dt} + \frac{V}{R} = 0$$

$$V(t) = V_o e^{-\frac{t}{RC}}$$

$$\tau = RC$$

"RC time constant"



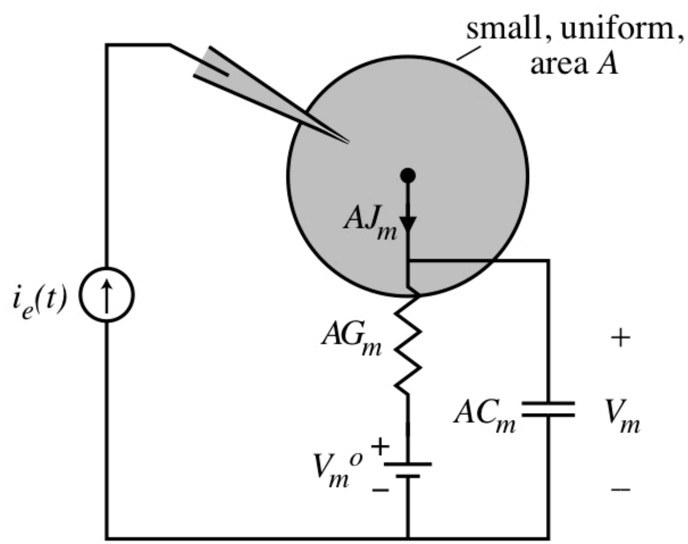
Complex impedance

AC (sinusoidally-driven at ω, steady-state) → KVL

$$Z = R - \frac{i}{\omega C}$$

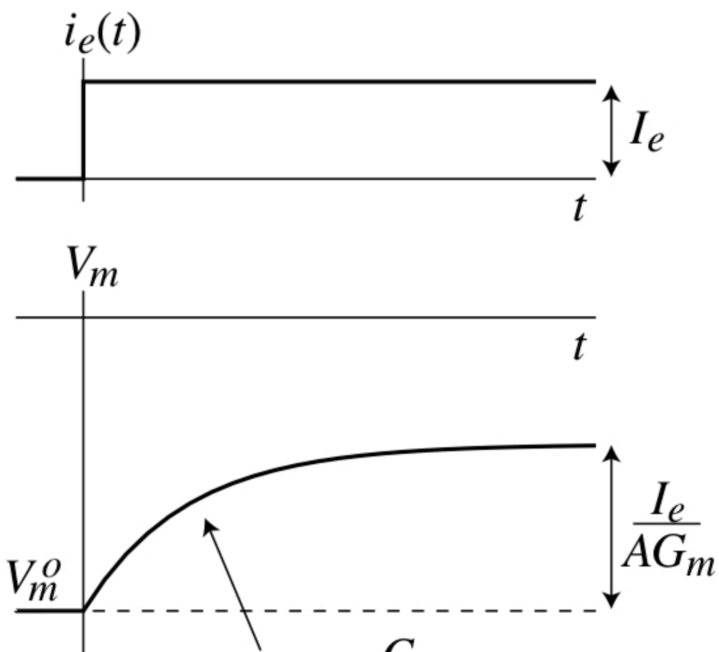
Think: RLC without the inductor

Wikipedia (RC circuit)



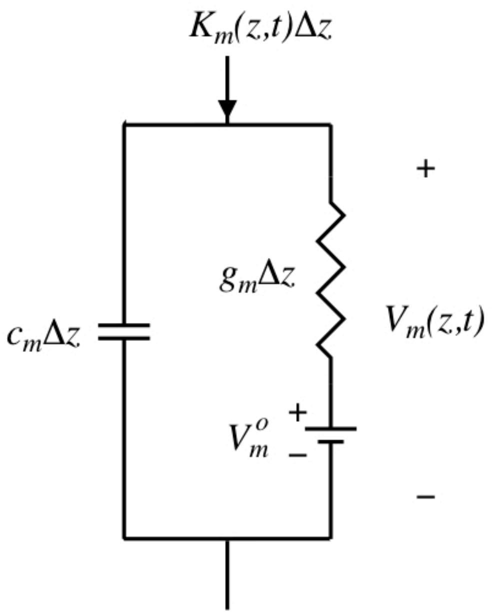
$$i_e(t) = AJ_m = AC_m \frac{dV_m}{dt} + AG_m(V_m - V_m^o)$$

$$\frac{AC_m}{AG_m}\frac{dV_m}{dt} + V_m = V_m^o + \frac{i_e(t)}{AG_m}$$



$$\tau_M = \frac{C_m}{G_m}$$ membrane time constant independent of cell size

see Weiss Figure 3.8



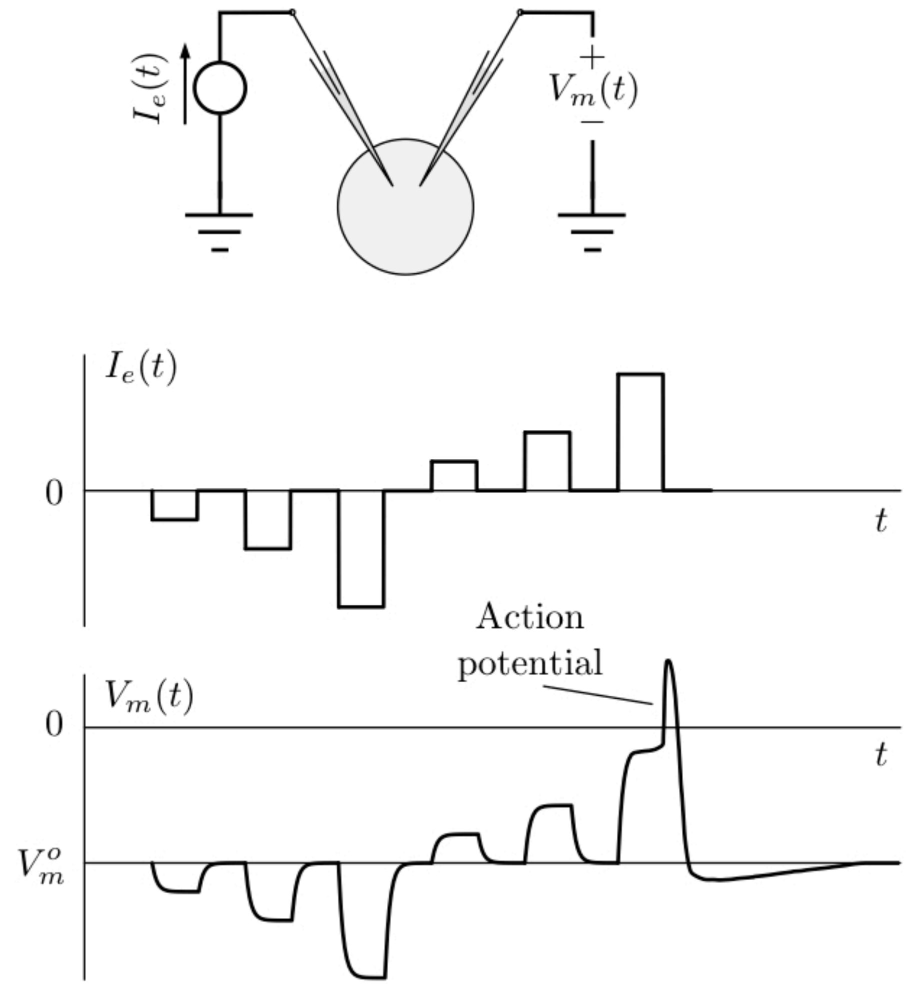


Figure 1.8

→ Delay explained

(but not action potentials yet!)

Cable Model - History

- First solved by William Thomson (aka Lord Kelvin) in ~1855

- Motivated by Atlantic submarine cable for intercontinental telegraphy

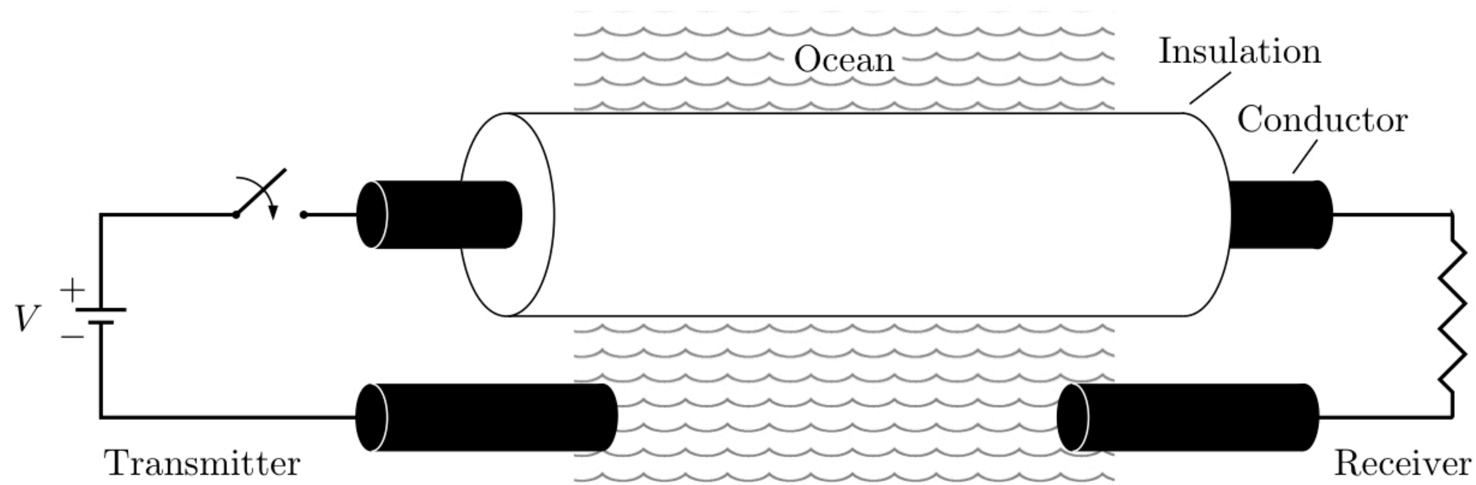


Figure 3.8

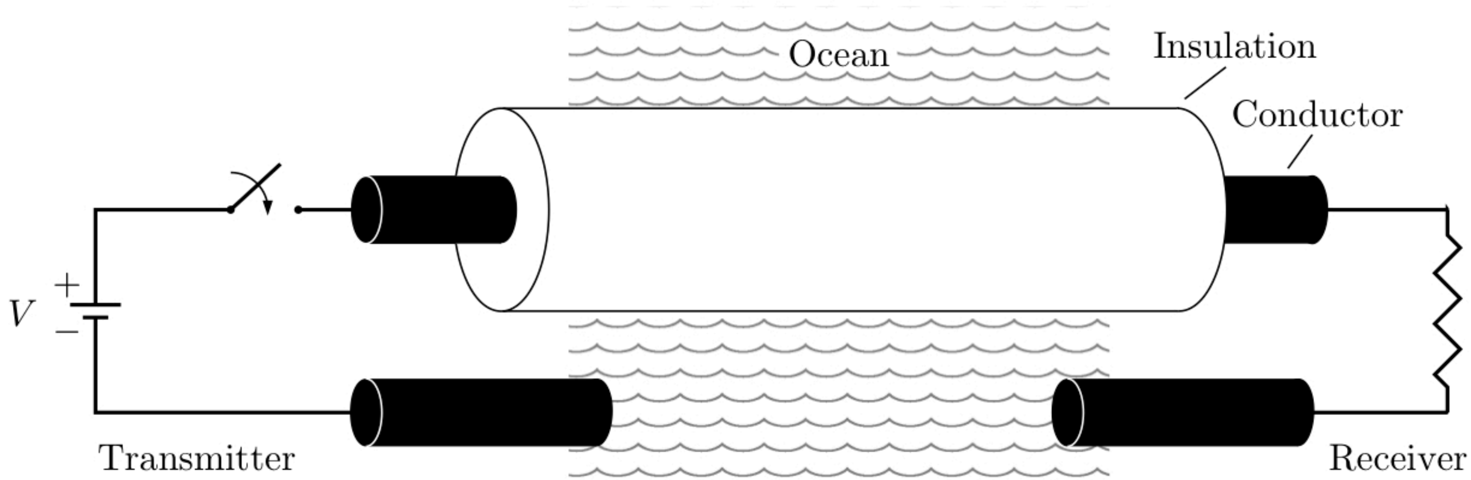


Figure 3.8

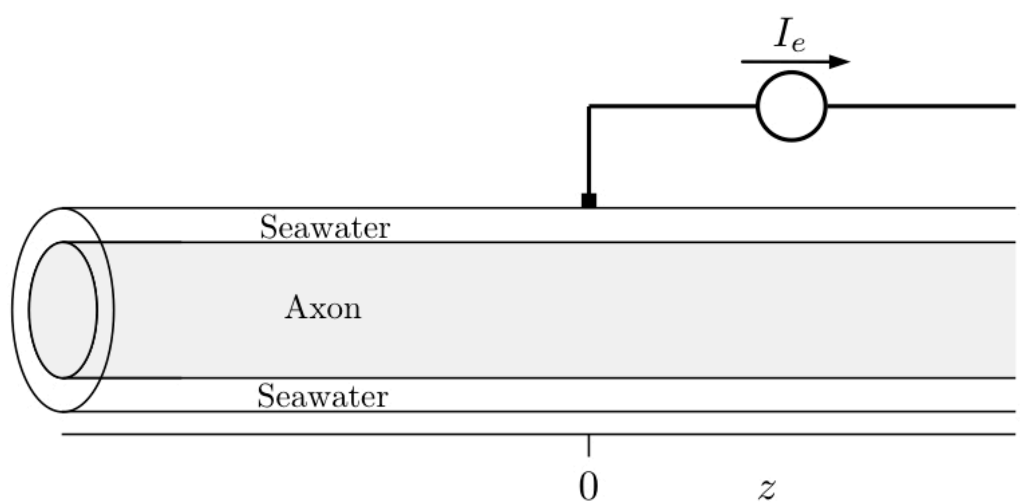


Figure 3.9

Cable Model - Overview

- Uses the Core Conductor model as underlying basis

- Assumes membrane that it can be described as a parallel capacitance and conductance

- Linear